APPLIED ARITHMETIC

THE THREE ESSENTIALS

BOOK 2

LENNES AND JENKINS

AP
T

The
gates
from
abide Happiness, Content and Comfort.

Based on Applied Arithmetic, they have a solid foundation on which to stand while forcing back the gates.

The theme of the border design includes the primitive counting method of the Indians, pictured in the knotted leather thongs. The artist has employed in the corners the devices of the ancient abacus and the modern type of mechanical calculator.

LENNES AND JENKINS

LIPPINCOTT'S SCHOOL TEXT SERIES

EDITED BY WILLIAM F. RUSSELL, PH.D.

DEAN, COLLEGE OF EDUCATION, STATE UNIVERSITY OF IOWA

APPLIED ARITHMETIC

THE THREE ESSENTIALS

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BOOK II

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PREFACE

THIS book is the second of a three-book series and is intended to cover the work in arithmetic of the fifth and sixth grades. The principles which have guided the authors may be grouped under three main heads:

1. *Selection and Organization of Subject Matter.* Recent discussion and practice, as revealed in the literature and in published curricula, seem to show substantial agreement as to what topics should be included in the earlier part of a course in Arithmetic, while there is yet considerable difference of opinion as to topics to be included in the later parts. All topics whose inclusion or exclusion is now being debated are placed among the supplementary topics at the end of books two and three. The main body of each book furnishes a minimum course which may be studied without break in continuity. At convenient points in the text supplementary topics may be taken up. It is believed that in this respect these books will serve each of many different needs just as effectively as if they were prepared to meet such need exclusively. The business of the maker of text books is to furnish teachers and supervising officers effective instruments for carrying out their purposes, rather than to seek to impose rigidly his own personal predilections.

It has been the purpose of the authors to arrange the subject matter in such manner that the greatest simplicity of treatment may be attained. In the second book this is exemplified by the consistent use of the principle of product and factors in the solution of problems, as shown on pages 24–29, 88, 89, 176, 177, 206–216. The problems considered on these pages constitute the most important applications of arithmetic within the scope of this book, and their solution is effected by means of the simple ideas developed in book one in connection with such simple combinations as $3 \times 4 = 12$ and $2 \times 3 \times 4 = 24$.

In book three this unifying and simplifying of the subject matter is carried out consistently.

2. *Derivation and Application.* The subject matter of Arithmetic may be divided into two parts:

(*a*) The four fundamental operations on integers and common and decimal fractions.

(*b*) The application of these operations to the solution of problems

In this book the fundamental operations are completed by a systematic study of common and decimal fractions. These are developed from simple objects well known to all children and from drawings which every child can make without trouble (see pages 34–38, 41, 42, 44, 45, 64, 78, 98, 112, 113). Few mistakes in teaching are more common or more pernicious than the subjection of the child to special and transient experiences for the sake of illustrative material. It has been the purpose of the authors to find the basis for arithmetic in the simple, obvious facts of the stream of life that flows about the child.

In the applications of arithmetic the only new element is the situation which gives rise to the problem. In percentage there is nothing new except the fact that *hundreths* or *per cents* are used very generally in practical life. In discount, the only new element is the fact that a reduction in price stated as a certain number of hundredths or per cents of the original price is often made. Similar remarks apply to commission, interest, profit and loss, taxes, insurance and other applications of percentage. A serious effort has been made to make this clear to the child and to put him in the way of obtaining solid information about the only new matter with which he is confronted as each topic is taken up. On this point see pages, 134, 135, 136, 160, 161, 178, 182, 186, 188, 220, 221, 276, 278, 280, 282, 285.

3. *Motivation.* The subject matter of arithmetic can be motivated most effectively only when the freest possible use is made of the child's many spontaneous interests. The authors believe they have not neglected any opportunity to interest the child in the

subject matter itself and its manifold applications. They have recognized, however, that it is possible so to connect the learning of arithmetic with other activities which in themselves are of compelling interest to the child, that the combination will be a source of joy and life when the arithmetic elements alone would lead to sadness and forced labor. For this reason systematic use has been made of games of group competition. On this point see the pages referred to in the index under the head, "Games used in drills."

The most effective applications of arithmetic can be made only when considerable local material is brought in. In the upper grades such material is also an important element in motivation. The child is becoming more and more interested in the activities of the grown-up. He is curious to know how things are really done, and if properly directed will gladly gather information from his own environment. For this reason many suggestions for bringing in local material are made throughout the book. See for example pages 20, 27, 69, 71, 93, 103, 104, 105, 107, 109, 136, 178, 187, 190, 194, 200.

Some other points may be mentioned. It is found on investigation that the fractions which are in common use are very simple. Denominators other than 2, 3, 4, 8, 12, 16, 32, occur so seldom as to be almost negligible. In fractions to be added, subtracted or divided they may be said not to occur at all. A problem like $\frac{3}{5}+\frac{2}{7}$ is as rare in practical life as the buffalo on our western prairies. For this reason much practice has been given on the manipulation of very simple fractions, while the more complicated fractions have been given less space. See pages 39, 40, 42, 43, 51, 52, 78, 96, 97, 98, 100, 154, 155.

As in book one, the two standard methods of subtraction have been given equal prominence. The addition method has been carried through consistently in fractions and in denominate numbers. See pages 54, 55, 247.

No effort has been spared to make the books attractive in

appearance and convenient in the arrangement of subject matter on the page.

The reason for producing this series lies, not in any one of the features mentioned here, but in the belief that by careful and systematic use of all that is best in present knowledge and usage it should be possible to produce a series of texts that would more adequately meet the requirement of the modern school than is done by any texts now in existence.

These books have been built leisurely. The first draft was made nearly ten years ago. During the intervening time the work of selecting what has proved most certainly valuable, both in what is old and what is new, and in organizing and relating the various parts, has been in constant progress. It is difficult, if not impossible to make proper acknowledgement to all who have helped in this work. The most prominent among these, however, have been Dr. Theodore Lindquist, Head of the Department of Mathematics in the State Normal School at Emporia, Kansas, Mr. H. C. Pearson, Principal of the Horace Mann School in New York City, and several of the Horace Mann teachers.

JULY 5, 1919. THE AUTHORS.

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APPLIED ARITHMETIC

THE THREE ESSENTIALS

BOOK II

CHAPTER I

1. The first part of this book consists of a review of what you have already learned about Arithmetic, together with many new applications which will show you more of the uses that people all about you are making of this important subject.

2. **Digits.** The numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, are called digits.

3. **Place Value.** In a number with more than five digits the digit in the 6th place represents hundreds of thousands, the digit in the 7th place represents millions, and so on, as indicated below.

billions	hundred millions	ten millions	millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones
5	3	9	7	4	8	2	6	8	4
Billions Period.		Millions Period.			Thousands Period.			Ones Period.	

For convenience in reading, the digits of large numbers are separated by commas into groups of 3 digits each. Thus, the above number is written 5,397,482,684. These groups are called *periods*.

In a whole number the digit in the right-hand place represents *ones*, the digit in the next place represents *tens*, and so on.

State what is represented by each digit in the number 5,397,482,684.

4. Reading Numbers. The number 8,604,896 is read *eight million, six hundred four thousand, eight hundred ninety-six.*

Notice that the zero is not read. In this number it signifies *no ten thousands.*

ORAL EXERCISES

Read the following:

1. In the year 1918 there were produced in the United States,
   918,920,000 bushels of wheat,
   3,159,494,000 bushels of corn,
   11,816,000 bales of cotton,
   634,594,000 tons of coal.

2. In the year 1910 the populations of the world's most important countries were:

United States *	91,972,267
British Empire	396,294,758
Germany	64,903,423
Russian Empire	160,095,200
France	38,961,945
Austria	49,418,596
Italy	32,475,253
Spain	19,503,008
China	439,214,000
Japan	53,875,390

WRITTEN EXERCISES

Write the following numbers, using figures:

3. One million, six hundred thirty-five thousand, three hundred ninety-eight.

4. Twelve million, one hundred twenty-eight thousand, six hundred twenty-one.

---

* Not including any foreign possessions.

5. One hundred eight million, thirty-seven thousand, five hundred nine.

6. Seventy-four million, eight hundred two thousand, seventy-six.

5. **Roman Numerals.** Roman numerals are formed by means of the seven letters:

Letters	I	V	X	L	C	D	M
Value	1	5	10	50	100	500	1000

1. Read the Roman numerals II, IV, VI, IX, XI, XII.

2. What is the difference between IV and VI? Between IX and XI?

Explain the system according to which the Roman numerals are formed.

3. Read the Roman numerals XXV, XL, IL, LIX, LXXIV, CCXXIX, MCLXXXVII, MCMXVI.

4. Write the following dates in Roman numerals: 1492, 1776, 1861, 1865, 1907, 1917.

6. **Addition, Addends, Sum.** The process of taking two numbers together to form one number is called *addition*. The numbers added are called *addends*. The result is called the *sum*.

The sign +, is the sign of addition, and is read *plus*.

Thus $9+6=15$ is read "9 plus 6 equals 15." In this combination 9 and 6 are the addends and 15 is the sum.

Make a list of all the addition combinations (45 in all) of two numbers, each below 10.

You should know each of these instantly and without having to think of any other combination.

Thus, for example, as soon as you hear the numbers 9 and 6 or see them you should know that their sum is 15. Unless you know these combinations thoroughly you will not make a good record in your number work this year.

Add:	
853	The sum of the first column is 25. Explain what you do with the 5 ones and the 2 tens. The sum of the second column, including the 2 tens *carried* is how many tens? Explain fully what you do with this number. How many hundreds are *carried* to the hundreds' column?
70	
66	
98	
438	

Test by adding each column both ways. In adding the first column say 3, 9, 17, 25. Do not say $3+6=9$, $9+8=17$, $17+8=25$.

In practice we do not even *think* that we carry so many *tens, hundreds* or *thousands*, but simply put down the digit to the right in each sum and carry the other.

WRITTEN EXERCISES

Add the following and keep a record of the time in which you do it.

1.	2.	3.	4.
938	7184	43742	99437
5873	76148	634134	549312
174	1972	9182	191563
96	30050	431876	10690
484	17436	85734	302715
8300	23456	91360	487659

5.	6.	7.	8.
2467	4984	28040	12134
841	617	1060	53143
84976	174836	467891	34567
87284	68475	59387	21098
29437	457893	86579	53174
59848	9748	165893	256

9.	10.	11.	12.
434	87065	97635	627
5678	126936	8976	29189
3219	90531	340698	577
46897	78964	69784	910
1071	1547	25652	8101
9140	13702	7814	25170

Below are the populations of the States of the Union as given by the census of 1910. First find the population of each group of States, and then find the total population.

New England States:

Maine	742,371
New Hampshire	430,572
Vermont	355,956
Massachusetts	3,366,416
Rhode Island	542,610
Connecticut	1,114,765

Middle Atlantic States:

New York	9,113,614
New Jersey	2,537,167
Pennsylvania	7,665,111

South Atlantic States:

Delaware	202,322
Maryland	1,295,346
District of Columbia	331,069
Virginia	2,061,612
West Virginia	1,221,119
North Carolina	2,306,287
South Carolina	1,515,400
Georgia	2,609,121
Florida	752,619

East Central States:

Ohio	4,767,121
Indiana	2,700,876
Illinois	5,638,591
Michigan	2,810,173
Wisconsin	2,333,860
Minnesota	2,075,708

South Central States:

Kentucky	2,289,905
Tennessee	2,184,789
Alabama	2,138,093
Mississippi	1,797,114
Louisiana	1,656,388
Texas	3,896,542
Arkansas	1,574,449
Oklahoma	1,657,155

West Central States:

Iowa	2,224,771
Missouri	3,293,335
North Dakota	577,056
South Dakota	583,888
Nebraska	1,192,214
Kansas	1,690,949

Mountain States:

Montana	376,053
Idaho	325,594
Wyoming	145,965
Colorado	799,024
Utah	373,351
Nevada	81,875
New Mexico	327,301
Arizona	204,354

Pacific Coast States:

California	2,377,549
Oregon	672,765
Washington	1,141,990

7. **The Decimal Number System.** The most important thing about our number system is that we count to *ten*, then to *twenty*, or two times ten, then to *thirty* or three times ten, and so on to *ten times ten* or *one hundred*, and so on. We thus see that the number *ten* is of special importance in our number system. For this reason it is called the *decimal number system*. The word decimal comes from the Latin word *decem*, meaning ten.

8. Subtraction. *Subtraction consists in taking one number away from another.*

The sign −, is the sign of subtraction, and is read *minus*.

Subtraction is also defined as the process of finding the difference between two numbers; or as finding how much must be added to one number to make the sum equal to another number.

Thus, to subtract 8 from 17 consists in taking 8 from 17, or in finding the difference between 8 and 17, or in finding how much must be added to 8 to make 17. That is, $17-8=9$ *because* $9+8=17$.

9. Minuend, Subtrahend, Remainder. The number before the minus sign is called the *minuend,* the number after the minus sign is called the *subtrahend,* and the result is called the *difference* or *remainder.*

ORAL EXERCISES.

In each of the following state what process must be used and why:

1. How much more is 92 than 34?
2. John and Henry together have 27 cents. How much has John if Henry has 16 cents?
3. John has 22 cents and Henry has 14 cents. How much more has John than Henry?
4. By how much does 25 exceed 16?
5. If the sum of the populations in the cities of St. Paul and Minneapolis is given, and also the population of St. Paul, how would you find the population of Minneapolis?
6. If you know how much a man has put into a bank and also how much he has left in the bank, how would you find how much he has drawn out?
7. If you know how much older a father is than his son, and also the age of the father, how do you find the son's age?

Example. From 3204 subtract 1846.

First method (adding or Austrian method): *

3204	*Ones.*	$14=6+8$	Write *8* Carry 1
1846	*Tens.*	$10=5+5$	Write *5* Carry 1
1358	*Hundreds.*	$12=9+3$	Write *3* Carry 1
	Thousands.	$3=2+1$	Write *1*

In each column we find what must be added to the subtrahend to make the sum equal to the minuend (increased, if necessary, by a 1 in the next higher place). A fuller explanation is given in Book III.

Second method (taking away method):

Since we can not take 6 from 4 we take 1 hundred from the 2 hundred and regard it as a $90+10$. The 10 is added to the 4. Then 6 from 14 leaves 8, 4 (tens) from 9 (tens) leaves 5 (tens).

Again, we can not take 8 (hundreds) from 1 (hundred). Hence, we take 1 (thousand) from the 3 (thousands) and add to the 1 (hundred). Then 8 from 11 leaves 3, and 1 from 2 leaves 1.

WRITTEN EXERCISES.

In this manner subtract the following:

1. 3198	2. 7490	3. 5514	4. 6542
1824	579	1678	1376
5. 1914	6. 4917	7. 7910	8. 9400
265	1728	2837	3782

9. How much must be added to 346 to make the sum 691?

10. The sum of two numbers is 8691. One of the numbers is 3649. Find the other number.

11. What is the difference between 24,791 and 8,691?

* Use the method which the child has already learned and pay no attention to the other method.

This man is depositing money in the bank. The bank keeps the money for him until he needs it, when he draws it out again. This man owns a store, and deposits his money each day.

WRITTEN EXERCISES.

1. Find how much money this man put in the bank in one week if he deposited \$186.50 on Monday, \$371.40 on Tuesday, \$249.30 on Wednesday, \$517.80 on Thursday, \$398.60 on Friday, and \$492.40 on Saturday.

Find the total deposits for each of these men if they made deposits as follows:

2. C. J. Hicks.

Monday	\$481.50	Tuesday	\$468.60
Wednesday	532.25	Thursday	425.30
Friday	416.80	Saturday	694.40

3. Gilbert A. Harris.

Monday	\$769.50	Tuesday	\$531.00
Wednesday	567.50	Thursday	604.75
Friday	512.25	Saturday	648.00

4. James B. Ashton.

Monday	\$476.00	Tuesday	\$321.60
Wednesday	367.90	Thursday	619.75
Friday	390.65	Saturday	405.40

5. John R. Walker.

Monday	\$1984.72	Tuesday	\$1580.75
Wednesday	1267.80	Thursday	1460.50
Friday	894.65	Saturday	2170.70

WRITTEN EXERCISES.

At the end of each month, or when asked to do so, the bank gives each depositor a list of all the money he has withdrawn since the last list was made.

A woman who paid the running expenses of her household with money drawn from the bank received the following lists of withdrawals:

January	February	March	April
$35.00	$35.00	$21.26	$35.00
12.55	5.40	35.00	17.50
8.70	11.76	8.53	8.30
7.60	5.00	5.00	2.60
5.00	15.00	4.23	4.70
4.60	4.54	2.64	24.60
10.00	2.31	4.69	9.10
20.00	3.86	10.00	7.40
13.40	10.00	2.64	14.75
11.60	15.00	62.25	7.13
5.00	3.45	7.80	9.40
2.00	5.35	4.60	12.30
4.00	4.40	21.30	7.90

WRITTEN EXERCISES.

1. How much did this woman withdraw from the bank altogether? (*Suggestion:* First find out how much she withdrew each month.)

2. If this woman deposited $250 in January, did she draw out more or less than she deposited that month? How much?

3. This woman deposited $185 in February, $210 in March, and $217.50 in April. Did she draw out more or less than she deposited in each of these months? How much for each month?

10. Multiplication, Multiplicand, Multiplier, Product. Multiplying a number by an integer gives the same result as adding the number to itself as often as indicated by the integer.

The number multiplied is called the *multiplicand.*

The number by which we multiply is called the *multiplier.*

The result obtained in multiplication is called the *product.*

The sign $\times$, is the sign of multiplication, and is read *times.*

Thus "$5\times15=75$" is read "5 times 15 equals 75."

In this example 15 is the *multiplicand,* 5 is the *multiplier,* and 75 is the *product.*

Example. Multiply 437 by 6.

437	(multiplicand)	Also find 6×437 by addition, and thus show
6	(multiplier)	that multiplication may be regarded as a
2622	(product)	short way of adding.

In practice the process of multiplying one number by another is based on the multiplication table. The number facts of this table must be known instantly and separately.

ORAL EXERCISES.

Read rapidly and supply the missing numbers in each of the following:

$8\times8=?$	$5\times9=?$	$10\times7=?$	$5\times7=?$
$7\times9=?$	$6\times5=?$	$2\times6=?$	$10\times4=?$
$6\times6=?$	$5\times8=?$	$10\times8=?$	$9\times2=?$
$4\times8=?$	$6\times7=?$	$3\times7=?$	$8\times9=?$
$4\times9=?$	$7\times7=?$	$9\times3=?$	$7\times8=?$
$6\times4=?$	$9\times9=?$	$10\times9=?$	$3\times6=?$
$10\times5=?$	$10\times6=?$	$7\times2=?$	$9\times6=?$
$2\times8=?$	$10\times4=?$	$4\times7=?$	$6\times8=?$

If any of the combinations of the multiplication table cause you to hesitate in the least, make a list of these and practice on them till you know them well.

11. **The Unit.** A single object or a group of objects taken together as one, and used for the purposes of counting or measuring is called a *unit.*

Thus, one book, one pencil, one desk, are units.

One dozen, or one herd of cattle, may also be regarded as units.

12. **Concrete and Abstract Numbers.** When a number refers to a definite kind of thing, the number is said to be *concrete.* Otherwise, it is said to be *abstract.*

Thus, 6 horses, 14 men, 10 dollars, 3 days, are concrete numbers, while 6, 14, 10, 3, are abstract numbers.

13. **The Multiplier Abstract.** We can multiply 8 dollars by 4, getting 32 dollars, but we cannot multiply 4 by 8 dollars.

That is, the multiplier must be an abstract number. If the multiplicand is a concrete number, the product will be a concrete number of the same kind as the multiplicand.

14. **The Method Used in Practice.** In actual practice, however, numbers are multiplied as if they were all abstract, the smaller number usually being used as the multiplier.

There is seldom any difficulty in telling what kind of number the product is.

Thus, to find the amount earned in 274 days at $4.00 a day, we multiply 274 by 4, and not $4 by 274. No practical business man would ever solve this problem by multiplying $4 by 274.

ORAL EXERCISES

1. Name several concrete and several abstract numbers.

2. A farmer bought 575 sheep at $9 a head. How do you find how much he paid for all the sheep?

3. How would you solve the following problem: If the average yield of corn in certain parts of Illinois is 58 bu. to the acre, what corn crop may be expected in a county where 149,800 acres are planted in corn?

Example 1. Multiply 4 by 6. Then multiply 4 by 3 and the product by 2. Compare the two products.

From this we see that to multiply a number by 3 and the product by 2 gives the same result as to multiply it by 2×3 or 6.

Example 2. Multiply 4 by 15. Also multiply 4 by 3 and the product by 5. Compare results.

In the same manner as in these examples we may multiply a number by 20 by multiplying it by 2 and the product by 10. To multiply a number by 700, we may multiply it by 7 and the product by 100.

ORAL EXERCISES

1. How do we multiply a number by 10? by 100? by 1000?
2. What is a convenient way of multiplying a number by 30? by 40? by 50? by 60? by 70? by 80? by 90?
3. What is a convenient way of multiplying a number by 200? by 300? by 400? by 500? by 600? by 800? by 900?
4. What is a convenient way of multiplying a number by 3000? by 8000? by 9000? by 4000?

WRITTEN EXERCISES

Find the products of the following:

1. $50\times165=?$	9. $800\times396=?$	17. $8000\times89=?$
2. $80\times642=?$	10. $4000\times297=?$	18. $7000\times298=?$
3. $500\times42=?$	11. $7000\times834=?$	19. $900\times574=?$
4. $200\times76=?$	12. $6000\times397=?$	20. $600\times853=?$
5. $40\times308=?$	13. $500\times219=?$	21. $3000\times947=?$
6. $700\times531=?$	14. $300\times807=?$	22. $4000\times576=?$
7. $400\times59=?$	15. $600\times598=?$	23. $700\times197=?$
8. $70\times197=?$	16. $700\times379=?$	24. $900\times643=?$

Example. 1. Multiply 6492 by 32.

6492	Multiply 6492 by 2, then by 30, and add the products.
32	How do we multiply by 30?
12984	Note, that when multiplying by 30, the first figure is written in tens' column and the zero is omitted. It is clear that this amounts to the same thing as to multiply by 30 and writing the first figure (zero) in ones' place.
19476	
207744	

Example. 2 Multiply 4502 by 207.

4502	Multiply 4502 by 7, then by 200, and add the products. Instead of multiplying by 2 and then by 100 by annexing 2 zeros we simply multiply by 2 and write the first figure in hundreds' place and omit the zeros.
207	
31514	
9004	
931914	

WRITTEN EXERCISES

1. Explain fully the multiplication of 489 by 648. Check your work by multiplying 648 by 489.

Multiply the following and check the work:

2. 876 × 154	3. 498 × 576	4. 398 × 680	5. 642 × 857
6. 297 × 806	7. 897 × 587	8. 408 × 796	9. 5987 × 846
10. 4934 × 902	11. 1398 × 789	12. 4572 × 597	13. 3984 × 276

14. Multiply $14.75 by 68.

(*Suggestion:* Multiply as if there were no decimal point, and place a decimal point in the product directly under the decimal point in the multiplicand.)

15. The area of the city of Chicago is about 190 square miles. How many acres is this? (One square mile = 640 acres.) How many farms averaging 100 acres would this make? If these farms had an average population of 8 persons to the farm, how many people would be living on this land?

John and Eleanor are helping their mother canning vegetables for the winter.

ORAL EXERCISES

1. At 4 cents a pound how much did 50 pounds of unshelled peas cost?

2. After shelling and drying, these peas filled 8 quart-jars. How much did the dried peas cost per quart?

3. At five cents a pound, how much did 120 pounds of string beans cost?

4. If these beans filled 40 quart-jars, how much did the canned beans cost per quart?

5. At 25 cents for 1 dozen bunches, how much did 8 dozen bunches of beets cost?

6. If these beets filled 8 quart-jars, how much did canned beets cost per quart?

7. At 20 cents per dozen ears, how much did 3 sacks of corn cost if each sack contained 15 dozen ears?

WRITTEN EXERCISES

1. Cherries may be bought for 55 cents a gallon when delivered to the house, and for 20 cents a gallon when bought on the trees. How much did John and Eleanor save by picking 14 gallons of cherries?

2. At 20 cents a gallon for cherries how much will it cost to can 30 quarts, if 3 cents' worth of sugar is used for each quart and if 35 cents' worth of gas is used for the 30 quarts? The value of the labor is not counted.

3. During the summer this family put down 45 dozen eggs at a total cost of 32 cents per dozen. If bought at the store during the winter, these eggs would cost on an average 48 cents a dozen. How much do the family save by putting down the eggs?

4. John's mother bought currants for 35 cents a gallon. If one gallon of currants made 12 glasses of jelly, how much did she pay for enough currants to make 60 glasses of jelly?

15. Division, Dividend, Divisor, Quotient. If the product of two numbers is given and one of the numbers, the process of finding the other number is called *division.*

The given product is called the *dividend.*

That one of the two numbers which is given is called the *divisor.* The one which is to be found is called the *quotient.*

Thus in $3\times ?=12$, the finding of the missing number is *division,* 12 is the *dividend,* 3 the *divisor,* and the missing number, 4, is the *quotient.*

It is important that this definition of division should be grasped thoroughly, as it will be a great help in solving practical problems.

ORAL EXERCISES

The finding of the missing numbers in each of the following is a problem in division. Give the dividend, the divisor and the quotient of each:

1.	$3\times ?=6$	$5\times ?=10$	$3\times ?=15$	$?\times 3=27$
2.	$7\times ?=21$	$2\times ?=16$	$?\times 4=20$	$5\times ?=35$

3. If the product of two numbers is known and also one of the numbers, how do you find the other?

Example. Divide 8786 by 6.

$$6\overline{)8786}$$
1464 remainder 2.

Explain this process as fully as you can. See if you can understand it better now than you did in the fourth grade.

WRITTEN EXERCISES

Divide:

1. $4\overline{)859}$	5. $9\overline{)2137}$	9. $7\overline{)3864}$	13. $7\overline{)4593}$
2. $7\overline{)9340}$	6. $5\overline{)1984}$	10. $7\overline{)4891}$	14. $8\overline{)3794}$
3. $8\overline{)9136}$	7. $4\overline{)18291}$	11. $5\overline{)2376}$	15. $9\overline{)5287}$
4. $6\overline{)1479}$	8. $3\overline{)2960}$	12. $8\overline{)6920}$	16. $6\overline{)4593}$

Example 1. Divide 71496 by 9, using short division. Then divide by using long division.

Compare these processes to see that they are alike, except that in long division more of the work is written down.

Example 2. Divide 9478 by 600.

$$\begin{array}{l} \ \ \ \ \ \ 15, \text{ remainder } 478. \\ 600\overline{)9478} \end{array}$$

Explain this process as fully as you can.

WRITTEN EXERCISES

Divide:

1. $80\overline{)3978}$ 2. $90\overline{)7842}$ 3. $70\overline{)6587}$ 4. $60\overline{)4987}$

5. $40\overline{)5730}$ 6. $800\overline{)39186}$ 7. $600\overline{)8749}$ 8. $900\overline{)53760}$

16. Estimating Quotients in Long Division. The most difficult step in long division is to find the quotient figures. The process is shown in the following:

Example 1. Divide 567 by 92.

$$\begin{array}{r} 6 \\ 92\overline{)567} \\ \underline{552} \\ 15 \end{array}$$

Taking quotient of tens, $56 \div 9 = 6$ with a remainder. On multiplying, 6 is found to be the correct quotient.

Example 2. Divide 784 by 88.

$$\begin{array}{r} 8 \\ 88\overline{)784} \\ \underline{704} \\ 80 \end{array}$$

Taking quotients of tens, $78 \div 9 = 8$ with a remainder. On multiplying, 8 is found the correct quotient.

ORAL AND WRITTEN EXERCISES

Estimate quotients in the following and make a list of your estimates. After having estimated all of them, test to see whether they are right.

1. $92\overline{)857}$ 2. $78\overline{)394}$ 3. $69\overline{)479}$ 4. $53\overline{)427}$

5. $83\overline{)674}$ 6. $74\overline{)498}$ 7. $86\overline{)792}$ 8. $79\overline{)687}$

16. Three-Figure Divisors. Estimating the quotient figures when dividing by three-figure divisors may be done by taking the *quotient of hundreds.* This is illustrated in the following:

Example 1. Divide 4865 by 718.

$$\begin{array}{r} 6 \\ 718\overline{)4865} \\ \underline{4308} \\ 557 \end{array}$$

Taking quotient of hundreds, 48÷7=6 with a remainder.

Is 6 the correct quotient? Why?

Example 2. Divide 7194 by 897.

$$\begin{array}{r} 7 \\ 897\overline{)7194} \\ \underline{6279} \\ 915 \end{array}$$

Taking quotient of hundreds, 71÷9=7 with a remainder.

On multiplying, 7 is found *not* to be the correct quotient. Explain.

ORAL EXERCISES

1. Explain "taking quotient of tens" (*a*) when the second figure in the divisor is small; (*b*) when the second figure in divisor is large.

2. Explain "taking quotient of hundreds" (*a*) when the second figure from the left in the divisor is small; (*b*) when this figure is large.

3. May the process in example 1 above give too small a quotient? Too large? Explain.

4. May the process in example 2 above give too small a quotient? Too large? Explain.

Estimate and make a list of quotients in the following:

5. 794 ÷ 84	6. 8247 ÷ 987	7. 4795 ÷ 826
8. 917 ÷ 346	9. 1024 ÷ 199	10. 5947 ÷ 643

WRITTEN EXERCISES

Test the correctness of the quotients just estimated.

17. **Dividing Numbers Representing Money.** The division of a number representing money by a concrete number is shown in the following examples:

Example 1. Divide \$786.40 by 16.

```
     $49.15
16 )$786.40
     64
     146
     144
       24
       16
       80
```

The division is carried out exactly as if there were no decimal point in the dividend. Then a decimal point is put into the quotient directly above the decimal point in the dividend.

Example 2. Divide \$1496.75 by 24.

```
    $62.36
24 )1496.75
    144
      56
      48
       87
       72
       153
       144
         9
```

In dividing we find a quotient of \$62.36 and a remainder of 9. Since 9 is less than one-half of 24 (the divisor), \$62.36 is the quotient correct to the nearest cent. If in this case we had a remainder of 12 or more (one-half the divisor) the quotient would be increased by one cent, that is, it would be \$62.37.

WRITTEN EXERCISES

1. Check example 1 above.
2. How do you check division when there is a remainder? Check example 2 above.

Divide and check each of the following:

3. 43839÷235	9. 3978÷349	15. 79456÷386
4. 59046÷418	10. 58741÷647	16. 87603÷830
5. 60807÷546	11. 94309÷875	17. 94006÷765
6. 59061÷794	12. 35046÷578	18. 98010÷410
7. 64083÷625	13. 37184÷806	19. 14968÷374
8. 43857÷541	14. 95384÷647	20. 97342÷826

WRITTEN EXERCISES

1. In a house 18 tons of soft coal are used each year for heating. At $4.65 a ton, what is the cost of this coal?

2. The same house could be heated with 11 tons of hard coal, costing $8.45 a ton. Which would be more expensive, hard coal or soft coal, and how much. (See Example 1.)

3. In the same house 3 cords of wood at $4.75 a cord and two loads of kindling at $3.25 a load are used in the fireplaces. What is the cost of this wood?

4. In this house the monthly bills for electric current are: $5.20, $6.10, $5.40, $5.10, $4.60, $3.25, $2.90, $3.70, $4.10, $4.80, $5.60, $5.40. What is the total yearly cost of lighting this house?

5. In the power station where the electric current is generated they burn 35 tons of coal a day. At $3.60 a ton what is the cost of this coal?

6. Gas for cooking is sold for 95 cents per thousand cubic feet. One month 5000 cubic feet are consumed. What is the gas bill for this month?

(*Suggestion:* Since 5000 cubic feet are used. multiply 95 by 5. This will give the result in cents.)

7. The monthly gas bills for the year are: $3.95, $3.75, $4.10, $4.70, $3.85, $4.10, $4.60, $5.80, $4.50, $3.70, $3.85, $4.15. What is the total gas bill for the year?

8. Find the cost of coal, gas, and electric current in your town, and make up problems on the cost of lighting and heating a house.

WRITTEN EXERCISES

1. John and Eleanor received 50 cents a week each from their mother for helping her during the summer vacation. How much did each of them receive in 14 weeks?

2. John bought a pair of skates for $2.10, a sweater for $2.85, a cap for 35 cents, and a pair of mittens for 50 cents. How much did these cost? How much did he have left of his $7.00?

3. Eleanor bought a pair of skates for $1.95, a sweater for $3.50, and a cap for 75 cents. How much did she have left of her $7.00?

4. John received a pair of skis for Christmas, and Eleanor received a new sled. The skis cost $3.45, and the sled cost $2.85. How much did they both cost?

5. John has an electric train which cost $4.25, a set of struct iron which cost $3.75, and other toys costing $2.60. How much do all these cost?

6. Eleanor has a set of play dishes and a stove costing $2.35, a doll's carriage costing $1.75, a set of doll's furniture costing $2.85, and a doll costing $1.15. How much do all these cost?

18. Even and Odd Numbers. The numbers 0, 2, 4, 6, 8, 10, 12, and so on, are called *even numbers*. The numbers 1, 3, 5, 7, 9, and so on, are called *odd numbers*.

19. Divisibility of Numbers. A number is divisible by another number if it can be divided by that number giving a whole number for a quotient and no remainder. All even numbers are divisible by 2.

Thus, $0 \div 2 = 0$, $2 \div 2 = 1$, $4 \div 2 = 2$, $6 \div 2 = 3$, and so on.

Odd numbers are not divisible by 2.

20. Tests of Divisibility. We will now give some tests of divisibility.

I. A number is divisible by 2 if its last figure is an even number, otherwise not.

II. A number is divisible by 5 if its last figure is 5 or 0. It is divisible by 10 if the last figure is 0.

III. A number is divisible by 3 if the sum of all its digits is divisible by 3.

Thus, 5493624 is divisible by 3 because the sum $5+4+9+3+6+2+4=33$ is divisible by 3.

36424 is not divisible by 3 because the sum $3+6+4+2+4=19$ is not divisible by 3.

IV. A number is divisible by 6 if it is divisible by both 2 and 3, otherwise not.

ORAL EXERCISES

1. Which of the numbers 346, 75, 23690, 3007, 39504, are divisible by 2?

2. Which of the numbers 845, 75, 360, 307, 950, 745, are divisible by 5? Which are divisible by 10?

3. For each of the numbers, 39, 78, 345, 5294, 70548, 9431, 89100, state whether it is divisible by 2, 3, 5, 6, 10.

WRITTEN EXERCISES

In cases of divisibility find the quotients in example 3 above.

(This page may be omitted if the teacher thinks best.)

21. **Multiplication and Division Compared.** In multiplication the multiplicand represents a group of things, and the multiplier states how many times this group is taken.

Multiplier × multiplicand = product.

A certain group taken so many times = product.

In division the product is always given. We also have given either the size of the groups, in which case we are required to find how many groups there are, or we have given the number of groups and are required to find the size of each group.

22. **Measuring and Grouping or Partitioning.** The following examples illustrate two kinds of division:

Example 1. At $3 a day, how many men can I pay for a day's work with $36?

The size of each group, $3, is given, and the question is as to the number of such groups in $36. We say we *measure* $36 by $3, and find that the measure is contained 12 times.

Example 2. If I have $36 to divide equally among 12 men, how much will each man get?

In this case the number of groups, 12, is given, and we are required to find the size of each group. The $36 are said to be divided or partitioned into 12 equal parts.

The process of division may therefore be regarded in some cases as a process of *measuring*, and in others as a process of *partitioning*. The work of carrying out the division is the same, however, in both cases.

If the size of the group is to be found, that is, in case of partitioning, the quotient represents the same kind of thing as the dividend. In case of measuring the quotient is abstract.

ORAL EXERCISES

Give examples in which division is to be regarded as measuring; others in which it is to be regarded as partitioning.

23. The Definition of Division and the Solution of Problems. In $3 \times 4 = 12$ we may leave out any one of the three numbers and thus obtain three distinct problems.

To find the missing number in $3 \times 4 = ?$ is a problem in multiplication.

To find the missing numbers in $3 \times ? = 12$, and $? \div 4 = 12$, are problems in division.

This simple idea forms the basis for the solution of a large number of problems.

If you know the length and the width of a rectangle how do you find its area?

The answer is that the area is found by multiplying the length by the width.

This may be written:

$$\textit{width} \times \textit{length} = \textit{area}.$$

If the area is not known we write:

$$\textit{width} \times \textit{length} = ?$$

We then have a problem in multiplication like $3 \times 4 = ?$

If the area and the width are known, but not the length, we write:

$$\textit{width} \times ? = \textit{area}.$$

To find the length is then a problem in division like finding the missing number in $4 \times ? = 12$.

If the area and the length are known, but not the width, we write:

$$? \times \textit{length} = \textit{area}.$$

To find the width is then a problem in division like finding the missing number in $? \times 3 = 12$.

You should make sure that you understand this page perfectly, since the same ideas will be used frequently hereafter. In the next few pages some of their simpler applications will be given.

It must be made clear that, before the rule *length* $\times$ *width* = *area* can be used, the length and width must be in the same units, and the unit of area must be a square of which the unit of length is a side.

WRITTEN EXERCISES

1. Find the area of a rectangle 60 feet wide and 80 feet long.

2. A rectangle is 67 feet long, and its area is 3685 square feet. Find the width.

3. A rectangle is 35 feet wide, and its area is 2240 square feet. Find its length.

4. A lot which contains 2072 square feet is 74 feet deep. How wide is it?

5. A field contains 3200 square rods. If the field is 40 rods wide, how long is it?

6. A maufacturer is building a factory in which he wants a floor space of 2400 square feet on.each floor. The width of his lot permits him to make the building just 30 feet wide inside. What must be the inside length of the building?

7. A man wants 8000 square feet of land for a factory site. How wide a piece of land must he buy if it is 125 feet deep?

8. A lot in the city of New York, 24 feet wide and 85 feet deep, sold for $1.20 per square foot. What was the selling price?

9. A field 120 rods long contains 30 acres. How wide is it?
(*Suggestion*: First find the number of square rods in 30 acres.)

10. At 75 cents a square yard, how much will it cost to cover a lot 60 feet by 120 feet with black dirt?
(*Suggestion*: First find the number of square yards in the lot.)

11. How long will it take a man to cut a field of grain 40 rods by 80 rods, if he cuts 8 acres a day?

A man bought 12 sheep at $9.00 a head. How much did they cost him?

If you know the number of things bought, and the price paid for each, how do you find the total cost? We may write this:

number of things bought $\times$ *price* = *total cost.*

Again, as in $3\times4=12$, we may leave out any one of the three numbers, thus obtaining three problems. That is,

(1) *number of things bought* $\times$ *price* = ?
(2) *number of things bought* $\times$? = *cost.*
(3) .... ? .... $\times$ *price* = *cost.*

ORAL EXERCISES

1. State each of these problems in words, and tell how to solve it. If you have difficulty in doing this, compare with the problems $3\times4=?$, $3\times?=12$, and $?\times4=12$.

2. How can you find the cost per acre of a piece of land, if you know the total cost and the number of acres?

3. How can you find the number of acres in a piece of land, if you know the total cost and the price per acre?

WRITTEN EXERCISES

Solve the following problems and state whether they come under (1), (2), or (3) of the forms given above:

1. A man bought 85 acres of land for $120 an acre. How much did the land cost him?

2. A man bought 175 head of sheep for $2450. How much did he pay per head for the sheep?

3. A man has $1170 with which to buy corn that is selling for 65 cents a bushel. How much corn can he buy?

(*Suggestion:* To find how many times 65 cents is contained in $1170, write the $1170 as 117000 cents and then divide.)

WRITTEN EXERCISES

1. At $56 per acre, how many acres can be bought for $7840?

2. At $90 per head, how many cows can be bought for $1080?

3. A man buys a flock of sheep for $450. What is the price per head if there are 75 sheep in the flock?

4. At $45 per acre, how much land can a farmer buy for $3825?

The total cost of a street railway system of 37 miles consists of the following items:

Track, $629,000; Power House, $148,000;
Overhead Trolley, $296,000; 21 cars at $2590.

5. What is the total cost of the 21 cars?

6. What is the cost per mile of the track? of the overhead trolley?

7. What is the total cost of this railway system, including all four items? What is the total cost per mile of this system?

8. A farmer buys a herd of 56 head of cattle for $3352, and later sells the herd for $4304. What is the increase in the price of the herd? What is the increase per head?

9. In a large city a lot 30 feet by 90 feet sells for $3,000. What is the price per square foot?

10. At 90 cents per square foot, how many square feet of land can be bought for $12,500?

11. At $175 per acre, what is the value of a farm 120 rods wide and 210 rods long?

12. At an auction sale a lady bought a bolt of sheeting containing 65 yards for $12.50. What was the cost per yard?

13. Make other problems like those given on this page, and solve them by means of the forms given on page 26.

If you walk 9 miles in 3 hours, how many miles per hour do you walk?

If a driving horse goes 28 miles in 4 hours, how many miles per hour does he go?

The distance covered in a unit of time is called *speed.* The distance and the time may be expressed in many different units. When we say we walk 3 *miles per hour* the unit of distance is the mile, and the unit of time is the hour. When we say a railway train goes a *mile a minute,* the unit of distance is the mile. A speed must always be given in certain units of time and distance.

ORAL EXERCISES

1. In each of the following, state in what units the speed is given: A rifle bullet goes 700 yards in a second; a passenger train goes 52 miles per hour; sound goes 1080 feet per second.

2. At an average speed of 20 miles an hour, how far will an automobile go in 4 hours?

3. If you know the speed and also the time, how do you find the distance covered? We write this:

$$speed \times time = distance.$$

As on the preceding pages, we may leave out any one of these three numbers, thus obtaining three problems:

(1) $speed \times time = ?$
(2) $speed \times ? = distance.$
(3) $? \times time = distance.$

State each of these problems in words, and tell how to solve it.

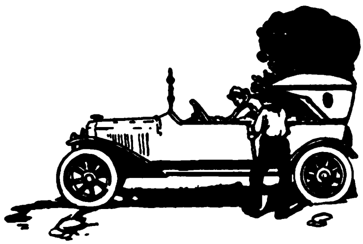
Notice that when you once know that

$$speed \times time = distance$$

you need no further rule for finding any one of these members when the other two are given.

WRITTEN EXERCISES

1. How long does it require a train going 45 miles an hour to go 270 miles?



2. What is the average speed of an auto that covers 360 miles in 20 hours?

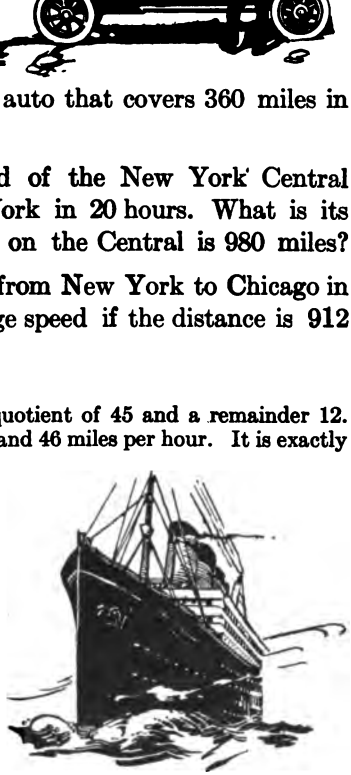
3. The Twentieth Century Limited of the New York Central goes from Chicago to New York in 20 hours. What is its average speed if the distance on the Central is 980 miles?

The Pennsylvania Limited goes from New York to Chicago in 20 hours. What is the average speed if the distance is 912 miles?

Solution:

$$\begin{array}{r} 45 \\ 20\,\overline{)\,912} \\ 80 \\ \hline 112 \\ 100 \\ \hline 12 \end{array}$$

Dividing 912 by 20 we get a quotient of 45 and a remainder 12. Hence, the speed is between 45 and 46 miles per hour. It is exactly $45\frac{3}{5}$ miles per hour.



5. In making the fastest journey ever made across the Atlantic Ocean, the steamship Mauretania steamed 2782 nautical miles in 107 hours. What was her average speed per hour?

The nautical mile is longer than the ordinary English mile, since the English mile = 5280 feet, while the nautical mile is nearly 6080 feet; 60 nautical miles make nearly 69 English miles.

24. Drills. Frequent drills are necessary in order to develop and retain reasonable speed and accuracy in performing the fundamental operations. Much interest is added to such drills by using games of competition. The games described here should be used as indicated later on.

1. *Girls vs. Boys.* Exercises are dictated by the teacher and copied by the pupils (or else given the pupils in mimeographed or printed form). At a given signal the pupils go to work, and then stop working at another signal. The pupils exchange papers and mark them as the teacher reads the correct answers. The number of correct answers obtained by the girls are added, and also those obtained by the boys. The side having the largest score wins. If the numbers of boys and girls differ, the average score of each side is taken.

2. *One Side of Room vs. the Other.* Play just as above except that one side of the room plays against the other side.

3. *Field Meet.* The room is divided into several teams, which play as in No. 1. Captains are selected first, and they choose the teams. There are several events—one for each of several successive days. The total scores for all the events of the meet are added. Thus, a meet may consist of four events, using addition, subtraction, multiplication and division, respectively. The same teams play in all the events of the meet.

4. *Cross-Country Race.* The class is divided into six or seven teams as in No. 3. When the first pupil finishes he holds up his hand and the teacher gives him the number 1, which he writes on his paper, and immediately turns it down on his desk. The pupil who finishes next is given the number 2 as his score, and so on. If two pupils hold up their hands at the same time, they are given the same score. When one-half or two-thirds of the class have finished the others stop working, and are all given the same score. The papers are marked by the pupils as in No. 1. If an example is not done at all, or if the answer is wrong, 5 is added to the score. If two are wrong 10 is added to the score, and so on. The team that gets the smallest total score wins.

The numbers on pages 32, 33, 46, 47 will be used for the games described on this page. The teacher will tell you just what numbers are to be used each time.

Even when trying very hard to work rapidly care must be taken to write the numbers reasonably well. If you get into the habit of writing numbers badly you will be sure to get mixed up and get wrong answers.

Add and check by adding each column both ways:

1.	2.	3.	4.	5.
67358	1642	1819	5416	5355
92100	18970	2797	3940	6113
84890	1748	10120	5764	1895
21924	9371	1914	5432	4974
33426	12463	6873	6789	7034
47597	7418	4422	1011	7149
55600	9214	4046	1315	2571
28230	17291	7970	3451	5118
64274	31000	9116	1718	3641
86450	10471	7474	8171	6618
91493	11495	2081	9456	1519
41765	12671	3841	5926	2406

Multiply and check by interchanging multiplier and multiplicand:

6. 378	7. 917	8. 356	9. 319	10. 246
265	827	497	874	396
11. 573	12. 719	13. 753	14. 413	15. 642
826	287	649	987	639
16. 677	17. 385	18. 347	19. 384	20. 496
852	638	595	981	926

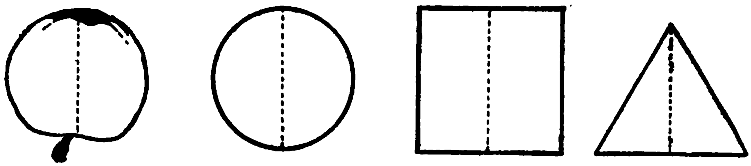
Divide and check by multiplying the quotient by the divisor and adding the remainder:

21. 43)71940	27. 541)9378	33. 820)94287
22. 284)374000	28. 97)47680	34. 9320)761400
23. 548)198700	29. 217)19476	35. 329)17604
24. 63)491700	30. 724)57647	36. 680)40670
25. 968)789100	31. 427)34678	37. 397)79340
26. 869)198700	32. 592)47626	38. 576)34970

		A	B	C	D	E
I.	1.	56,342	43,571	3492	315	26
	2.	37,802	29,634	2451	109	34
	3.	15,638	13,402	7631	410	45
	4.	78,281	56,491	1819	102	38
	5.	65,812	45,302	3402	560	25
II.	6.	34,291	32,615	4605	324	24
	7.	27,032	18,926	3204	116	39
	8.	35,604	39,324	8160	417	86
	9.	91,244	78,506	7251	352	59
	10.	63,007	58,029	3014	708	79
III.	11.	32,051	27,609	8407	342	63
	12.	69,107	54,346	1532	591	72
	13.	38,293	37,491	3040	682	17
	14.	62,812	47,394	2914	317	26
	15.	73,415	65,705	1857	827	83
IV.	16.	47,251	28,348	3756	326	87
	17.	75,432	29,256	2845	925	75
	18.	64,835	32,917	7156	273	65
	19.	65,374	44,512	4265	754	84
	20.	89,125	75,632	9463	934	67
V.	21.	35,637	33,981	7463	542	64
	22.	48,952	35,764	6173	384	89
	23.	78,334	56,473	5429	268	27
	24.	95,637	89,763	7638	712	38
	25.	84,732	84,648	5249	432	59
VI.	26.	59,481	51,627	2478	891	79
	27.	95,814	83,916	6071	673	82
	28.	89,184	34,670	3462	817	68
	29.	76,437	59,347	5432	231	53
	30.	67,347	71,408	7641	462	86

		A	B	C	D	E
VII.	31.	45,063	38,407	4536	238	57
	32.	89,603	72,635	7029	507	63
	33.	58,620	49,724	6385	920	84
	34.	72,806	35,209	1407	763	58
	35.	38,745	29,347	6819	647	78
VIII.	36.	58,674	38,579	5685	842	85
	37.	64,095	24,607	4328	567	79
	38.	85,306	77,845	9638	945	27
	39.	95,483	65,248	7536	847	48
	40.	67,538	64,729	6308	904	56
IX.	41.	95,632	49,745	8327	618	73
	42.	94,072	89,638	5429	527	45
	43.	35,608	33,849	6328	439	36
	44.	27,419	26,193	7437	508	40
	45.	38,516	27,498	6056	319	72
X.	46.	81,970	21,671	1982	598	92
	47.	39,181	39,842	3765	276	73
	48.	54,391	91,627	1917	782	86
	49.	24,217	84,207	7248	937	64
	50.	62,390	73,617	8294	876	27
XI.	51.	93,718	17,621	2891	895	29
	52.	27,938	24,893	5673	672	37
	53.	54,817	72,619	7191	287	68
	54.	61,902	70,248	8427	739	46
	55.	53,190	71,637	4928	678	72
XII.	56.	54,970	27,945	5416	342	31
	57.	21,929	92,912	2892	719	92
	58.	31,467	76,413	3767	243	87
	59.	84,891	19,848	6145	917	78
	60.	39,374	47,393	2982	826	29

25. Fractions of One Thing. Fractions are of frequent use in every-day life, and we are now to study them more fully than we have done before. We will first study fractions of one thing.

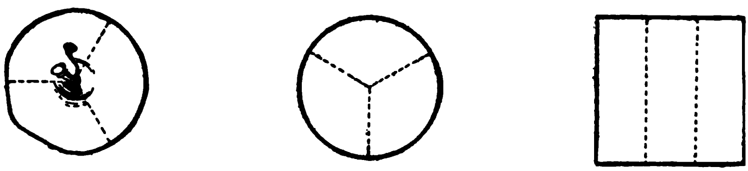


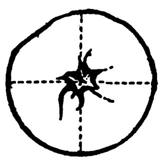
ORAL EXERCISES

1. If an apple is cut into two equal parts, what is each part called?

2. How many half circles are there in a whole circle?

3. If a square or a triangle is cut into two equal parts, what is each part called? How many halves are there in a whole?





4. If an apple is cut into three equal parts, what is each part called? How many thirds are there in a whole?

5. Point to $\frac{1}{3}$ of the circle, $\frac{1}{3}$ of the square, $\frac{2}{3}$ of the circle, $\frac{2}{3}$ of the square.

6. When a whole is divided into four equal parts, what is each part called?

7. Point to $\frac{1}{4}$ of the apple, $\frac{2}{4}$ of the apple.

8. Draw a circle and divide it into fourths. Point to $\frac{1}{2}$ of the circle, to $\frac{1}{4}$, to $\frac{2}{4}$, to $\frac{3}{4}$, to $\frac{4}{4}$.

26. Fractions of a Group of Things. To take a fraction of a group of things, the group must be divided into several equal groups. Then one or more of these groups must be taken.

ORAL EXERCISES

1. How many dots are there in this group? • •
   Point to ½ of them, to ¼ of them. • •

2. How many dots are there in this group? • • •
   Point to ½ of them, to ⅓ of them. • • •

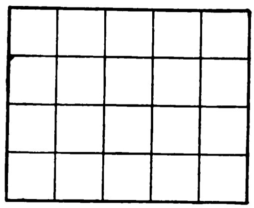
3. How many dots are there in this group? • • • •
   Point to ½ of them, to ¼ of them. • • • •

4. How many dots are there in this group? • • • • • • • •
   Point to ½ of them, ¼, ⅛, 1/16. • • • • • • • •

5. Point to ¾ of these dots, to ⅝, to ⅞.

6. Point to 3/16 of these dots, to 5/16, to 7/16, to 9/16, to 11/16, to 13/16, to 15/16.

7. Draw a rectangle like this. Show 1/20 of it, 3/20 of it, 4/20 of it. Also show ⅕ of the rectangle. How does ⅕ compare with 4/20?



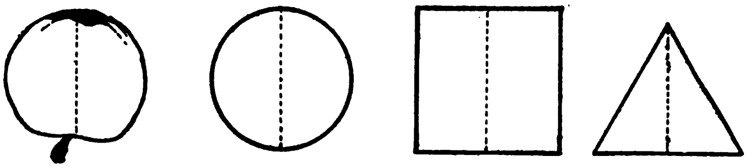
8. Cut out a paper 3″ x 3″ and fold it to show 1/9 of it. Also show 2/9 of it, 3/9. How does ⅓ compare with 3/9?

(3″ means 3 in., and 3″ x 3″ means 3 in. long and 3 in. wide.)

9. Cut out a paper 5″ x 2″ and fold it to show 1/10 of it. Also show 3/10 of it, and 5/10 of it. How does ½ compare with 5/10?

10. Cut out a paper 4″ x 3″ and fold it to show 1/12 of it. Also show ⅓ of it and ¼ of it. Compare ⅓ and 4/12. Also compare ¼ and 3/12.

25. Fractions of One Thing. Fractions are of frequent use in every-day life, and we are now to study them more fully than we have done before. We will first study fractions of one thing.

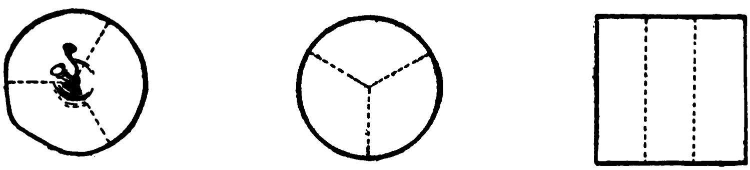


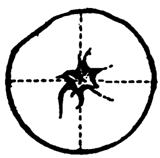
ORAL EXERCISES

1. If an apple is cut into two equal parts, what is each part called?

2. How many half circles are there in a whole circle?

3. If a square or a triangle is cut into two equal parts, what is each part called? How many halves are there in a whole?





4. If an apple is cut into three equal parts, what is each part called? How many thirds are there in a whole?

5. Point to $\frac{1}{3}$ of the circle, $\frac{1}{3}$ of the square, $\frac{2}{3}$ of the circle, $\frac{2}{3}$ of the square.

6. When a whole is divided into four equal parts, what is each part called?

7. Point to $\frac{1}{4}$ of the apple, $\frac{2}{4}$ of the apple.

8. Draw a circle and divide it into fourths. Point to $\frac{1}{2}$ of the circle, to $\frac{1}{4}$, to $\frac{2}{4}$, to $\frac{3}{4}$, to $\frac{4}{4}$.

26. **Fractions of a Group of Things.** To take a fraction of a group of things, the group must be divided into several equal groups. Then one or more of these groups must be taken.

ORAL EXERCISES

1. How many dots are there in this group? • •
Point to $\frac{1}{2}$ of them, to $\frac{1}{4}$ of them. • •

2. How many dots are there in this group? • • •
Point to $\frac{1}{2}$ of them, to $\frac{1}{3}$ of them. • • •

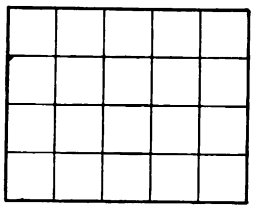
3. How many dots are there in this group? • • • •
Point to $\frac{1}{2}$ of them, to $\frac{1}{4}$ of them. • • • •

4. How many dots are there in this group? • • • • • • • •
Point to $\frac{1}{2}$ of them, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$. • • • • • • • •

5. Point to $\frac{3}{4}$ of these dots, to $\frac{5}{8}$, to $\frac{7}{8}$.

6. Point to $\frac{3}{16}$ of these dots, to $\frac{5}{16}$, to $\frac{7}{16}$, to $\frac{9}{16}$, to $\frac{11}{16}$, to $\frac{13}{16}$, to $\frac{15}{16}$.

7. Draw a rectangle like this. Show $\frac{1}{20}$ of it, $\frac{3}{20}$ of it, $\frac{4}{20}$ of it. Also show $\frac{1}{5}$ of the rectangle. How does $\frac{1}{5}$ compare with $\frac{4}{20}$?



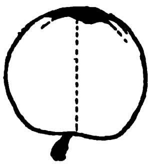
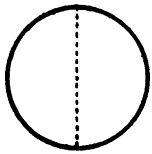
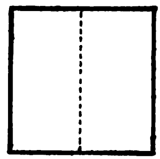
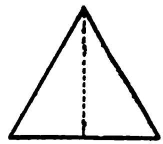
8. Cut out a paper 3″ x 3″ and fold it to show $\frac{1}{9}$ of it. Also show $\frac{2}{9}$ of it, $\frac{3}{9}$. How does $\frac{1}{3}$ compare with $\frac{3}{9}$?

(3″ means 3 in., and 3″ x 3″ means 3 in. long and 3 in. wide.)

9. Cut out a paper 5″ x 2″ and fold it to show $\frac{1}{10}$ of it. Also show $\frac{3}{10}$ of it, and $\frac{5}{10}$ of it. How does $\frac{1}{2}$ compare with $\frac{5}{10}$?

10. Cut out a paper 4″ x 3″ and fold it to show $\frac{1}{12}$ of it. Also show $\frac{1}{3}$ of it and $\frac{1}{4}$ of it. Compare $\frac{1}{3}$ and $\frac{4}{12}$. Also compare $\frac{1}{4}$ and $\frac{3}{12}$.

25. Fractions of One Thing. Fractions are of frequent use in every-day life, and we are now to study them more fully than we have done before. We will first study fractions of one thing.

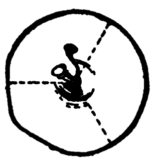
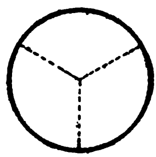
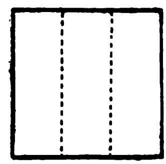
   

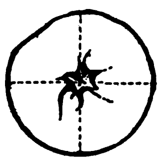
ORAL EXERCISES

1. If an apple is cut into two equal parts, what is each part called?

2. How many half circles are there in a whole circle?

3. If a square or a triangle is cut into two equal parts, what is each part called? How many halves are there in a whole?



4. If an apple is cut into three equal parts, what is each part called? How many thirds are there in a whole?

5. Point to $\frac{1}{3}$ of the circle, $\frac{1}{3}$ of the square, $\frac{2}{3}$ of the circle, $\frac{2}{3}$ of the square.

6. When a whole is divided into four equal parts, what is each part called?

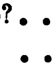
7. Point to $\frac{1}{4}$ of the apple, $\frac{2}{4}$ of the apple.

8. Draw a circle and divide it into fourths. Point to $\frac{1}{2}$ of the circle, to $\frac{1}{4}$, to $\frac{2}{4}$, to $\frac{3}{4}$, to $\frac{4}{4}$.

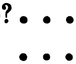
26. **Fractions of a Group of Things.** To take a fraction of a group of things, the group must be divided into several equal groups. Then one or more of these groups must be taken.

ORAL EXERCISES

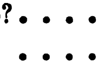
1. How many dots are there in this group? Point to $\frac{1}{2}$ of them, to $\frac{1}{4}$ of them.



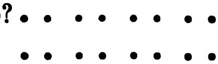
2. How many dots are there in this group? Point to $\frac{1}{2}$ of them, to $\frac{1}{3}$ of them.



3. How many dots are there in this group? Point to $\frac{1}{2}$ of them, to $\frac{1}{4}$ of them.



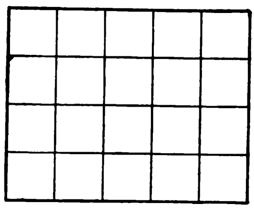
4. How many dots are there in this group? Point to $\frac{1}{2}$ of them, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$.



5. Point to $\frac{3}{4}$ of these dots, to $\frac{5}{8}$, to $\frac{7}{8}$.

6. Point to $\frac{3}{16}$ of these dots, to $\frac{5}{16}$, to $\frac{7}{16}$, to $\frac{9}{16}$, to $\frac{11}{16}$, to $\frac{13}{16}$, to $\frac{15}{16}$.

7. Draw a rectangle like this. Show $\frac{1}{20}$ of it, $\frac{3}{20}$ of it, $\frac{4}{20}$ of it. Also show $\frac{1}{5}$ of the rectangle. How does $\frac{1}{5}$ compare with $\frac{4}{20}$?



8. Cut out a paper 3″ x 3″ and fold it to show $\frac{1}{9}$ of it. Also show $\frac{2}{9}$ of it, $\frac{3}{9}$. How does $\frac{1}{3}$ compare with $\frac{3}{9}$?

(3″ means 3 in., and 3″ x 3″ means 3 in. long and 3 in. wide.)

9. Cut out a paper 5″ x 2″ and fold it to show $\frac{1}{10}$ of it. Also show $\frac{3}{10}$ of it, and $\frac{5}{10}$ of it. How does $\frac{1}{2}$ compare with $\frac{5}{10}$?

10. Cut out a paper 4″ x 3″ and fold it to show $\frac{1}{12}$ of it. Also show $\frac{1}{3}$ of it and $\frac{1}{4}$ of it. Compare $\frac{1}{3}$ and $\frac{4}{12}$. Also compare $\frac{1}{4}$ and $\frac{3}{12}$.

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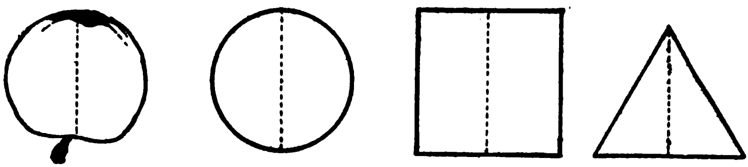

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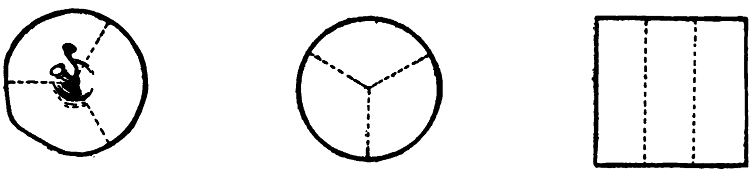
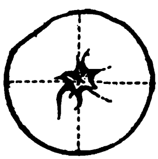
1. If an apple is cut into two equal parts, what is each part called?

2. How many half circles are there in a whole circle?

3. If a square or a triangle is cut into two equal parts, what is each part called? How many halves are there in a whole?





4. If an apple is cut into three equal parts, what is each part called? How many thirds are there in a whole?

5. Point to $\frac{1}{3}$ of the circle, $\frac{1}{3}$ of the square, $\frac{2}{3}$ of the circle, $\frac{2}{3}$ of the square.

6. When a whole is divided into four equal parts, what is each part called?

7. Point to $\frac{1}{4}$ of the apple, $\frac{2}{4}$ of the apple.

8. Draw a circle and divide it into fourths. Point to $\frac{1}{2}$ of the circle, to $\frac{1}{4}$, to $\frac{2}{4}$, to $\frac{3}{4}$, to $\frac{4}{4}$.

26. **Fractions of a Group of Things.** To take a fraction of a group of things, the group must be divided into several equal groups. Then one or more of these groups must be taken.

ORAL EXERCISES

1. How many dots are there in this group? • •
   Point to $\frac{1}{2}$ of them, to $\frac{1}{4}$ of them. • •

2. How many dots are there in this group? • • •
   Point to $\frac{1}{2}$ of them, to $\frac{1}{3}$ of them. • • •

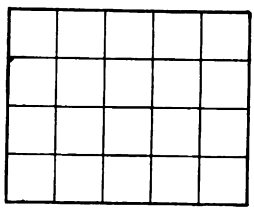
3. How many dots are there in this group? • • • •
   Point to $\frac{1}{2}$ of them, to $\frac{1}{4}$ of them. • • • •

4. How many dots are there in this group? • • • • • • • •
   Point to $\frac{1}{2}$ of them, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$. • • • • • • • •

5. Point to $\frac{3}{4}$ of these dots, to $\frac{5}{8}$, to $\frac{7}{8}$.

6. Point to $\frac{3}{16}$ of these dots, to $\frac{5}{16}$, to $\frac{7}{16}$, to $\frac{9}{16}$, to $\frac{11}{16}$, to $\frac{13}{16}$, to $\frac{15}{16}$.

7. Draw a rectangle like this. Show $\frac{1}{20}$ of it, $\frac{3}{20}$ of it, $\frac{4}{20}$ of it. Also show $\frac{1}{5}$ of the rectangle. How does $\frac{1}{5}$ compare with $\frac{4}{20}$?



8. Cut out a paper 3″ x 3″ and fold it to show $\frac{1}{9}$ of it. Also show $\frac{2}{9}$ of it, $\frac{3}{9}$. How does $\frac{1}{3}$ compare with $\frac{3}{9}$?

(3″ means 3 in., and 3″ x 3″ means 3 in. long and 3 in. wide.)

9. Cut out a paper 5″ x 2″ and fold it to show $\frac{1}{10}$ of it. Also show $\frac{3}{10}$ of it, and $\frac{5}{10}$ of it. How does $\frac{1}{2}$ compare with $\frac{5}{10}$?

10. Cut out a paper 4″ x 3″ and fold it to show $\frac{1}{12}$ of it. Also show $\frac{1}{3}$ of it and $\frac{1}{4}$ of it. Compare $\frac{1}{3}$ and $\frac{4}{12}$. Also compare $\frac{1}{4}$ and $\frac{3}{12}$.

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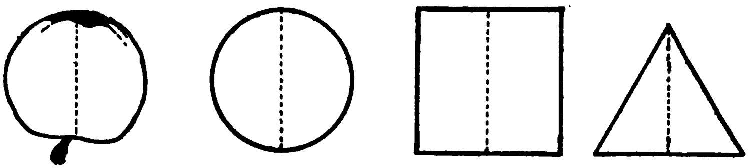
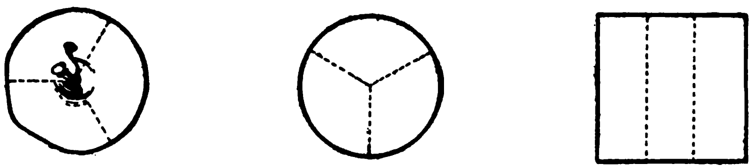


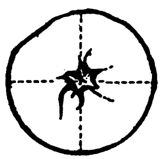
ORAL EXERCISES

1. If an apple is cut into two equal parts, what is each part called?

2. How many half circles are there in a whole circle?

3. If a square or a triangle is cut into two equal parts, what is each part called? How many halves are there in a whole?





4. If an apple is cut into three equal parts, what is each part called? How many thirds are there in a whole?

5. Point to $\frac{1}{3}$ of the circle, $\frac{1}{3}$ of the square, $\frac{2}{3}$ of the circle, $\frac{2}{3}$ of the square.

6. When a whole is divided into four equal parts, what is each part called?

7. Point to $\frac{1}{4}$ of the apple, $\frac{2}{4}$ of the apple.

8. Draw a circle and divide it into fourths. Point to $\frac{1}{2}$ of the circle, to $\frac{1}{4}$, to $\frac{2}{4}$, to $\frac{3}{4}$, to $\frac{4}{4}$.

26. **Fractions of a Group of Things.** To take a fraction of a group of things, the group must be divided into several equal groups. Then one or more of these groups must be taken.

ORAL EXERCISES

1. How many dots are there in this group? • •
Point to $\frac{1}{2}$ of them, to $\frac{1}{4}$ of them. • •

2. How many dots are there in this group? • • •
Point to $\frac{1}{2}$ of them, to $\frac{1}{3}$ of them. • • •

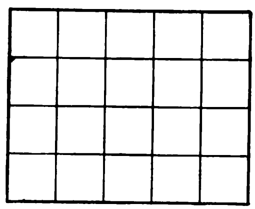
3. How many dots are there in this group? • • • •
Point to $\frac{1}{2}$ of them, to $\frac{1}{4}$ of them. • • • •

4. How many dots are there in this group? • • • • • • • •
Point to $\frac{1}{2}$ of them, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$. • • • • • • • •

5. Point to $\frac{3}{4}$ of these dots, to $\frac{5}{8}$, to $\frac{7}{8}$.

6. Point to $\frac{3}{16}$ of these dots, to $\frac{5}{16}$, to $\frac{7}{16}$, to $\frac{9}{16}$, to $\frac{11}{16}$, to $\frac{13}{16}$, to $\frac{15}{16}$.

7. Draw a rectangle like this. Show $\frac{1}{20}$ of it, $\frac{3}{20}$ of it, $\frac{4}{20}$ of it. Also show $\frac{1}{5}$ of the rectangle. How does $\frac{1}{5}$ compare with $\frac{4}{20}$?



8. Cut out a paper 3″ x 3″ and fold it to show $\frac{1}{9}$ of it. Also show $\frac{2}{9}$ of it, $\frac{3}{9}$. How does $\frac{1}{3}$ compare with $\frac{3}{9}$?

(3″ means 3 in., and 3″ x 3″ means 3 in. long and 3 in. wide.)

9. Cut out a paper 5″ x 2″ and fold it to show $\frac{1}{10}$ of it. Also show $\frac{3}{10}$ of it, and $\frac{5}{10}$ of it. How does $\frac{1}{2}$ compare with $\frac{5}{10}$?

10. Cut out a paper 4″ x 3″ and fold it to show $\frac{1}{12}$ of it. Also show $\frac{1}{3}$ of it and $\frac{1}{4}$ of it. Compare $\frac{1}{3}$ and $\frac{4}{12}$. Also compare $\frac{1}{4}$ and $\frac{3}{12}$.

27. Definitions. We are now able to understand the more formal statements given below. Such statements are called definitions.

A fraction represents one or more equal parts of anything.

In a fraction the number below the line is called the *denominator*, and *indicates into how many parts an object or group is divided.*

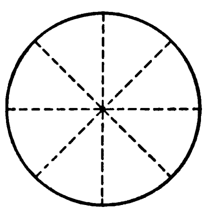
The number above the line is called the *numerator*, and *indicates how many of the equal parts are taken.*

Thus, the fraction $\frac{3}{8}$ indicates 3 of the 8 equal parts of one. It is read *three-eighths*, and also *three divided by eight.*

The numerator and denominator of a fraction are called the *terms of a fraction.*

28. Halves, Fourths, Eighths. Of all the fractions, halves, fourths, and eighths are in most frequent use.

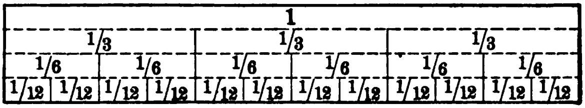
1							
½				½			
¼		¼		¼		¼	
⅛	⅛	⅛	⅛	⅛	⅛	⅛	⅛



ORAL EXERCISES

1. How many halves are there in 1? How many 4ths? How many 8ths?
2. In the above figure point out $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$. How many 8ths make a 4th? A half? How many 4ths make a half?
3. Show $\frac{1}{2}$ of the circle, $\frac{1}{4}$ of it, and $\frac{1}{8}$ of it.
4. How many 8ths are there in $\frac{2}{4}$? in $\frac{3}{4}$?
5. Read each fraction on this page in two ways.

29. **Thirds, Sixths, Twelfths.** From the figure we see that a third may be changed into sixths and twelfths.



ORAL EXERCISES

1. How many 3rds are there in 1? How many 6ths? How many 12ths?

2. In the figure point out $1/3$, $1/6$, and $1/12$. How many 6ths make a 3rd? How many 12ths make a 6th? How many 12ths make a 3rd?

3. Supply the missing numbers in the following: $\frac{1}{3}=\frac{?}{6}$, $\frac{1}{6}=\frac{?}{12}$, $\frac{1}{3}=\frac{?}{12}$.

4. In the figure point out $2/3$. How many 6ths are there in $2/3$?

5. How many 12ths are there in $2/3$?

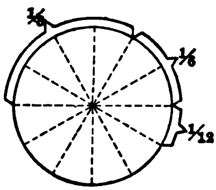
6. In the figure point out $2/6$, $3/6$, $4/6$, $5/6$.

7. How many 12ths are there in $5/6$?

8. Which is larger, $1/3$ or $1/6$? How many 6ths are needed to make one 3d?

9. Which is larger, $1/6$ or $1/12$? How many 12ths are needed to make one 6th?

10. Which is larger, $1/3$ or $1/12$? How many 12ths are needed to make one 3d?



11. In the circle point out $1/3$, $1/6$ and $1/12$.

12. Show $1/2$ of $1/3$. Also $1/2$ of $1/6$. How many 12ths are there in $\frac{1}{2}$ of $\frac{1}{3}$? How many 6ths? How many 12ths are there in $\frac{1}{2}$ of $\frac{1}{6}$?

Helen's mother sent her to the store to buy spices. She bought a 2-ounce can of paprica, a 4-ounce can of pepper, and an 8-ounce can of cinnamon.

ORAL EXERCISES

1. How many ounces are there in one pound? One ounce is what fraction of a pound?
2. How many 16ths of a pound are there in 2 ounces? In 4 ounces? In 8 ounces?
3. How many ounces are there in $\frac{1}{2}$ of a pound? $\frac{1}{2}$ is equal to how many 16ths?
4. How many 4-ounce cans will make a pound? $\frac{1}{4}$ of a pound is how many ounces? $\frac{1}{4}$ is equal to how many 16ths?
5. How many 2's are there in 16? How many 2-ounce cans will make a pound? $\frac{1}{8}$ of a pound is how many ounces? $\frac{1}{8}$ is equal to how many 16ths?
6. Supply the missing numbers in the following:
$\frac{1}{2}=\frac{?}{16}$, $\frac{1}{4}=\frac{?}{16}$, $\frac{1}{8}=\frac{?}{16}$.

7. One dozen is how many? One is what fraction fo a dozen?
8. How many in $\frac{1}{2}$ of a dozen? In $\frac{1}{3}$ of a dozen? In $\frac{1}{4}$ of a dozen? In $\frac{1}{6}$ of a dozen? In $\frac{1}{12}$ of a dozen.
9. Supply the missing numbers in the following:
$\frac{1}{2}=\frac{?}{12}$, $\frac{1}{3}=\frac{?}{12}$, $\frac{1}{4}=\frac{?}{12}$, $\frac{1}{6}=\frac{?}{12}$.
10. Read each of the following fractions in two ways:
$\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{3}$, $\frac{1}{6}$, $\frac{5}{6}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$.

30. **Multiplying Both Terms of a Fraction.** We have found that $\frac{1}{2}=\frac{2}{4}$, $\frac{1}{2}=\frac{4}{8}$, $\frac{1}{3}=\frac{2}{6}$, $\frac{1}{6}=\frac{2}{12}$, $\frac{1}{3}=\frac{4}{12}$, and so on.

In each of these the second fraction may be obtained by multiplying both terms of the first fraction by the same number.

That is, $\frac{1}{2}=\frac{2\times1}{2\times2}=\frac{2}{4}$, $\frac{1}{2}=\frac{4\times1}{4\times2}=\frac{4}{8}$, $\frac{1}{3}=\frac{4\times1}{4\times3}=\frac{4}{12}$

These examples illustrate the following rule:

> *Both terms of a fraction may be multiplied by the same number without changing the value of the fraction.*

ORAL EXERCISES

Supply the missing numbers:

1. $\frac{1}{3}=\frac{?}{9}$ $\frac{1}{4}=\frac{?}{8}$ $\frac{3}{5}=\frac{?}{15}$ $\frac{2}{3}=\frac{?}{12}$ $\frac{5}{6}=\frac{?}{18}$
2. $\frac{3}{7}=\frac{?}{21}$ $\frac{6}{7}=\frac{?}{28}$ $\frac{3}{8}=\frac{?}{24}$ $\frac{5}{9}=\frac{?}{18}$ $\frac{4}{9}=\frac{?}{27}$
3. $\frac{4}{5}=\frac{?}{25}$ $\frac{5}{8}=\frac{?}{56}$ $\frac{3}{11}=\frac{?}{44}$ $\frac{5}{12}=\frac{?}{60}$ $\frac{7}{8}=\frac{?}{80}$

31. **Dividing both Terms of a Fraction.** Since $\frac{1}{4}=\frac{3}{12}$ we can change $\frac{3}{12}$ to $\frac{1}{4}$. This is done by dividing both terms of $\frac{3}{12}$ by 3.

The following is a general rule:

> *Both terms of a fraction may be divided by the same number without changing the value of the fraction.*

ORAL EXERCISES

Read the following and supply the missing numbers:

1. $\frac{3}{9}=\frac{?}{3}$ $\frac{4}{10}=\frac{?}{5}$ $\frac{4}{12}=\frac{?}{3}$ $\frac{8}{16}=\frac{?}{2}$ $\frac{6}{18}=\frac{?}{3}$
2. $\frac{3}{12}=\frac{?}{4}$ $\frac{8}{12}=\frac{?}{3}$ $\frac{6}{12}=\frac{?}{6}$ $\frac{5}{20}=\frac{?}{4}$ $\frac{6}{18}=\frac{?}{9}$
3. $\frac{6}{12}=\frac{?}{4}$ $\frac{4}{12}=\frac{?}{3}$ $\frac{6}{12}=\frac{?}{2}$ $\frac{4}{18}=\frac{?}{9}$ $\frac{3}{21}=\frac{?}{7}$
4. $\frac{6}{10}=\frac{?}{5}$ $\frac{8}{10}=\frac{?}{5}$ $\frac{2}{10}=\frac{?}{5}$ $\frac{6}{16}=\frac{?}{8}$ $\frac{6}{30}=\frac{?}{10}$

32. A Factor of a Number. *A whole number which exactly divides another number is a factor of that number.*

Thus, 2 is a factor of 4, 6, 8, 10, and also of 2. 2 is not a factor of 1, 3, 5, 7, 9. Similarly, 3 is a factor of 3, 12, 15, 18, but not of 8, 14 or 17.

33. Common Factors. *Two numbers which have the same factor are said to have a common factor.*

Thus, 2 and 4 have the common factor of 2, and 6 and 9 have the common factor 3.

The numbers 8 and 9 have no common factor except 1. We usually disregard the factor 1, and say that these numbers have no common factor.

34. Fractions in Lowest Terms. *A fraction is in its lowest terms if the numerator and denominator have no common factor.*

The following rule is important:

> *A fraction may be reduced to its lowest terms by dividing the numerator and denominator by common factors.*

Fractions in a final result should be reduced to the lowest terms.

ORAL EXERCISES

Which of the following fractions are in their lowest terms? Reduce the others to lowest terms.

1. $\frac{3}{9}$	7. $\frac{3}{12}$	13. $\frac{3}{15}$	19. $\frac{7}{19}$	25. $\frac{8}{28}$
2. $\frac{7}{8}$	8. $\frac{5}{20}$	14. $\frac{4}{14}$	20. $\frac{8}{48}$	26. $\frac{8}{30}$
3. $\frac{6}{16}$	9. $\frac{4}{18}$	15. $\frac{2}{17}$	21. $\frac{9}{45}$	27. $\frac{8}{34}$
4. $\frac{4}{9}$	10. $\frac{4}{21}$	16. $\frac{4}{15}$	22. $\frac{9}{33}$	28. $\frac{12}{14}$
5. $\frac{2}{10}$	11. $\frac{7}{28}$	17. $\frac{8}{32}$	23. $\frac{8}{24}$	29. $\frac{12}{18}$
6. $\frac{4}{12}$	12. $\frac{5}{24}$	18. $\frac{6}{18}$	24. $\frac{8}{26}$	30. $\frac{12}{24}$

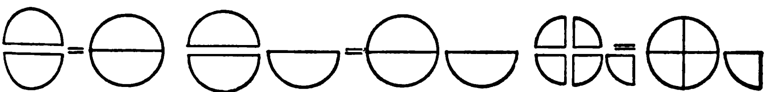
Drill in Fundamentals. Play game No. 4, page 30, using examples in multiplication.

35. **Proper and Improper Fractions.** A fraction which is less than 1 is called a *proper fraction,* while other fractions are called *improper fractions.*

Thus, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$ are proper fractions, while $\frac{2}{2}$, $\frac{4}{4}$, $\frac{3}{2}$, $\frac{6}{5}$, are improper fractions.

36. **Mixed Numbers.** A number consisting of a whole number and a fraction is called a *mixed number.*

Thus, $1\frac{1}{2}$, $2\frac{1}{3}$, $3\frac{5}{6}$, are mixed numbers.



$\frac{2}{2}=1$ $\frac{3}{2} = 1\frac{1}{2}$ $\frac{5}{4}=1\frac{1}{4}$

An improper fraction may always be reduced to an integer or to a mixed number, as in $\frac{2}{2}=1$, $\frac{4}{4}=1$, $\frac{3}{2}=1\frac{1}{2}$, $\frac{5}{4}=1\frac{1}{4}$.

A number is in the *simplest form* if it is an integer or if its fractional part is a proper fraction in its lowest terms.

Example 1. Reduce $\frac{72}{4}$.

$$4\overline{)72}\quad 18$$

Since $\frac{72}{4}$ means 72 divided by 4, we carry out the indicated division. The quotient is 18, with no remainder. Hence the result is 18.

Example 2. Reduce $\frac{64}{5}$.

$$5\overline{)64}\quad 12\frac{4}{5}$$

Dividing 64 by 5, the quotient is 12, and the remainder is 4. Dividing 4 by 5, we have $\frac{4}{5}$. Hence the result is $12\frac{4}{5}$.

ORAL EXERCISES

Which of the following are in the simplest form. Reduce the others to the simplest form:

1. $1\frac{2}{3}$	4. $\frac{16}{3}$	7. $\frac{12}{5}$	10. $\frac{45}{6}$	13. $3\frac{7}{21}$
2. $\frac{4}{8}$	5. $\frac{24}{7}$	8. $\frac{24}{5}$	11. $\frac{57}{9}$	14. $3\frac{9}{6}$
3. $\frac{8}{4}$	6. $\frac{26}{8}$	9. $\frac{34}{8}$	12. $1\frac{4}{12}$	15. $8\frac{9}{21}$

37. Fractions Having a Common Denominator. Fractions which have the same denominator are said to have a *common denominator*. Such fractions are also called *similar* or *like fractions*. Thus, $\frac{3}{8}$ and $\frac{5}{8}$ are similar, or like, while $\frac{3}{8}$ and $\frac{7}{12}$ are not similar or like.

38. Addition of Fractions Having a Common Denominator. In the manner shown in the diagram, any two fractions having the same denominator may be added.

$$\underbrace{1/8 \quad 1/8}_{2/8} \; + \; \underbrace{1/8 \quad 1/8 \quad 1/8}_{3/8} \quad 1/8 \quad 1/8 \quad 1/8 \qquad 2/8 + 3/8 = 5/8$$

ORAL EXERCISES

Add the following and reduce sums to simplest forms:

1.	$\frac{1}{2}+\frac{1}{2}$	$\frac{1}{3}+\frac{1}{3}$	$\frac{1}{4}+\frac{3}{4}$	$\frac{1}{5}+\frac{3}{5}$	$\frac{1}{6}+\frac{5}{6}$
2.	$\frac{3}{8}+\frac{5}{8}$	$\frac{1}{8}+\frac{5}{8}$	$\frac{1}{10}+\frac{1}{10}$	$\frac{2}{9}+\frac{4}{9}$	$\frac{5}{9}+\frac{1}{9}$
3.	$\frac{1}{16}+\frac{3}{16}$	$\frac{3}{16}+\frac{5}{16}$	$\frac{5}{16}+\frac{7}{16}$	$\frac{3}{16}+\frac{7}{16}$	$\frac{1}{16}+\frac{7}{16}$
4.	$\frac{5}{32}+\frac{3}{32}$	$\frac{5}{32}+\frac{7}{32}$	$\frac{7}{32}+\frac{9}{32}$	$\frac{5}{32}+\frac{11}{32}$	$\frac{9}{32}+\frac{11}{32}$

39. Addition of Fractions Not Having a Common Denominator. To add the fractions $\frac{1}{2}$ and $\frac{1}{4}$, the $\frac{1}{2}$ must be reduced to 4ths. Thus, $\frac{1}{2}+\frac{1}{4}=\frac{2}{4}+\frac{1}{4}=\frac{3}{4}$.

Again, to add $\frac{1}{2}$ and $\frac{1}{6}$, the $\frac{1}{2}$ must be reduced to 6ths. Thus, $\frac{1}{2}+\frac{1}{6}=\frac{3}{6}+\frac{1}{6}=\frac{4}{6}=\frac{2}{3}$.

ORAL EXERCISES

Add the following and reduce sums to simplest forms:

1.	$\frac{1}{2}+\frac{1}{4}$	$\frac{1}{2}+\frac{1}{8}$	$\frac{1}{2}+\frac{3}{8}$	$\frac{1}{4}+\frac{1}{8}$	$\frac{1}{4}+\frac{3}{8}$
2.	$\frac{1}{8}+\frac{1}{16}$	$\frac{1}{8}+\frac{3}{16}$	$\frac{1}{8}+\frac{5}{16}$	$\frac{1}{8}+\frac{7}{16}$	$\frac{3}{8}+\frac{1}{16}$
3.	$\frac{7}{8}+\frac{1}{16}$	$\frac{3}{8}+\frac{3}{16}$	$\frac{3}{8}+\frac{5}{16}$	$\frac{3}{8}+\frac{7}{16}$	$\frac{5}{8}+\frac{5}{16}$
4.	$\frac{2}{3}+\frac{1}{6}$	$\frac{1}{3}+\frac{1}{9}$	$\frac{2}{3}+\frac{1}{9}$	$\frac{1}{3}+\frac{2}{9}$	$\frac{2}{3}+\frac{2}{9}$

Example. Add $\frac{1}{2}$ and $\frac{2}{3}$.

$$\frac{1}{2}=\frac{3}{6}$$
$$\frac{2}{3}=\frac{4}{6}$$
$$\frac{7}{6}=1\frac{1}{6}$$

We see at once that $\frac{1}{2}$ and $\frac{2}{3}$ may be reduced to 6ths. Multiply both terms of $\frac{1}{2}$ by 3, and both terms of $\frac{2}{3}$ by 2. Finally, add $\frac{3}{6}$ and $\frac{4}{6}$.

ORAL EXERCISES

Add and reduce each sum to the simplest form:

1. $\frac{1}{2}+\frac{1}{3}$	$\frac{1}{4}+\frac{1}{3}$	$\frac{3}{4}+\frac{1}{3}$	$\frac{3}{4}+\frac{2}{3}$	$\frac{1}{4}+\frac{1}{6}$
2. $\frac{1}{3}+\frac{1}{5}$	$\frac{2}{3}+\frac{3}{5}$	$\frac{2}{3}+\frac{4}{5}$	$\frac{1}{6}+\frac{1}{8}$	$\frac{5}{6}+\frac{1}{8}$
3. $\frac{1}{2}+\frac{1}{5}$	$\frac{1}{2}+\frac{3}{5}$	$\frac{1}{2}+\frac{1}{7}$	$\frac{1}{2}+\frac{3}{7}$	$\frac{1}{2}+\frac{6}{7}$
4. $\frac{2}{3}+\frac{1}{8}$	$\frac{1}{3}+\frac{3}{8}$	$\frac{1}{3}+\frac{5}{8}$	$\frac{1}{6}+\frac{1}{9}$	$\frac{5}{6}+\frac{1}{9}$

Example. Add $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$.

$$\frac{1}{2}=\frac{6}{12}$$
$$\frac{1}{3}=\frac{4}{12}$$
$$\frac{1}{4}=\frac{3}{12}$$
$$\frac{13}{12}=1\frac{1}{12}$$

We see at once that $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ may all be reduced to 12ths. We multiply both terms of $\frac{1}{2}$ by 6, both terms of $\frac{1}{3}$ by 4, and both terms of $\frac{1}{4}$ by 3. We know that we must multiply both terms of $\frac{1}{2}$ by 6 because $6\times2=12$.

WRITTEN EXERCISES

Add and reduce each sum to the simplest form:

1. $\frac{1}{2}+\frac{1}{3}+\frac{3}{4}$	7. $\frac{1}{3}+\frac{1}{4}+\frac{1}{6}$	13. $\frac{1}{4}+\frac{1}{8}+\frac{1}{16}$
2. $\frac{1}{2}+\frac{2}{3}+\frac{1}{4}$	8. $\frac{2}{3}+\frac{1}{4}+\frac{5}{6}$	14. $\frac{1}{4}+\frac{3}{4}+\frac{3}{16}$
3. $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}$	9. $\frac{1}{3}+\frac{3}{4}+\frac{1}{6}$	15. $\frac{1}{4}+\frac{5}{8}+\frac{5}{16}$
4. $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$	10. $\frac{2}{3}+\frac{3}{4}+\frac{5}{6}$	16. $\frac{3}{4}+\frac{7}{8}+\frac{1}{16}$
5. $\frac{1}{2}+\frac{1}{4}+\frac{3}{8}$	11. $\frac{2}{3}+\frac{3}{4}+\frac{5}{6}$	17. $\frac{3}{4}+\frac{5}{8}+\frac{7}{16}$
6. $\frac{1}{2}+\frac{3}{4}+\frac{1}{8}$	12. $\frac{1}{3}+\frac{1}{4}+\frac{5}{6}$	18. $\frac{3}{4}+\frac{3}{8}+\frac{9}{16}$

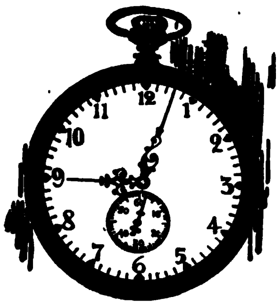
19. A board $\frac{5}{8}$ inches thick is covered with a veneer $\frac{3}{16}$ inches thick. How thick is the veneered board?

ORAL EXERCISES

1. Tell how many inches there are in each of the following:

$\frac{1}{2}$ of one foot
$\frac{1}{3}$ of one foot
$\frac{2}{3}$ of one foot
$\frac{1}{4}$ of one foot
$\frac{3}{4}$ of one foot

$\frac{1}{6}$ of one foot
$\frac{5}{6}$ of one foot
$\frac{1}{12}$ of one foot
$\frac{5}{12}$ of one foot
$\frac{7}{12}$ of one foot



2. Tell how many minutes there are in each of the following:

$\frac{1}{2}$ of one hour
$\frac{1}{3}$ of one hour
$\frac{2}{3}$ of one hour
$\frac{1}{4}$ of one hour

$\frac{1}{5}$ of one hour
$\frac{2}{5}$ of one hour
$\frac{3}{5}$ of one hour
$\frac{4}{5}$ of one hour

3. Tell how many hours there are in each of the following, 24 hours being one day:

$\frac{1}{2}$ of one day
$\frac{1}{3}$ of one day
$\frac{2}{3}$ of one day

$\frac{1}{4}$ of one day
$\frac{3}{4}$ of one day
$\frac{1}{6}$ of one day

4. If 52 weeks are regarded as one year, tell how many weeks there are in each of the following:

$\frac{1}{2}$ of one year
$\frac{1}{4}$ of one year

$\frac{3}{4}$ of one year
$\frac{1}{13}$ of one year

5. If a school year is 40 weeks, tell how many weeks there are in each of the following:

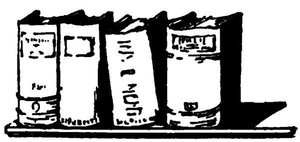
$\frac{1}{2}$ of a school year
$\frac{1}{4}$ of a school year
$\frac{3}{4}$ of a school year

$\frac{1}{5}$ of a school year
$\frac{2}{5}$ of a school year
$\frac{3}{5}$ of a school year

Drill in Fundamentals. Run a cross-country race. (See page 30.) Use examples in long division.

PROBLEMS

1. Three boards are laid on top of one another. How thick are they altogether if the boards are $\frac{3}{4}$, $\frac{7}{8}$, and $\frac{3}{16}$ of an inch thick respectively?



2. Four books stand together on a shelf. They are $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{2}$, and $\frac{7}{8}$ of an inch thick respectively. How wide a space do they occupy on the shelf?

3. Add $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{5}$ of a dollar. Then reduce each of these fractions of a dollar to cents and add. Compare the results.

4. Add $\frac{1}{4}$, $\frac{13}{16}$, and $\frac{5}{8}$ of a pound. Then reduce each of these fractions of a pound to ounces and add. Compare the results.

5. Add $\frac{5}{6}$, $\frac{3}{4}$ and $\frac{2}{3}$ of a foot. Then reduce each of these fractions of a foot to inches and add. Compare the results.

6. Add $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{8}$ of a peck. Then reduce each of these fractions of a peck to quarts and add. Compare the results.

7. Add $\frac{3}{4}$, $\frac{7}{16}$ and $\frac{11}{32}$ of a bushel. Then reduce each of these fractions of a bushel to quarts and add. Compare the results.

8. Add $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{5}{6}$ of an hour. Then reduce each of these fractions of an hour to minutes and add. Compare the results.

9. In a truck garden $\frac{3}{8}$ of an acre is in corn, $\frac{1}{4}$ of an acre in beets, and $\frac{1}{2}$ of an acre in beans. How much land is used for all these?

Look over the problems on this page again to see how many you can do without using pencil and paper.

		A	B	C	D	E	F	G	K
I.	1.	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{8}$	2
	2.	$\frac{5}{6}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{5}{8}$	$\frac{7}{16}$	3
	3.	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{5}{12}$	$\frac{1}{36}$	$\frac{7}{8}$	$\frac{5}{16}$	6
	4.	$\frac{15}{16}$	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{7}{12}$	$\frac{1}{9}$	$\frac{5}{16}$	$\frac{7}{32}$	8
	5.	$\frac{11}{12}$	$\frac{5}{6}$	$\frac{2}{3}$	$\frac{5}{8}$	$\frac{1}{16}$	$\frac{27}{32}$	$\frac{2}{9}$	12
II.	6.	$\frac{14}{15}$	$\frac{4}{5}$	$\frac{7}{10}$	$\frac{3}{5}$	$\frac{1}{20}$	$\frac{9}{10}$	$\frac{3}{10}$	15
	7.	$\frac{11}{18}$	$\frac{5}{9}$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{1}{9}$	$\frac{1}{8}$	$\frac{5}{12}$	18
	8.	$\frac{13}{16}$	$\frac{3}{4}$	$\frac{7}{10}$	$\frac{2}{3}$	$\frac{1}{12}$	$\frac{3}{16}$	$\frac{9}{16}$	8
	9.	$\frac{25}{32}$	$\frac{13}{16}$	$\frac{3}{5}$	$\frac{7}{10}$	$\frac{1}{2}$	$\frac{4}{5}$	$\frac{14}{35}$	6
	10.	$\frac{5}{6}$	$\frac{2}{3}$	$\frac{5}{8}$	$\frac{9}{16}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{15}{32}$	12
III.	11.	$4\frac{5}{6}$	$5\frac{3}{8}$	$10\frac{1}{5}$	$5\frac{1}{4}$	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{7}{8}$	6
	12.	$3\frac{5}{8}$	$4\frac{2}{3}$	$8\frac{2}{3}$	$3\frac{2}{3}$	$\frac{5}{6}$	$\frac{5}{8}$	$\frac{3}{4}$	4
	13.	$4\frac{3}{4}$	$6\frac{3}{4}$	$4\frac{3}{5}$	$6\frac{1}{3}$	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{3}{5}$	5
	14.	$7\frac{2}{3}$	$2\frac{1}{2}$	$6\frac{7}{8}$	$5\frac{3}{8}$	$\frac{4}{5}$	$\frac{3}{8}$	$\frac{7}{12}$	3
	15.	$8\frac{7}{8}$	$5\frac{1}{6}$	$12\frac{1}{2}$	$4\frac{7}{8}$	$\frac{1}{2}$	$\frac{5}{12}$	$\frac{11}{20}$	2
IV.	16.	$12\frac{2}{3}$	$18\frac{2}{3}$	$6\frac{1}{2}$	$5\frac{1}{8}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{5}{16}$	8
	17.	$8\frac{3}{4}$	$24\frac{1}{8}$	$7\frac{3}{4}$	$12\frac{3}{5}$	$\frac{5}{12}$	$\frac{5}{6}$	$\frac{2}{8}$	12
	18.	$5\frac{1}{6}$	$3\frac{5}{8}$	$8\frac{2}{3}$	$6\frac{7}{8}$	$\frac{7}{8}$	$\frac{7}{8}$	$\frac{3}{4}$	5
	19.	$7\frac{1}{8}$	$6\frac{2}{3}$	$5\frac{1}{6}$	$15\frac{2}{3}$	$\frac{5}{8}$	$\frac{5}{6}$	$\frac{1}{2}$	3
	20.	$3\frac{5}{6}$	$3\frac{5}{9}$	$12\frac{1}{4}$	$18\frac{7}{8}$	$\frac{3}{4}$	$\frac{5}{16}$	$\frac{5}{8}$	4
V.	21.	$8\frac{3}{5}$	$2\frac{3}{8}$	$1\frac{4}{5}$	$3\frac{5}{8}$	$\frac{5}{9}$	$\frac{2}{5}$	$\frac{5}{16}$	6
	22.	$6\frac{2}{3}$	$4\frac{3}{4}$	$1\frac{7}{8}$	$2\frac{5}{16}$	$\frac{3}{7}$	$\frac{3}{5}$	$\frac{7}{16}$	12
	23.	$4\frac{5}{8}$	$5\frac{7}{8}$	$2\frac{5}{9}$	$6\frac{7}{16}$	$\frac{7}{9}$	$\frac{5}{8}$	$\frac{2}{3}$	9
	24.	$6\frac{3}{8}$	$7\frac{3}{16}$	$3\frac{7}{16}$	$9\frac{9}{16}$	$\frac{5}{9}$	$\frac{7}{8}$	$\frac{3}{7}$	8
	25.	$5\frac{5}{6}$	$4\frac{9}{16}$	$4\frac{5}{16}$	$7\frac{3}{16}$	$\frac{3}{7}$	$\frac{3}{16}$	$\frac{5}{8}$	7

		A	B	C	D	E
I.	1.	2434.54	24.684	24.501	.168	.25
	2.	1426.3	323.47	32.48	1.238	.24
	3.	5368.13	53.760	54.164	3.74	6.7
	4.	7021.5	564.90	78.13	2.63	5.6
	5.	6842.75	293.813	65.127	.784	.93
II.	6.	9025.6	312.56	72.18	.685	.74
	7.	156.32	14.785	9.337	.024	1.2
	8.	6024.5	473.29	12.08	4.13	.32
	9.	5803.6	513.24	85.61	.632	.43
	10.	4150.3	37.292	7.142	.029	.26
III.	11.	5632.4	415.24	48.25	.136	.27
	12.	146.45	36.365	7.362	.024	.38
	13.	789.07	718.05	6.045	.167	9.7
	14.	824.8	613.34	19.24	2.32	5.5
	15.	5813.7	329.05	8.307	5.48	.082
IV.	16.	348.07	315.62	25.32	.407	.32
	17.	1456.38	14.607	7.015	3.26	5.4
	18.	3406.79	215.67	63.24	2.05	.38
	19.	8019.63	3152.48	5.038	3.14	.27
	20.	7905.48	728.54	12.48	508	4.7
V.	21.	453.12	329.18	63.42	.307	.73
	22.	6713.5	5048.7	8.597	1.42	.09
	23.	24.324	18.052	71.062	5.46	1.48
	24.	613.98	508.38	83.507	7.05	2.07
	25.	532.46	407.22	19.207	.432	5.3
VI.	26.	817.15	51.84	139.4	5.102	0.871
	27.	1147.08	621.29	8270.82	1.046	1.93
	28.	29.671	71.64	934.81	0.780	8.46
	29.	387.802	37.80	642.92	0.194	2.31
	30.	819.27	2.59	21.31	8.191	8.09

40. Multiples of Numbers. The numbers 10, 15, 20, 25 are said to be *multiples* of 5 because they are products obtained by multiplying 5 by integers. The table of 5's gives all multiples of 5 up to $10\times5=50$. Similarly the table of 8's gives all multiples of 8 up to $10\times8=80$.

41. The Least Common Multiple. The number 24 is said to be a *common multiple* of 6 and 8 because it is a multiple of 6 and also of 8. 48 is also a common multiple of 6 and 8. 24 is *their least common multiple.*

Instead of least common multiple we write L. C. M.

ORAL EXERCISES

Find the L. C. M. of each of the following:

1. 2, 3, 4.	2. 2, 5, 10.	3. 3, 4, 6.
4. 2, 6, 12.	5. 4, 8, 16.	6. 5, 10, 4.

In most practical problems the L. C. M. may be found directly by inspection as in the above exercises. In a few more complicated cases the method shown in the following example may be used:

Example. Find the L. C. M. of 6, 8, 20.

Solution: Write down a series of multiples of the largest number, 20, and notice the smallest of these which is a multiple of the others.

Thus, in the series 40, 60, 80, 100, 120, 140, we notice that 120 is the smallest which is a multiple of both 6 and 8. Hence 120 is the L. C. M. of 6, 8, 20.

WRITTEN EXERCISES

Find the L. C. M. of each of the following:

1. 3, 5, 8.	4. 5, 8, 12.	7. 8, 9, 36.
2. 15, 18.	5. 2, 7, 9.	8. 5, 8, 12.
3. 8, 12, 16.	6. 4, 6, 9.	9. 6, 9, 12.

42. **The Least Common Denominator.** Two fractions such as $\frac{1}{2}$ and $\frac{1}{3}$ may be reduced to 6ths, or to 12ths, 18ths, etc. The smallest of these numbers is their least common denominator. Instead of least common denominator we write L. C. D. Notice that 6 is the L. C. M. of 2 and 3, and that 6 is also the L. C. D. of $\frac{1}{2}$ and $\frac{1}{3}$.

The general rule is:

To find the L. C. D. of fractions find the L. C. M. of their denominators.

The process to be used in more difficult examples is shown in the following;

Example. Reduce $\frac{3}{4}$, $\frac{5}{9}$ and $\frac{7}{16}$ to a common denominator.

$$\frac{3}{4}=\frac{36\times 3}{36\times 4}=\frac{108}{144}$$

$$\frac{5}{9}=\frac{16\times 5}{16\times 9}=\frac{80}{144}$$

$$\frac{7}{16}=\frac{9\times 7}{9\times 16}=\frac{63}{144}$$

(1) Find by the method shown on page 48 that 144 is the L. C. M. of 4, 9, 16. Hence the fractions must be reduced to 144ths.

(2) Divide 144 by 4 and multiply both terms of $\frac{3}{4}$ by the quotient.

(3) Divide 144 by 9 and multiply both terms of $\frac{5}{9}$ by the quotient.

(4) Divide 144 by 16 and multiply both terms of $\frac{7}{16}$ by the quotient.

WRITTEN EXERCISES

Reduce the following fractions to a common denominator and add them. Find the L. C. D. by inspection when you can.

1. $\frac{5}{6}+\frac{3}{8}+\frac{7}{12}$	4. $\frac{3}{8}+\frac{7}{16}+\frac{5}{30}$	7. $\frac{3}{5}+\frac{5}{6}+\frac{7}{10}$
2. $\frac{1}{6}+\frac{7}{8}+\frac{3}{10}$	5. $\frac{5}{12}+\frac{7}{16}+\frac{3}{8}$	8. $\frac{3}{7}+\frac{1}{4}+\frac{3}{16}$
3. $\frac{11}{12}+\frac{5}{18}+\frac{1}{8}$	6. $\frac{1}{4}+\frac{3}{32}+\frac{5}{64}$	9. $\frac{5}{8}+\frac{5}{12}+\frac{3}{16}$

Drill in Fundamentals. Play game No. 4, page 30. Use examples in adding fractions.

Example. Add $1\frac{1}{2}+3\frac{2}{3}+2\frac{1}{4}$.

$$\begin{array}{ll} 1\frac{1}{2} & \frac{6}{12} \\ 3\frac{2}{3} & \frac{8}{12} \\ 2\frac{1}{4} & \frac{3}{12} \\ \hline 7\frac{5}{12} & \frac{17}{12} \text{ or } 1\frac{5}{12} \end{array}$$

First add the fractions separately, and then the integers. If possible, the fractions should be added mentally, since that saves one copying of the numbers. The sum of the fractions is $\frac{17}{12}$ or $1\frac{5}{12}$. The 1 is added to the integers.

WRITTEN EXERCISES

In this manner add each of the following:

1. $3\frac{1}{4}+2\frac{3}{8}+1\frac{1}{2}$
2. $5\frac{3}{4}+4\frac{5}{6}+\frac{1}{3}$
3. $2\frac{4}{5}+3\frac{7}{10}+2\frac{1}{2}$
4. $6\frac{1}{2}+2\frac{3}{8}+4\frac{3}{4}$
5. $1\frac{5}{12}+4\frac{5}{6}+2\frac{1}{2}$
6. $5\frac{1}{4}+2\frac{7}{8}+4\frac{9}{16}$
7. $4\frac{3}{7}+2\frac{1}{3}+4\frac{5}{14}$
8. $3\frac{5}{9}+2\frac{2}{3}+3\frac{5}{6}$
9. $4\frac{1}{4}+2\frac{1}{8}+4\frac{1}{16}$

10. In a sewing class a girl made an apron, a cap and a pair of loose sleeves. She used $1\frac{1}{16}$ yards of cloth for the apron, $\frac{1}{2}$ yard for the cap and $\frac{1}{3}$ yard for the sleeves. How many yards of cloth did she use?

11. She does $3\frac{3}{4}$ yards of hemming on the apron, and $\frac{3}{4}$ yards of hemming on each sleeve. How many yards of hemming does she do on the sleeves and apron?

12. One girl in the sewing class made four lace handkerchiefs for Christmas. For each of two handkerchiefs she used $1\frac{1}{4}$ yards of lace, and for each of the other two she used $1\frac{1}{3}$ yards of lace. How many yards of lace did she use in all?

43. **Subtraction of Fractions.** When two fractions have a common denominator, it is clear that we can subtract one from the other by subtracting the numerators, leaving the denominator unchanged.

That is, $\frac{7}{12}-\frac{5}{12}=\frac{2}{12}=\frac{1}{6}$.

ORAL EXERCISES

In each of the following examples give the remainder reduced to the simplest form.

1. $\frac{3}{8}-\frac{1}{8}$	7. $\frac{7}{16}-\frac{3}{16}$	13. $\frac{5}{12}-\frac{3}{12}$
2. $\frac{3}{4}-\frac{1}{4}$	8. $\frac{9}{16}-\frac{4}{16}$	14. $\frac{7}{12}-\frac{3}{12}$
3. $\frac{7}{8}-\frac{3}{8}$	9. $\frac{13}{16}-\frac{5}{16}$	15. $\frac{7}{15}-\frac{2}{15}$
4. $\frac{5}{8}-\frac{3}{8}$	10. $\frac{15}{16}-\frac{7}{16}$	16. $\frac{11}{15}-\frac{6}{16}$
5. $\frac{5}{8}-\frac{3}{8}$	11. $\frac{11}{16}-\frac{5}{16}$	17. $\frac{17}{32}-\frac{9}{32}$
6. $\frac{4}{5}-\frac{1}{5}$	12. $\frac{13}{16}-\frac{3}{16}$	18. $\frac{27}{32}-\frac{3}{32}$

If the minuend and subtrahend are not like fractions, they must first be reduced to such fractions.

Example. From $\frac{1}{3}$ subtract $\frac{1}{4}$.

$\frac{1}{3}=\frac{4}{12}$ First reduce $\frac{1}{3}$ and $\frac{1}{4}$ to 12ths and then subtract as shown.

$\frac{1}{4}=\frac{3}{12}$

$\frac{1}{12}$ remainder.

WRITTEN EXERCISES

Find the remainders in the following:

1. $\frac{1}{2}-\frac{1}{4}$	6. $\frac{3}{4}-\frac{5}{8}$	11. $\frac{13}{16}-\frac{3}{4}$
2. $\frac{3}{4}-\frac{1}{2}$	7. $\frac{5}{6}-\frac{2}{8}$	12. $\frac{15}{16}-\frac{5}{8}$
3. $\frac{1}{3}-\frac{1}{6}$	8. $\frac{7}{8}-\frac{3}{4}$	13. $\frac{13}{16}-\frac{5}{8}$
4. $\frac{1}{4}-\frac{1}{8}$	9. $\frac{9}{16}-\frac{1}{2}$	14. $\frac{11}{32}-\frac{3}{8}$
5. $\frac{1}{2}-\frac{1}{8}$	10. $\frac{11}{16}-\frac{1}{4}$	15. $\frac{25}{32}-\frac{7}{16}$

ORAL EXERCISES

Give the remainders in the following. Also add each pair of fractions.

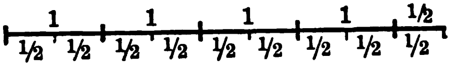
1. $\frac{1}{2}-\frac{1}{3}$	9. $\frac{5}{6}-\frac{1}{4}$	17. $\frac{4}{5}-\frac{2}{3}$
2. $\frac{2}{3}-\frac{1}{2}$	10. $\frac{5}{6}-\frac{3}{4}$	18. $\frac{1}{6}-\frac{1}{8}$
3. $\frac{1}{3}-\frac{1}{4}$	11. $\frac{1}{3}-\frac{1}{5}$	19. $\frac{3}{8}-\frac{1}{6}$
4. $\frac{2}{3}-\frac{1}{4}$	12. $\frac{2}{3}-\frac{1}{5}$	20. $\frac{5}{8}-\frac{1}{6}$
5. $\frac{3}{4}-\frac{1}{3}$	13. $\frac{2}{3}-\frac{2}{5}$	21. $\frac{7}{8}-\frac{1}{6}$
6. $\frac{3}{4}-\frac{2}{3}$	14. $\frac{2}{3}-\frac{3}{5}$	22. $\frac{7}{8}-\frac{5}{6}$
7. $\frac{1}{4}-\frac{1}{6}$	15. $\frac{3}{5}-\frac{1}{3}$	23. $\frac{5}{6}-\frac{3}{8}$
8. $\frac{3}{4}-\frac{1}{6}$	16. $\frac{4}{5}-\frac{1}{3}$	24. $\frac{5}{6}-\frac{5}{8}$

WRITTEN EXERCISES

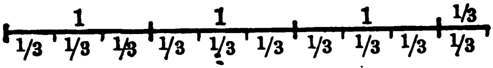
In the following subtract and reduce all results to the simplest form. Also add each pair of fractions.

25. $\frac{4}{5}-\frac{1}{6}$	37. $\frac{1}{7}-\frac{1}{8}$	49. $\frac{2}{9}-\frac{1}{6}$
26. $\frac{3}{5}-\frac{3}{8}$	38. $\frac{3}{7}-\frac{1}{8}$	50. $\frac{5}{6}-\frac{4}{9}$
27. $\frac{5}{6}-\frac{3}{8}$	39. $\frac{5}{7}-\frac{5}{8}$	51. $\frac{1}{12}-\frac{1}{16}$
28. $\frac{5}{8}-\frac{2}{5}$	40. $\frac{6}{7}-\frac{3}{8}$	52. $\frac{5}{12}-\frac{3}{16}$
29. $\frac{7}{8}-\frac{2}{3}$	41. $\frac{7}{8}-\frac{6}{7}$	53. $\frac{7}{12}-\frac{5}{16}$
30. $\frac{3}{7}-\frac{1}{9}$	42. $\frac{1}{8}-\frac{1}{12}$	54. $\frac{9}{16}-\frac{5}{12}$
31. $\frac{5}{9}-\frac{2}{5}$	43. $\frac{5}{8}-\frac{7}{12}$	55. $\frac{1}{8}-\frac{1}{9}$
32. $\frac{4}{7}-\frac{3}{5}$	44. $\frac{5}{12}-\frac{3}{8}$	56. $\frac{5}{8}-\frac{4}{9}$
33. $\frac{7}{8}-\frac{4}{5}$	45. $\frac{7}{8}-\frac{5}{12}$	57. $\frac{5}{9}-\frac{3}{8}$
34. $\frac{3}{7}-\frac{2}{5}$	46. $\frac{11}{12}-\frac{7}{8}$	58. $\frac{7}{8}-\frac{5}{9}$
35. $\frac{3}{8}-\frac{2}{9}$	47. $\frac{1}{6}-\frac{1}{9}$	59. $\frac{8}{9}-\frac{7}{8}$
36. $\frac{3}{10}-\frac{2}{9}$	48. $\frac{5}{6}-\frac{2}{9}$	60. $\frac{5}{6}-\frac{13}{16}$

ORAL EXERCISES



1. How many halves are there in 1? In $1\frac{1}{2}$? In 2? In $2\frac{1}{2}$? In 3? In $3\frac{1}{2}$? In 4? In $4\frac{1}{2}$? Point to these in the figure.



2. How many 3ds are there in 1? In $1\frac{1}{3}$? In $1\frac{2}{3}$? In 2? In $2\frac{1}{3}$? In $2\frac{2}{3}$? In $3\frac{1}{3}$? Point to these in the figure.

3. How many 4ths are there in 1? In 4? In 8? In $2\frac{1}{4}$? In $3\frac{3}{4}$? In $10\frac{3}{4}$?

4. Give a rule for reducing a whole number to 3ds, to 4ths, to 5ths, to 6ths.

5. Reduce to improper fractions: $4\frac{1}{3}$, $3\frac{4}{5}$, $5\frac{3}{8}$, $4\frac{3}{4}$, $7\frac{3}{4}$, $2\frac{7}{16}$, $3\frac{5}{12}$, $4\frac{7}{7}$.

Sometimes it is necessary to reduce only part of a whole number to a fraction. Thus, $4=3\frac{6}{6}$, $2\frac{3}{4}=1\frac{7}{4}$, $5\frac{7}{8}=4\frac{15}{8}$.

6. $3\frac{1}{4}$ equals 2 and how many 4ths?

7. $4\frac{2}{3}$ equals 3 and how many 3ds?

8. $5\frac{2}{5}$ equals 4 and how many 5ths?

9. $6\frac{3}{7}$ equals 5 and how many 7ths?

10. $5\frac{3}{8}$ equals 4 and how many 8ths?

11. Give the missing numbers in $4\frac{7}{8}=3\frac{?}{8}$, $8\frac{5}{7}=7\frac{?}{7}$, $4\frac{3}{16}=3\frac{?}{16}$.

Drill in Fundamentals. On three succeeding days run off a field meet. Use examples in addition for the first event, examples in subtraction for the second, and examples in multiplication for the third.

Example 1. From $12\frac{3}{5}$ subtract 8.

$12\frac{3}{5}$ Subtract 8 from 12 and bring down the $\frac{3}{5}$.

$$\begin{array}{r} 12\frac{3}{5} \\ 8 \\ \hline 4\frac{3}{5} \end{array}$$

Example 2. From 2 subtract $\frac{3}{4}$.

$$\begin{array}{r} 2\frac{4}{4} \\ \frac{3}{4} \\ \hline 1\frac{1}{4} \end{array}$$

Add 1 or $\frac{4}{4}$ to minuend. Then subtract $\frac{3}{4}$ from $\frac{4}{4}$ and carry 1 to subtrahend. } First method.

$$\begin{array}{r} 1\frac{4}{4} \\ \frac{3}{4} \\ \hline 1\frac{1}{4} \end{array}$$

Change 2 to $1\frac{4}{4}$, then subtract $\frac{3}{4}$ from $\frac{4}{4}$. } Second method.

ORAL EXERCISES

1. $5\frac{3}{4}-2$	5. $2-\frac{2}{3}$	9. $8-\frac{5}{9}$	13. $8-\frac{13}{16}$
2. $9\frac{7}{9}-4$	6. $4-\frac{3}{4}$	10. $7-\frac{5}{12}$	14. $9-\frac{7}{12}$
3. $8\frac{3}{7}-6$	7. $8-\frac{6}{7}$	11. $6-\frac{9}{16}$	15. $10-\frac{5}{7}$
4. $6\frac{2}{3}-4$	8. $9-\frac{4}{5}$	12. $3-\frac{9}{10}$	16. $14-\frac{13}{16}$

Example 3. From $3\frac{3}{4}$ subtract $1\frac{1}{4}$.

Subtract the fractions and then the whole numbers.

$$\begin{array}{l} 3\frac{3}{4} \\ 1\frac{1}{4} \\ \hline 2\frac{2}{4} = 2\frac{1}{2} \end{array}$$

Example 4. From 4 subtract $1\frac{4}{5}$.

(*Suggestion:* Add 1 or $\frac{5}{5}$ to minuend and carry 1 to subtrahend, or change 4 into $3\frac{5}{5}$. Then subtract as in example 3.)

WRITTEN EXERCISES

1. $2-1\frac{1}{2}$	5. $5-1\frac{5}{7}$	9. $12-7\frac{4}{7}$	13. $14-7\frac{4}{9}$
2. $3-1\frac{3}{4}$	6. $9-4\frac{3}{8}$	10. $14-8\frac{5}{9}$	14. $16-6\frac{7}{8}$
3. $4-2\frac{7}{8}$	7. $8-3\frac{7}{32}$	11. $23-17\frac{3}{10}$	15. $21-14\frac{9}{16}$
4. $5-2\frac{7}{16}$	8. $10-2\frac{9}{32}$	12. $23-16\frac{3}{16}$	16. $42-28\frac{7}{24}$

Example 1. From $5\frac{1}{3}$ subtract $2\frac{1}{4}$.

$$\begin{array}{r} 5\frac{1}{3}=5\frac{4}{12} \\ 2\frac{1}{4}=2\frac{3}{12} \\ \hline 3\frac{1}{12} \end{array}$$

First subtract the fractions separately. If possible, do this mentally.

Example 2. From $3\frac{1}{3}$ subtract $1\frac{1}{2}$.

$$\begin{array}{rr} 3\frac{1}{3}=3\frac{2}{6} & 3\frac{8}{6} \\ 1\frac{1}{2}=1\frac{3}{6} & 2\frac{3}{6} \\ \hline & 1\frac{5}{6} \end{array}$$

After reducing $\frac{1}{3}$ and $\frac{1}{2}$ to 6ths we find that $\frac{2}{6}$ is less than $\frac{3}{6}$. Hence add $\frac{6}{6}$ to minuend and carry 1 to subtrahend. } First method.

$$\begin{array}{rr} 3\frac{1}{3}=3\frac{2}{6} & 2\frac{8}{6} \\ 1\frac{1}{2}=1\frac{3}{6} & 1\frac{3}{6} \\ \hline & 1\frac{5}{6} \end{array}$$

Change $3\frac{2}{6}$ to $2\frac{8}{6}$ and subtract as in example 1. } Second method.

WRITTEN EXERCISES

In the manner of example 1 subtract the following:

1. $3\frac{1}{3}-1\frac{1}{12}$	6. $6\frac{1}{3}-2\frac{1}{8}$	11. $12\frac{7}{8}-4\frac{3}{4}$
2. $5\frac{2}{3}-2\frac{1}{4}$	7. $9\frac{1}{4}-3\frac{3}{16}$	12. $18\frac{5}{8}-12\frac{1}{3}$
3. $6\frac{3}{4}-3\frac{5}{8}$	8. $15\frac{2}{3}-7\frac{1}{6}$	13. $25\frac{7}{16}-4\frac{13}{32}$
4. $7\frac{2}{3}-4\frac{3}{5}$	9. $25\frac{3}{5}-12\frac{7}{15}$	14. $108\frac{3}{10}-45\frac{1}{4}$
5. $5\frac{3}{4}-1\frac{3}{8}$	10. $35\frac{4}{7}-21\frac{5}{14}$	15. $5\frac{1}{4}-2\frac{1}{3}$

In the manner of example 2 subtract the following:

16. $7\frac{1}{3}-2\frac{3}{4}$	21. $28\frac{5}{8}-13\frac{5}{6}$	26. $124\frac{7}{12}-94\frac{5}{6}$
17. $9\frac{2}{3}-4\frac{3}{4}$	22. $19\frac{7}{8}-6\frac{15}{16}$	27. $67\frac{5}{12}-42\frac{11}{24}$
18. $12\frac{4}{5}-6\frac{9}{10}$	23. $17\frac{3}{8}-4\frac{4}{9}$	28. $39\frac{3}{8}-14\frac{7}{10}$
19. $15\frac{3}{4}-4\frac{7}{8}$	24. $49\frac{3}{7}-21\frac{2}{3}$	29. $64\frac{2}{3}-36\frac{7}{8}$
20. $17\frac{3}{5}-4\frac{2}{3}$	25. $59\frac{4}{5}-31\frac{6}{7}$	30. $81\frac{3}{7}-47\frac{5}{6}$

31. In a sewing class a girl is to do 7 yards of hemming. How many yards has she left to do after hemming $3\frac{5}{8}$ yards?

Example 1. Perform the indicated operations in $1\frac{1}{2}+3\frac{3}{4}-1\frac{1}{5}$.

Add:
$$\begin{array}{r} 1\frac{1}{2}=1\frac{2}{4} \\ 3\frac{3}{4}=\underline{3\frac{3}{4}} \\ 4\frac{5}{4}=5\frac{1}{4} \end{array}$$

Solution: First add $1\frac{1}{2}$ and $3\frac{3}{4}$ obtaining $5\frac{1}{4}$ as the sum.

Subtract:
$$\begin{array}{r} 5\frac{1}{4}=5\frac{5}{20} \\ 1\frac{1}{5}=\underline{1\frac{4}{20}} \\ 4\frac{1}{20} \end{array}$$

From $5\frac{1}{4}$ subtract $1\frac{1}{5}$. The result is $4\frac{1}{20}$.

Example 2. Perform the indicated operations in $\frac{3}{4}+5\frac{2}{3}-2\frac{1}{3}$.

Subtract:
$$\begin{array}{r} 5\frac{2}{3} \\ \underline{2\frac{1}{3}} \\ 3\frac{1}{3} \end{array}$$

Since $5\frac{2}{3}$ and $2\frac{1}{3}$ contain like fractions it is simpler to first subtract $2\frac{1}{3}$ from $5\frac{2}{3}$ and then add $\frac{3}{4}$.

Add:
$$\begin{array}{r} 3\frac{1}{3}=3\frac{4}{12} \\ \frac{3}{4}=\underline{\frac{9}{12}} \\ 4\frac{1}{12} \end{array}$$

WRITTEN EXERCISES

In the following perform the indicated operations in the simplest way as suggested above:

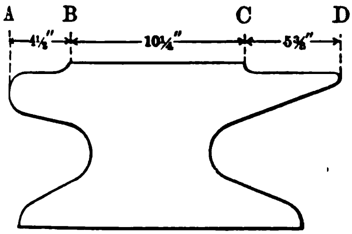
1. $1\frac{2}{3}+4\frac{3}{4}-2\frac{1}{6}$
2. $1\frac{1}{2}+4\frac{1}{3}-5\frac{1}{4}$
3. $5\frac{3}{4}+2\frac{1}{2}-3\frac{1}{3}$
4. $7\frac{1}{4}+5\frac{5}{6}-9\frac{3}{4}$
5. $24\frac{7}{8}+12\frac{3}{4}-30\frac{1}{2}$
6. $7\frac{2}{3}+7\frac{5}{6}-8\frac{5}{8}$
7. $35\frac{3}{8}-16\frac{1}{4}+2\frac{1}{2}$
8. $16\frac{1}{3}-4\frac{2}{3}+2\frac{5}{6}$
9. $31\frac{3}{7}+5\frac{1}{2}-28\frac{6}{7}$
10. $47\frac{3}{5}+25\frac{2}{3}-3\frac{1}{2}$
11. $14\frac{2}{3}+6\frac{3}{4}-2\frac{1}{6}$
12. $71\frac{5}{6}-19\frac{2}{3}+8\frac{1}{2}$
13. $41\frac{5}{8}-17\frac{3}{4}-3\frac{1}{2}$
14. $29\frac{1}{4}-6\frac{1}{3}+5\frac{1}{12}$
15. $13\frac{3}{8}-8\frac{3}{4}+2\frac{7}{16}$
16. $58\frac{7}{12}-8\frac{5}{8}+16\frac{5}{6}$

Drill in Fundamentals. Drill on denominate number facts, as 1 foot = ? inches. Let the boys play against the girls.

The signs ′ and ″ are often used to indicate *feet* and *inches*. Thus, 3′ 6″ means 3 feet 6 inches.

1. A picture is $6\frac{1}{4}''$ by $8\frac{3}{4}''$ inside the frame, and the frame is $\frac{5}{8}''$ wide. What are the dimensions of the picture, including the frame?

2. From the data given in the picture, find the length from A to D.



3. A sheet of letter paper is $6\frac{1}{4}$ inches wide. How long must the envelope be to allow $\frac{1}{8}$ of an inch at each end?

4. A lady bought $1\frac{1}{4}$ pounds of meat on Monday, $\frac{7}{8}$ of a pound on Tuesday, $2\frac{1}{2}$ pounds on Wednesday, $1\frac{5}{8}$ pounds on Friday, and $3\frac{1}{2}$ pounds on Saturday. How many pounds of meat did she buy during the whole week?

5. A lady bought $\frac{1}{2}$ of a yard of velvet for trimming her coat, $\frac{3}{4}$ of a yard for her dress, and $\frac{7}{8}$ of a yard for her hat. How many yards of velvet did she buy all together?

6. Mrs. West bought $8\frac{1}{2}$ yards of goods for school dresses for her daughters Mary and Elsie. For Mary's dress she used $3\frac{1}{2}$ yards, and for Elsie's $3\frac{3}{4}$ yards. How much goods was left? Give the result in yards and inches?

7. A board $\frac{7}{8}$ inches thick is screwed to a plank $3\frac{1}{4}$ inches thick. How thick are the two together? By how much does a screw $2\frac{1}{2}$ inches long fail to reach through the two?

8. A lady bought $\frac{5}{8}$ of a yard of silk. Is this more or less than $\frac{2}{3}$ of a yard? How much? Reduce the difference to inches.

44. A Fraction Multiplied by an Integer. To multiply a number by an integer gives the same result as adding as many times as the multiplier contains one.

Thus, since $\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$, we have $2\times\frac{1}{3}=\frac{2}{3}$.

And $3\times\frac{2}{7}=\frac{6}{7}$, because $\frac{2}{7}+\frac{2}{7}+\frac{2}{7}=\frac{6}{7}$.

Example. $7\times\frac{2}{3}=\frac{14}{3}=4\frac{2}{3}$, and $8\times\frac{5}{7}=\frac{40}{7}=5\frac{5}{7}$.

ORAL EXERCISES

In this manner find the products in the following and reduce each to the simplest form:

1. $3\times\frac{3}{4}$	5. $5\times\frac{3}{4}$	9. $9\times\frac{2}{5}$	13. $6\times\frac{4}{5}$
2. $4\times\frac{1}{3}$	6. $3\times\frac{5}{8}$	10. $4\times\frac{5}{7}$	14. $4\times\frac{5}{9}$
3. $5\times\frac{2}{3}$	7. $3\times\frac{2}{5}$	11. $9\times\frac{3}{4}$	15. $8\times\frac{3}{5}$
4. $4\times\frac{3}{5}$	8. $5\times\frac{3}{8}$	12. $5\times\frac{7}{8}$	16. $7\times\frac{3}{8}$

45. Multiplying an Integer by a Fraction. We have frequently noticed that when we multiply two numbers together the order in which they are taken does not affect the product. That is, $2\times4=4\times2$, and $3\times5=5\times3$. In the same manner $\frac{3}{4}\times5=5\times\frac{3}{4}=\frac{15}{4}=3\frac{3}{4}$, and $\frac{5}{7}\times4=4\times\frac{5}{7}=\frac{20}{7}=2\frac{6}{7}$.

ORAL EXERCISES

In this manner find the products in the following:

1. $\frac{4}{5}\times2$	3. $\frac{7}{8}\times3$	5. $\frac{4}{9}\times5$	7. $\frac{6}{7}\times6$
2. $\frac{5}{8}\times5$	4. $\frac{5}{12}\times5$	6. $\frac{2}{5}\times6$	8. $\frac{3}{5}\times8$

Examples like those given on this page may all be solved by using the following rule:

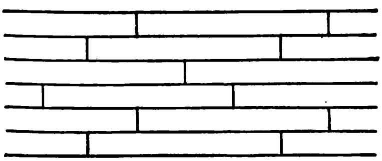
To find the product of a fraction and an integer, multiply the numerator of the fraction by the integer and reduce the result to the simplest form.

Drill in Fundamentals. Play a game like No. 1 on page 30, using examples in multiplying fractions by integers.

In the following reduce the products to the simplest form:

1. $28\times\frac{4}{5}$	8. $\frac{7}{16}\times81$	15. $75\times\frac{8}{4}$
2. $\frac{7}{32}\times95$	9. $49\times\frac{9}{16}$	16. $35\times\frac{7}{8}$
3. $\frac{9}{16}\times43$	10. $\frac{13}{16}\times49$	17. $14\times\frac{3}{5}$
4. $\frac{7}{24}\times73$	11. $49\times\frac{5}{32}$	18. $24\times\frac{7}{13}$
5. $64\times\frac{9}{31}$	12. $63\times\frac{9}{16}$	19. $15\times\frac{3}{7}$
6. $\frac{25}{32}\times9$	13. $\frac{11}{12}\times52$	20. $41\times\frac{3}{5}$
7. $\frac{9}{16}\times41$	14. $35\times\frac{7}{32}$	21. $79\times\frac{2}{3}$

22. In a sewing class there are 25 girls. Each girl makes an apron, a cap and a pair of sleeves, using $\frac{7}{8}$ of a yard of cloth for the apron, $\frac{1}{2}$ of a yard for the cap, and $\frac{3}{8}$ of a yard for the sleeves. How many yards does the class use for aprons? For caps? For sleeves?



23. The facing bricks used in this wall are 2 inches thick. If each course of mortar is $\frac{1}{8}$ of an inch thick, what will be the height of 6 such bricks and 6 layers of mortar?

(*Suggestion:* The thickness of one brick and one layer of mortar is $2\frac{1}{8}$ inches. Hence find $6\times2\frac{1}{8}$.)

24. The length of these bricks is $8\frac{1}{2}$ inches, and the thickness of the mortar between the ends is $\frac{3}{16}$ of an inch. What is the total length of 3 such bricks and 3 layers of mortar?

(*Suggestion:* The length of one brick including one layer of mortar in inches is $8\frac{1}{2}+\frac{3}{16}=8\frac{11}{16}$.)

25. The width of these bricks is $4\frac{1}{4}$ inches. How thick is a wall made of 3 widths of brick and two $\frac{3}{16}$-inch layers of mortar?

46. Cancellation. We have already seen that both terms of a fraction may be divided by the same number without changing the value of the fraction.

That is, $\frac{3}{6}=\frac{1}{2}$, $\frac{4}{12}=\frac{1}{3}$, $\frac{10}{15}=\frac{2}{3}$.

In each of these cases the numerator and denominator have a common factor which is removed by division. We call this process *cancelling*. In $\frac{3}{6}=\frac{1}{2}$ the factor 3 is cancelled. In $\frac{4}{12}=\frac{1}{3}$ the factor 4 is cancelled. In $\frac{10}{15}=\frac{2}{3}$ the factor 5 is cancelled.

> The taking out of a common factor of the numerator and denominator of a fraction is called *cancelling*.

The rule above may now be stated as follows:

A common factor in the numerator and denominator of a fraction may be cancelled without changing the value of the fraction.

Consider now the example $6\times\frac{4}{15}=\frac{24}{15}=\frac{8}{5}=1\frac{3}{5}$. In this case 3 is cancelled after multiplying. In practice, however, the 3 should be cancelled *before* multiplying as follows:

$$\overset{2}{\cancel{6}}\times\frac{4}{\underset{5}{\cancel{15}}}=\frac{8}{5}=1\frac{3}{5}$$

When the common factor 3 is cancelled 6 is divided by 3, giving a quotient 2, which is written above the 6, and 15 is divided by 3, giving a quotient 5, which is written below the 15.

WRITTEN EXERCISES

Find the products indicated below in two ways: (1) multiply the numerator by the integer before cancelling, (2) cancel before multiplying as in the example first given.

1. $8\times\frac{7}{12}$	5. $4\times\frac{5}{6}$	9. $12\times\frac{4}{9}$
2. $6\times\frac{5}{8}$	6. $3\times\frac{4}{9}$	10. $16\times\frac{3}{10}$
3. $9\times\frac{7}{15}$	7. $8\times\frac{5}{12}$	11. $24\times\frac{7}{16}$
4. $4\times\frac{3}{10}$	8. $5\times\frac{3}{16}$	12. $30\times\frac{5}{12}$

In an example like $5\times\frac{7}{10}$ the multiplier, 5, cancels out, leaving 1 as the multiplier. That is, $\overset{1}{\cancel{5}}\times\frac{7}{\underset{2}{\cancel{10}}}=1\times\frac{7}{2}=3\frac{1}{2}$.

In $8\times\frac{3}{4}$ the 4 cancels out, leaving 1 as the denominator. That is, $\overset{2}{\cancel{8}}\times\frac{3}{\underset{1}{\cancel{4}}}=2\times\frac{3}{1}=6$.

In practice we omit the 1's and write

$$\cancel{5}\times\frac{7}{\underset{2}{\cancel{10}}}=\frac{7}{2}=3\tfrac{1}{2} \text{ and } \overset{2}{\cancel{8}}\times\frac{3}{\cancel{4}}=6.$$

WRITTEN EXERCISES

In this manner find the products of the following:

1. $8\times\frac{7}{24}$	4. $3\times\frac{4}{9}$	7. $7\times\frac{12}{35}$	10. $12\times\frac{5}{8}$
2. $5\times\frac{3}{10}$	5. $\frac{4}{5}\times30$	8. $6\times\frac{5}{12}$	11. $48\times\frac{7}{12}$
3. $4\times\frac{17}{16}$	6. $\frac{3}{4}\times40$	9. $9\times\frac{2}{3}$	12. $\frac{5}{6}\times14$

47. **Product of an Integer and a Mixed Number.** To multiply a mixed number by an integer, we multiply the whole number and the fraction separately by the integer, and add the products.

Example. Multiply $2\frac{1}{2}$ by 5, and $3\frac{3}{4}$ by 6.

$$\begin{array}{r} 2\frac{1}{2} \\ 5 \\ \hline 10\frac{5}{2}=12\frac{1}{2} \end{array} \qquad\qquad \begin{array}{r} 3\frac{3}{4} \\ 6 \\ \hline 18\frac{18}{4}=22\frac{1}{2} \end{array}$$

WRITTEN EXERCISES

In this manner find the products of the following:

1. $5\times1\frac{1}{3}$	4. $1\frac{5}{6}\times4$	7. $5\times4\frac{3}{4}$	10. $16\frac{2}{3}\times6$
2. $7\times2\frac{3}{5}$	5. $2\frac{3}{7}\times5$	8. $12\frac{1}{2}\times6$	11. $33\frac{1}{3}\times6$
3. $6\times3\frac{1}{3}$	6. $6\times2\frac{3}{8}$	9. $3\frac{1}{3}\times12$	12. $6\frac{1}{4}\times8$

WRITTEN WORK

By turning a screw completely around once it goes into the wood as far as the distance between the threads. This distance is called the *pitch* of the screw.

1. A screw has a pitch of $\frac{1}{8}$ of an inch. How far is it driven into the wood by being turned 12 times?

2. A screw has a pitch of $\frac{3}{32}$ of an inch. How far is it driven into the wood by being turned around 16 times?

3. If the average weight of a business letter is $\frac{3}{5}$ of an ounce, how many ounces will 1000 such letters weigh? How many pounds is this?

4. A Sunday newspaper weighs 13 ounces ($\frac{13}{16}$ of a pound). How many pounds are there in an edition of 120,000 such papers? How many tons is this?

5. A certain steel plate is $\frac{5}{32}$ of an inch thick. How high is a pile of 150 such plates?

6. A text book weighs 18 ounces ($1\frac{2}{16}$ or $1\frac{1}{8}$ of a pound) when wrapped for shipment. One day a publishing house sent out 846 of these books by mail. What was the total weight of the shipment?

7. A certain family consumes on an average $1\frac{5}{8}$ pounds of meat per day. How many pounds is this for a month of 30 days? What is the cost of the meat if the average price is 24 cents a pound?

8. Two cards for mounting photographs are $6\frac{1}{4}'' \times 8\frac{1}{2}''$, and $8\frac{5}{8}'' \times 4\frac{3}{4}''$. What is the difference in the lengths of these cards? What is the difference in their widths?

In canning fruit the following amounts of sugar are used for one quart:

peaches	4 ounces	($\frac{4}{16}$, or $\frac{1}{4}$ of a pound)
blackberries	6 ounces	($\frac{6}{16}$, or $\frac{3}{8}$ of a pound)
strawberries	8 ounces	($\frac{8}{16}$, or $\frac{1}{2}$ of a pound)
quinces	10 ounces	($\frac{10}{16}$, or $\frac{5}{8}$ of a pound)

WRITTEN WORK

1. How much sugar is used in putting up 40 quarts of peaches?

2. How much sugar is used in putting up 24 quarts of strawberries?

3. How much sugar is used in putting up 30 quarts of blackberries?

4. How much sugar is used in putting up 12 quarts of quinces?

5. A hotel keeper figures that for a turkey dinner he needs on an average 10 ounces ($\frac{5}{8}$ pounds) of turkey for each guest. How many pounds of turkey does he buy if he expects 175 guests?

6. The basement of a house is 48 feet long and 38 feet wide. How much does it cost to lay a cement floor in this basement at $1.00 a square yard?

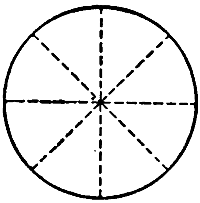
This problem may be solved by first finding the number of square feet and then dividing by 9, or by first reducing the dimensions of the basement to yards and then finding the area in square yards. Solve the problem both ways and compare results. How many yards are there in 48 feet? In 38 feet?

7. At $2.15 a square yard, what is the cost of laying a walk 120 feet long and 5 feet wide?

8. A rug is 13 feet long and $9\frac{3}{8}$ feet wide. Find the number of square feet in the rug.

48. Product of Two Fractions. The exercises on the next three pages lead to a rule for multiplying fractions.

ORAL EXERCISES

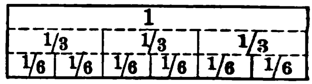


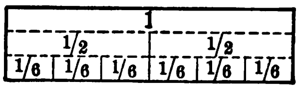
1. Point out $\frac{1}{2}$ of the circle, $\frac{1}{4}$ of it, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{4}{4}$, $\frac{1}{8}$, $\frac{2}{8}$, $\frac{3}{8}$, $\frac{4}{8}$, $\frac{5}{8}$, $\frac{6}{8}$, $\frac{7}{8}$, $\frac{8}{8}$.

2. Read and supply the missing numbers in the following and find each result in the circle: $\frac{1}{2}$ of $\frac{1}{2}=?$, $\frac{1}{2}$ of $\frac{1}{4}=?$, $\frac{1}{2}$ of $\frac{3}{4}=?$

3. In the next figure point out $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{6}$, $\frac{2}{6}$, $\frac{3}{6}$, $\frac{4}{6}$, $\frac{5}{6}$, $\frac{6}{6}$.

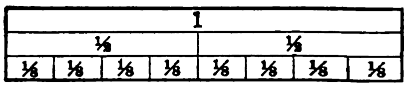
4. Supply the missing numbers in $\frac{1}{2}$ of $\frac{1}{3}=?$ and find the result in the figure.

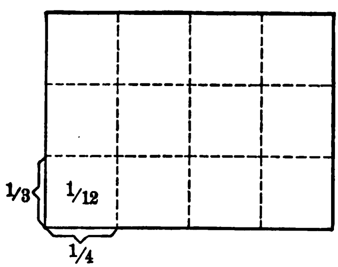




5. Supply the missing numbers in the following and find each result in the figure: $\frac{1}{3}$ of $\frac{1}{2}=?$ $\frac{2}{3}$ of $\frac{1}{2}=?$ Compare $\frac{1}{3}$ of $\frac{1}{2}$, and $\frac{1}{2}$ of $\frac{1}{3}$.

6. Supply the missing numbers in the following and find each result in the figure: $\frac{1}{4}$ of $\frac{1}{2}=?$ $\frac{3}{4}$ of $\frac{1}{2}=?$





7. Show $\frac{1}{3}$ of this rectangle. Also show $\frac{1}{4}$ of the rectangle. Show $\frac{1}{3}$ of $\frac{1}{4}$. What fraction of the whole rectangle is this? Show $\frac{1}{4}$ of $\frac{1}{3}$. What fraction of the rectangle is this? Supply the missing numbers in $\frac{1}{3}$ of $\frac{1}{4}=?$ $\frac{1}{4}$ of $\frac{1}{3}=?$

ORAL EXERCISES

1. Find $\frac{1}{2}$ of $\frac{2}{3}$, $\frac{1}{2}$ of $\frac{4}{5}$, $\frac{1}{2}$ of $\frac{2}{7}$.

2. Supply the missing numbers in the following, and draw a figure to show each result:

$\frac{1}{2}$ of $\frac{1}{5}$ = ? $\frac{1}{2}$ of $\frac{1}{6}$ = ? $\frac{1}{2}$ of $\frac{1}{8}$ = ? $\frac{1}{8}$ of $\frac{1}{2}$ = ?

3. Draw a figure to show the result in each of the following:

$\frac{1}{3}$ of $\frac{1}{3}$ = ? $\frac{1}{3}$ of $\frac{1}{4}$ = ? $\frac{1}{4}$ of $\frac{1}{3}$ = ? $\frac{1}{3}$ of $\frac{1}{5}$ = ?

4. Draw a figure to show the result in each of the following:

$\frac{1}{4}$ of $\frac{1}{4}$ = ? $\frac{1}{4}$ of $\frac{1}{5}$ = ? $\frac{1}{5}$ of $\frac{1}{4}$ = ? $\frac{1}{4}$ of $\frac{1}{6}$ = ?

Notice that to take $\frac{1}{2}$ of a fraction we divide its numerator by 2 or multiply its denominator by 2. That is, we take one-half of the parts or else make each part only one-half as large as it was before.

Similarly, to take $\frac{1}{3}$ of a fraction we divide the numerator by 3 or multiply the denominator by 3. Why do we do this?

5. How do we take $\frac{1}{4}$ of a fraction?

6. How do we take $\frac{1}{5}$ of a fraction? $\frac{1}{6}$ of it? $\frac{1}{7}$ of it?

When we take $\frac{1}{2}$ of a fraction we say we *multiply it by* $\frac{1}{2}$. Hence, instead of $\frac{1}{2}$ of $\frac{1}{2}$ we write $\frac{1}{2}\times\frac{1}{2}$. Similarly, $\frac{1}{3}$ of $\frac{1}{2}$ is written $\frac{1}{3}\times\frac{1}{2}$, and we say we *multiply* $\frac{1}{2}$ by $\frac{1}{3}$.

7. Read the following and supply the missing numbers:

$\frac{1}{5}\times\frac{3}{4}$ = ?	$\frac{1}{4}\times\frac{7}{12}$ = ?	$\frac{1}{3}\times\frac{5}{11}$ = ?
$\frac{1}{6}\times\frac{5}{7}$ = ?	$\frac{1}{3}\times\frac{7}{11}$ = ?	$\frac{1}{4}\times\frac{5}{12}$ = ?
$\frac{1}{3}\times\frac{7}{12}$ = ?	$\frac{1}{9}\times\frac{7}{12}$ = ?	$\frac{1}{12}\times\frac{5}{11}$ = ?

Drill in Fundamentals. Play game No. 3, page 30. Use examples, adding fractions for the first event, and examples in multiplying fractions for the second event.

We have already seen that to multiply a fraction by $\frac{1}{2}$ we multiply its denominator by 2; to multiply a fraction by $\frac{1}{3}$ we multiply its denominator by 3, etc.

Thus, $\frac{1}{3}\times\frac{2}{5}=\frac{2}{3\times5}=\frac{2}{15}$, and $\frac{1}{7}\times\frac{2}{3}=\frac{2}{7\times3}=\frac{2}{21}$

Many such examples may be simplified by cancellation.

Example. Multiply $\frac{4}{7}$ by $\frac{1}{6}$.

$$\frac{1}{6}\times\frac{4}{7}=\frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{6}}\times7}=\frac{2}{21}$$

After indicating the product cancel 2 and then multiply 7 by 3.

$$\frac{1}{\underset{3}{\cancel{6}}}\times\frac{\overset{2}{\cancel{4}}}{7}=\frac{2}{21}$$

In practice indicate the multiplication, and cancel as in the second form given here.

WRITTEN EXERCISES

Multiply the following, using the second form:

1. $\frac{1}{5}\times\frac{25}{22}$	5. $\frac{1}{3}\times\frac{6}{7}$	9. $\frac{1}{7}\times\frac{35}{48}$
2. $\frac{1}{7}\times\frac{14}{32}$	6. $\frac{1}{8}\times\frac{16}{17}$	10. $\frac{1}{9}\times\frac{27}{32}$
3. $\frac{1}{4}\times\frac{8}{9}$	7. $\frac{1}{4}\times\frac{12}{13}$	11. $\frac{1}{4}\times\frac{15}{16}$
4. $\frac{1}{2}\times\frac{4}{5}$	8. $\frac{1}{3}\times\frac{9}{32}$	12. $\frac{1}{5}\times\frac{15}{64}$

Since $\frac{2}{3}=2\times\frac{1}{3}$, to multiply by $\frac{2}{3}$ we multiply first by $\frac{1}{3}$ and then by 2.

Example. Multiply $\frac{2}{3}\times\frac{4}{5}$.

Solution: First take $\frac{1}{3}\times\frac{4}{5}$ and then multiply the product by 2. $\frac{1}{3}\times\frac{4}{5}=\frac{4}{15}$, $2\times\frac{4}{15}=\frac{8}{15}$.

The numerator 8 of the product equals the product of the numerators of the two given fractions (2×4), and the denominator, 15, of the product equals the product of the denominators of the two given fractions (3×5). From this we get the general rule for the multiplication of fractions. (See next page.)

49. **Rule:** *To find the product of two fractions multiply the numerators together for a new numerator, and multiply the denominators together for a new denominator, cancelling when possible.*

If you have any doubt as to the correctness of the rule, check results obtained by it, by finding the product as in the last example on page 66.

ORAL EXERCISES

Find the following products by the use of the above rule:

1. $\frac{2}{3}\times\frac{4}{5}$	5. $\frac{3}{4}\times\frac{3}{7}$	9. $\frac{3}{4}\times\frac{3}{5}$	13. $\frac{5}{6}\times\frac{1}{7}$
2. $\frac{3}{5}\times\frac{4}{7}$	6. $\frac{6}{5}\times\frac{2}{3}$	10. $\frac{2}{5}\times\frac{3}{5}$	14. $\frac{4}{5}\times\frac{6}{7}$
3. $\frac{3}{7}\times\frac{1}{5}$	7. $\frac{1}{8}\times\frac{3}{4}$	11. $\frac{1}{2}\times\frac{3}{7}$	15. $\frac{3}{8}\times\frac{5}{4}$
4. $\frac{6}{7}\times\frac{3}{5}$	8. $\frac{3}{5}\times\frac{6}{7}$	12. $\frac{2}{3}\times\frac{7}{9}$	16. $\frac{2}{7}\times\frac{5}{11}$

The last two examples can be reduced to lower terms by cancellation. In practice we should cancel *before* multiplying the numerator and denominator together.

Example. Find the product of $\frac{2}{3}\times\frac{6}{5}$.

Solution:

$$\frac{2}{\cancel{3}}\times\frac{\overset{2}{\cancel{6}}}{5}=\frac{4}{5}$$

$$\boxed{\frac{\overset{1}{\cancel{2}}}{\underset{2}{\cancel{4}}}\times\frac{\overset{1}{\cancel{2}}}{\underset{3}{\cancel{6}}}=\frac{1}{6}}$$

In multiplying fractions always look carefully for factors common to a numerator and a denominator. Also remember that a number divided by itself gives 1. Notice the example solved here.

WRITTEN EXERCISES

Find the following products in this manner:

1. $\frac{2}{3}\times\frac{6}{5}$	5. $\frac{2}{5}\times\frac{10}{11}$	9. $\frac{4}{5}\times\frac{7}{12}$	13. $\frac{3}{8}\times\frac{5}{9}$
2. $\frac{2}{3}\times\frac{6}{7}$	6. $\frac{2}{5}\times\frac{3}{4}$	10. $\frac{3}{8}\times\frac{4}{5}$	14. $\frac{8}{9}\times\frac{7}{16}$
3. $\frac{3}{4}\times\frac{6}{5}$	7. $\frac{5}{8}\times\frac{6}{7}$	11. $\frac{7}{8}\times\frac{5}{21}$	15. $\frac{3}{7}\times\frac{4}{9}$
4. $\frac{3}{5}\times\frac{5}{18}$	8. $\frac{7}{10}\times\frac{5}{7}$	12. $\frac{3}{4}\times\frac{2}{9}$	16. $\frac{5}{12}\times\frac{7}{16}$

We are frequently required to multiply a mixed number by a whole number. The following form is convenient:

Multiply $3\frac{2}{5}$ by 6. *Solution:* $6 \times 3 = 18$. Hence the product is $20\frac{2}{5}$.

$$\begin{array}{r} 6 \times 3 = 18 \\ 6 \times \frac{2}{5} = \frac{12}{5} = \underline{2\frac{2}{5}} \\ 20\frac{2}{5} \end{array}$$

WRITTEN EXERCISES

In this manner find the products of the following:

1. $5 \times 6\frac{3}{4}$	6. $9 \times 3\frac{5}{8}$	11. $7 \times 12\frac{7}{8}$
2. $7 \times 5\frac{2}{3}$	7. $8 \times 7\frac{9}{10}$	12. $9 \times 9\frac{11}{12}$
3. $9 \times 7\frac{3}{5}$	8. $9 \times 11\frac{3}{8}$	13. $8 \times 12\frac{13}{16}$
4. $8 \times 6\frac{4}{9}$	9. $7 \times 12\frac{7}{16}$	14. $10 \times 9\frac{7}{16}$
5. $7 \times 8\frac{6}{7}$	10. $8 \times 9\frac{4}{7}$	15. $12 \times 12\frac{7}{8}$

16. A beam is made by nailing together 3 planks each $1\frac{3}{4}$ inches thick. How thick is the beam?

17. Each volume of an encyclopedia is $2\frac{1}{8}$ inches thick. How many inches long must a shelf be to hold 24 such volumes?

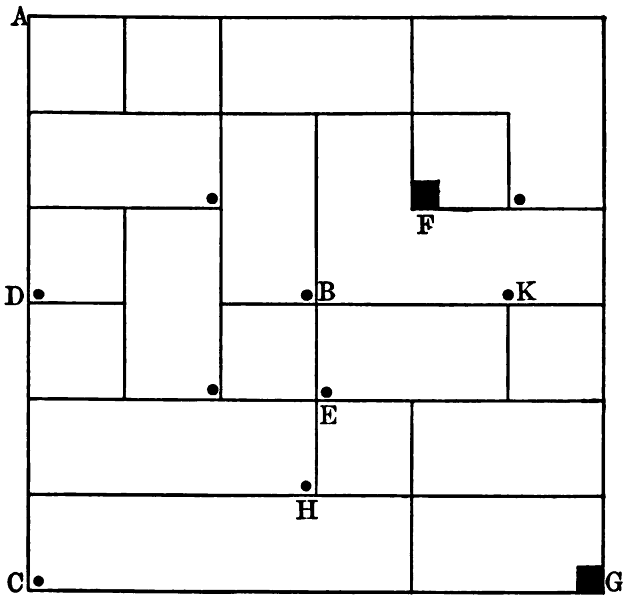
18. A man steps on an average $2\frac{4}{5}$ feet. How far does he go if he takes 2480 steps? Is this more or less than one mile?

19. If you step $2\frac{2}{3}$ feet, how far will you go if you take 2400 steps? Is this more or less than a mile? How much?

20. A wagon wheel goes $17\frac{1}{3}$ feet in making one complete revolution. How far does this wheel go in making 300 revolutions? Is this more or less than one mile? How much?

21. In a recent year the average yield of wheat in the United States was $15\frac{7}{8}$ bushels per acre. At this rate, how many bushels are raised on a field of 65 acres?

22. A man farming on shares gets $\frac{2}{3}$ of the crops. What is his share of a wheat crop of 1145 bushels?



1. With a ruler measure the distances between the following points: A and C, A and D, A and F, and C and F. In each case disregard fractions of an inch less than $\frac{1}{8}$.

2. If one inch on the map represents 2 miles of country, find the actual distances from A to C, from A to D, and from C to F. Notice that these are the distances in straight lines across the fields.

3. In this same map find the distances between A and G, A and B, A and H.

4. On a map of your state find how many miles are represented by one inch. Measure the distance on the map between two places you know. Then figure out how many miles there are between these places.

50. Product of a Mixed Number and a Fraction. The following example shows the process:

Example. Multiply $2\frac{1}{3}$ by $\frac{4}{5}$.

$$\begin{array}{ll} \frac{4}{5}\times 2=\frac{8}{5}=1\frac{3}{5}=1\frac{9}{15} & \\ \frac{4}{5}\times\frac{1}{3}=\frac{4}{15}= & \underline{\ \ \frac{4}{15}} \\ & 1\frac{13}{15} \end{array}$$

First multipy 2 by $\frac{4}{5}$, and then multiply $\frac{1}{3}$ by $\frac{4}{5}$. Add the products.

WRITTEN EXERCISES

Multiply the following:

1. $1\frac{3}{4}\times\frac{2}{3}$	4. $9\frac{1}{3}\times\frac{2}{5}$	7. $6\frac{2}{3}\times\frac{1}{2}$	10. $8\frac{3}{7}\times\frac{3}{8}$
2. $4\frac{1}{4}\times\frac{3}{4}$	5. $6\frac{1}{2}\times\frac{3}{4}$	8. $9\frac{1}{4}\times\frac{3}{5}$	11. $9\frac{3}{4}\times\frac{2}{5}$
3. $2\frac{4}{5}\times\frac{3}{7}$	6. $7\frac{1}{3}\times\frac{2}{3}$	9. $7\frac{3}{4}\times\frac{5}{6}$	12. $7\frac{7}{8}\times\frac{4}{5}$

51. Product of two Mixed Numbers. To multiply one mixed number by another, it is usually best to reduce both to improper fractions. If the numbers are large it is best, however, to use the four-step method. (See example 2 below.)

Example 1. Multiply $2\frac{3}{4}$ by $3\frac{1}{3}$.

$$2\frac{3}{4}=\frac{11}{4}$$
$$3\frac{1}{3}=\frac{10}{3}$$
$$\frac{11}{4}\times\frac{10}{3}=\frac{55}{6}=9\frac{1}{6}.$$

Reduce $2\frac{3}{4}$ and $3\frac{1}{3}$ to improper fractions, and then multiply.

Example 2. Multiply $32\frac{1}{2}$ by $8\frac{2}{3}$.

$$\begin{array}{rr} & 32\frac{1}{2} \\ & \underline{8\frac{2}{3}} \\ 8\times 32= & 256 \\ \frac{2}{3}\times 32= & 21\frac{1}{3} \\ 8\times\frac{1}{2}= & 4 \\ \frac{2}{3}\times\frac{1}{2}= & \underline{\frac{1}{3}} \\ & 281\frac{2}{3} \end{array}$$

(Four-step process)

(1) Multiply the integers $(8\times 32=256)$
(2) Multiply 32 by $\frac{2}{3}$ $(\frac{2}{3}\times 32=21\frac{1}{3})$
(3) Multiply $\frac{1}{2}$ by 8 $(8\times\frac{1}{2}=4)$
(4) Multiply $\frac{1}{2}$ by $\frac{2}{3}$ $(\frac{2}{3}\times\frac{1}{2}=\frac{1}{3})$
Add the partial products.

Also solve this problem by reducing to improper fractions and compare results.

WRITTEN EXERCISES

Using either of the processes on page 70, find the products of the following:

1. $1\frac{1}{4} \times 3\frac{3}{2}$	5. $8\frac{3}{5} \times 6\frac{4}{9}$	9. $8\frac{3}{4} \times 6\frac{1}{3}$
2. $6\frac{3}{4} \times 3\frac{2}{3}$	6. $4\frac{1}{3} \times 5\frac{1}{5}$	10. $2\frac{1}{4} \times 5\frac{1}{3}$
3. $9\frac{6}{7} \times 7\frac{3}{8}$	7. $6\frac{1}{2} \times 7\frac{1}{3}$	11. $7\frac{2}{3} \times 4\frac{1}{5}$
4. $4\frac{1}{3} \times 2\frac{3}{5}$	8. $3\frac{2}{3} \times 8\frac{1}{4}$	12. $10\frac{4}{9} \times 6\frac{5}{8}$

13. At $6.00 a week how much does a servant get for a month of 30 days? (30 days is $4\frac{2}{7}$ weeks.) How much does she get for a month of 31 days?

14. How many minutes are there in one hour? 30 minutes is what fraction of an hour? 15 minutes? 20 minutes? 10 minutes? 40 minutes? 45 minutes?

15. Express each of the following as hours and fractions of hours: 75 minutes, 90 minutes, 105 minutes.

16. If a child spends 45 minutes each school day on arithmetic, how many hours does the child spend on arithmetic in one school year of 200 days? (First reduce 45 minutes to a fraction of an hour.)

17. If a child spends 90 minutes of each school day on reading and language, how many hours does the child spend on reading and language in one school year of 200 days? (First reduce 90 minutes to one hour and a fraction of an hour.)

18. From your school program find how many minutes you spend each day on each of your subjects. Then find how many hours you spend on each subject in a school year. (First find how many days you go to school in one year.)

Drill in Fundamentals. Play game No. 4 on page 30. Use examples in multiplication of whole numbers.

52. Dividing Numbers of the Same Kind. To divide two numbers of the same kind we simply divide one of the numbers by the other, paying no attention to the kind of number, as shown in the following examples:

Example 1. A farmer hauls 50 bushels of wheat in one load. How many loads are there in his whole crop of 4350 bushels?

Solution:

$$5\cancel{0}\overline{)435\cancel{0}}\quad 87$$

We divide 4350 by 50, getting a quotient of 87, which is the number of loads.

Example 2. Divide $\frac{7}{20}$ by $\frac{2}{20}$.

$\frac{7}{20}\div\frac{2}{20}=7\div2=\frac{7}{2}=3\frac{1}{2}$. Since $\frac{7}{20}$ and $\frac{2}{20}$ are numbers of the same kind (twentieths) we simply divide 7 by 2.

53. Dividing Numbers of Different Kinds. If the dividend and divisor are different kinds of numbers, they must be changed to numbers of the same kind before dividing, as in the following examples:

Example 1. How many badges, each 7 inches long, may be cut from 12 yards of ribbon?

12 yards $=36\times12$ inches, or 432 inches.

$$7\overline{)432}\quad 61\text{, remainder 5.}$$

In this case we must reduce the 12 yards to inches before dividing. That is, the dividend and divisor must be reduced to numbers of the same kind. We then find by dividing that 61 badges may be cut, leaving a piece 5 inches long.

Example 2. Divide $\frac{3}{4}$ by $\frac{2}{5}$.

$\frac{3}{4}\div\frac{2}{5}=\frac{15}{20}\div\frac{8}{20}$.
$15\div8=1\frac{7}{8}$.

Reducing $\frac{3}{4}$ and $\frac{2}{5}$ to a common denominator we have $\frac{15}{20}\div\frac{8}{20}$. Since these numbers are of the same kind, we divide 15 by 8.

WRITTEN EXERCISES

Using the method of the last example on page 72, find the quotients of the following:

1. $\frac{2}{3} \div \frac{3}{4}$	3. $\frac{1}{4} \div \frac{1}{3}$	5. $\frac{1}{4} \div \frac{2}{3}$	7. $\frac{1}{2} \div \frac{3}{5}$
2. $\frac{1}{3} \div \frac{1}{4}$	4. $\frac{1}{2} \div \frac{1}{3}$	6. $\frac{3}{5} \div \frac{1}{2}$	8. $\frac{3}{8} \div \frac{3}{4}$

54. Dividing by a Fraction. A convenient method for dividing by a fraction is given in the following rule:

> *To divide by a fraction, invert the terms of the divisor and multiply.*

Thus, in the first example in the exercises above, instead of reducing $\frac{2}{3} \div \frac{3}{4}$ to $\frac{8}{12} \div \frac{9}{12}$ and then dividing 8 by 9, getting $\frac{8}{9}$, we may invert the terms of $\frac{3}{4}$ and multiply. That is, $\frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$.

Solve all of the above exercises in these two ways to see that they give the same result.

The same rule may also be used to divide by a whole number:

Thus, $\frac{3}{5} \div 2 = \frac{3}{5} \times \frac{1}{2} = \frac{3}{10}$, and $\frac{4}{7} \div 2 = \frac{4}{7} \times \frac{1}{2} = \frac{2}{7}$.

In practice, however, the last example would be solved by dividing the numerator 4 by 2 directly, writing $\frac{4}{7} \div 2 = \frac{2}{7}$.

WRITTEN EXERCISES

Divide:

1. $\frac{3}{4} \div \frac{3}{5}$	7. $\frac{6}{7} \div \frac{2}{5}$	13. $\frac{9}{16} \div \frac{5}{8}$	19. $\frac{5}{8} \div \frac{16}{9}$
2. $\frac{4}{5} \div \frac{3}{4}$	8. $\frac{4}{9} \div \frac{2}{3}$	14. $\frac{7}{12} \div \frac{5}{18}$	20. $\frac{5}{16} \div \frac{3}{8}$
3. $\frac{2}{3} \div \frac{3}{7}$	9. $\frac{3}{8} \div \frac{5}{16}$	15. $\frac{11}{15} \div \frac{3}{5}$	21. $\frac{5}{18} \div \frac{7}{12}$
4. $\frac{2}{5} \div \frac{1}{3}$	10. $\frac{5}{9} \div \frac{3}{7}$	16. $\frac{7}{16} \div \frac{3}{8}$	22. $\frac{3}{5} \div \frac{11}{15}$
5. $\frac{3}{7} \div \frac{2}{3}$	11. $\frac{7}{8} \div \frac{9}{13}$	17. $\frac{3}{32} \div \frac{5}{16}$	23. $\frac{3}{8} \div \frac{7}{16}$
6. $\frac{3}{8} \div \frac{1}{4}$	12. $\frac{5}{16} \div \frac{3}{32}$	18. $\frac{9}{13} \div \frac{7}{8}$	24. $\frac{5}{9} \div \frac{3}{7}$

55. Dividing a Fraction by an Integer. In some cases it is simpler to use a direct method for dividing a fraction by an integer.

From $\frac{4}{7} \div 2 = \frac{2}{7}$, and $\frac{3}{7} \div 2 = \frac{3}{14}$, try to formulate a rule for dividing a fraction by an integer.

ORAL EXERCISES

Find the quotients of the following in the simplest way:

1. $\frac{4}{5} \div 2$	3. $\frac{6}{7} \div 3$	5. $\frac{5}{8} \div 2$	7. $\frac{7}{8} \div 3$
2. $\frac{3}{7} \div 4$	4. $\frac{5}{7} \div 3$	6. $\frac{9}{16} \div 6$	8. $\frac{8}{15} \div 4$

9. Instead of the divisors $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{5}{6}$, $\frac{3}{7}$, $\frac{9}{10}$, $\frac{7}{16}$ what multipliers may be used.

ORAL AND WRITTEN EXERCISES

Solve as many as possible orally:

1. $27 \div \frac{3}{5}$	12. $72 \div \frac{3}{5}$	23. $\frac{3}{8} \div \frac{5}{16}$	34. $\frac{9}{16} \div \frac{1}{16}$
2. $15 \div \frac{3}{4}$	13. $72 \div \frac{9}{11}$	24. $\frac{4}{5} \div \frac{1}{2}$	35. $\frac{13}{16} \div \frac{1}{15}$
3. $8 \div \frac{3}{5}$	14. $84 \div \frac{7}{10}$	25. $\frac{3}{8} \div \frac{1}{3}$	36. $\frac{7}{8} \div \frac{1}{8}$
4. $7 \div \frac{4}{5}$	15. $20 \div \frac{2}{5}$	26. $\frac{7}{10} \div \frac{1}{9}$	37. $\frac{2}{3} \div \frac{4}{5}$
5. $63 \div \frac{3}{7}$	16. $24 \div \frac{5}{6}$	27. $\frac{5}{16} \div \frac{1}{12}$	38. $\frac{3}{7} \div \frac{2}{9}$
6. $54 \div \frac{5}{6}$	17. $30 \div \frac{3}{7}$	28. $\frac{3}{16} \div \frac{9}{10}$	39. $\frac{7}{8} \div \frac{7}{16}$
7. $27 \div \frac{9}{10}$	18. $45 \div \frac{9}{10}$	29. $\frac{4}{16} \div \frac{15}{32}$	40. $\frac{13}{16} \div \frac{5}{32}$
8. $9 \div \frac{3}{4}$	19. $64 \div \frac{4}{7}$	30. $\frac{4}{5} \div \frac{1}{3}$	41. $\frac{5}{7} \div \frac{3}{8}$
9. $12 \div \frac{4}{5}$	20. $54 \div \frac{9}{10}$	31. $\frac{5}{8} \div \frac{3}{7}$	42. $\frac{3}{8} \div \frac{5}{7}$
10. $12 \div \frac{5}{6}$	21. $48 \div \frac{6}{7}$	32. $\frac{7}{8} \div \frac{3}{32}$	43. $\frac{6}{7} \div \frac{3}{5}$
11. $16 \div \frac{3}{4}$	22. $\frac{2}{3} \div \frac{3}{4}$	33. $\frac{7}{32} \div \frac{1}{11}$	44. $\frac{9}{16} \div \frac{4}{5}$

Drill in Fundamentals. Play game No. 2, page 30. Use examples in division of fractions.

56. **Dividing by a Mixed Number.** If the divisor is a mixed number, first reduce it to an improper fraction before dividing.

Example 1. Divide 34 by $3\frac{4}{5}$.

$3\frac{4}{5}=\frac{19}{5}$

$34\div\frac{19}{5}=34\times\frac{5}{19}=\frac{170}{19}=8\frac{18}{19}$

Reduce $3\frac{4}{5}$ to $\frac{19}{5}$. Then $34\div 3\frac{4}{5}=34\div\frac{19}{5}$.

Example 2. Divide $18\frac{3}{4}$ by $2\frac{1}{3}$. $2\frac{1}{3}=\frac{7}{3}$. Then $18\frac{3}{4}\div 2\frac{1}{3}=18\frac{3}{4}\times\frac{3}{7}$.

$$\begin{aligned}\tfrac{3}{7}\times 18 &= \tfrac{54}{7}=7\tfrac{5}{7}\\ \tfrac{3}{7}\times\tfrac{3}{4} &= \quad\ \tfrac{9}{28}\\ \hline &\quad 7\tfrac{29}{28}=8\tfrac{1}{28}\end{aligned}$$

Reduce $2\frac{1}{3}$ to $\frac{7}{3}$. Then we need to divide by $\frac{7}{3}$ or multiply by $\frac{3}{7}$. To multiply $18\frac{3}{4}$ by $\frac{3}{7}$ use the method of the first example on page 70.

WRITTEN EXERCISES

In this manner obtain the results in the following:

1. $7\frac{1}{3}\div\frac{3}{5}$	4. $11\frac{2}{3}\div 7\frac{1}{2}$	7. $9\frac{4}{5}\div\frac{7}{10}$
2. $9\frac{9}{10}\div 3\frac{2}{3}$	5. $12\frac{1}{2}\div\frac{5}{8}$	8. $11\frac{4}{7}\div 4\frac{1}{2}$
3. $8\frac{1}{3}\div 6\frac{1}{4}$	6. $8\frac{2}{5}\div\frac{3}{7}$	9. $6\frac{2}{3}\div 4\frac{1}{7}$

57. **Dividing a Mixed Number by an Integer.** If the divisor is an integer we may proceed as in the following:

Example Divide $27\frac{2}{3}$ by 4.

$27\div 4=6$, remainder 3.
$3\frac{2}{3}\div 4=\frac{11}{3}\div 4=\frac{11}{12}$.
Hence, $27\frac{2}{3}\div 4=6\frac{11}{12}$.

4 is contained 6 times in 27, with remainder 3. Then divide $3\frac{2}{3}$ by 4.

In this manner obtain the results in the following:

1. $36\frac{2}{3}\div 5$	5. $28\frac{2}{3}\div 6$	9. $56\frac{4}{5}\div 6$
2. $67\frac{1}{2}\div 9$	6. $43\frac{5}{6}\div 5$	10. $71\frac{3}{16}\div 8$
3. $76\frac{2}{3}\div 8$	7. $71\frac{1}{4}\div 6$	11. $45\frac{7}{8}\div 9$
4. $84\frac{3}{7}\div 9$	8. $35\frac{3}{5}\div 8$	12. $59\frac{5}{8}\div 7$

58. Division of Fractions Related to Multiplication. By the definition of division, dividing 12 by 4 is the same as finding the missing number in $4 \times ? = 12$.

Similarly, to divide 6 by $\frac{1}{3}$ is the same as to find the missing number in $\frac{1}{3}$ of $? = 6$, or $\frac{1}{3} \times ? = 6$.

The problem in $\frac{1}{3} \times ? = 6$ is this: Find a number such that $\frac{1}{3}$ of (or times) it equals 6.

To divide 12 by $\frac{2}{3}$ is the same as to find the missing number in $\frac{2}{3} \times ? = 12$.

The problem stated in words is: "Find a number such that $\frac{2}{3}$ of (or times) it equals 12."

This problem may also be stated: "12 is $\frac{2}{3}$ of what number?"

To divide $\frac{2}{3}$ by $\frac{3}{4}$ is the same as finding the missing number in $\frac{3}{4} \times ? = \frac{2}{3}$.

The problem stated in words is: "Find a number such that $\frac{3}{4}$ of (or times) it equals $\frac{2}{3}$."

This problem may also be stated: "$\frac{2}{3}$ is $\frac{3}{4}$ of what number?"

WRITTEN EXERCISES

Find the missing numbers in the following, and state each problem in words:

1. $\frac{1}{3} \times ? = \frac{5}{6}$
2. $\frac{3}{4} \times ? = \frac{3}{7}$
3. $\frac{1}{5} \times ? = \frac{2}{3}$
4. $\frac{3}{4} \times ? = \frac{7}{8}$
5. $\frac{3}{7} \times ? = 12$
6. $\frac{5}{8} \times ? = 10$
7. $\frac{5}{6} \times ? = \frac{5}{8}$
8. $\frac{2}{7} \times ? = \frac{3}{5}$
9. $\frac{5}{8} \times ? = \frac{7}{8}$
10. $\frac{3}{5} \times ? = 3\frac{1}{2}$
11. $1\frac{1}{2} \times ? = 3\frac{3}{4}$
12. $4\frac{3}{4} \times ? = 7\frac{1}{2}$

Drill in Fundamentals. Play game No. 3, page 30. Use examples in multiplication of integers for the first event, and examples in long division for the second event.

59. **The Three Problems in Fractions.** By leaving out in succession each of the numbers in $3\times4=12$, we obtain three distinct problems which are solved by supplying the missing numbers in the following:

(1) $3\times4=?$ (2) $3\times?=12$ (3) $?\times4=12$

Precisely similar problems occur in fractions.

Thus, from $\frac{2}{3}\times\frac{5}{7}=\frac{10}{21}$ we obtain three problems which are solved by finding the missing number in

(1) $\frac{2}{3}\times\frac{5}{7}=?$ (2) $\frac{2}{3}\times?=\frac{10}{21}$ (3) $?\times\frac{5}{7}=\frac{10}{21}$

It is of fundamental importance that we understand the simplicity of these problems.

$3\times?=12$	(I) $\frac{2}{3}\times?=\frac{10}{21}$
Therefore $?=12\div3=4$	Therefore $?=\frac{10}{21}\div\frac{2}{3}=\frac{5}{7}$
$?\times4=12$	(II) $?\times\frac{5}{7}=\frac{10}{21}$
Therefore $?=12\div4=3$	Therefore $?=\frac{10}{21}\div\frac{5}{7}=\frac{2}{3}$

We now recall that $\frac{2}{3}\times\frac{5}{7}$ means the same as $\frac{2}{3}$ of $\frac{7}{5}$. Hence problems I and II may be read as follows:

(I) $\frac{2}{3}$ of what number equals $\frac{10}{21}$? or $\frac{10}{21}$ is $\frac{2}{3}$ of what number?

(II) What fraction (or part) of $\frac{5}{7}$ equals $\frac{10}{21}$? or $\frac{10}{21}$ is what fraction (or part) of $\frac{5}{7}$?

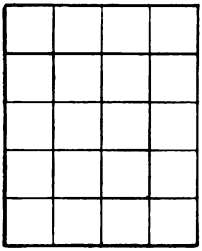
Sometimes the form of the statement of one of these problems as it turns up in practice may be confusing. A careful study of the statements given above will be of great help.

If you know how to add, subtract, multiply, and divide fractions, and if you understand thoroughly pages 76 and 77 of this book, you will seldom meet with a problem in fractions that you can not solve.

WRITTEN EXERCISES

Make examples like (I) and (II) above and solve them.

ORAL EXERCISES



1. Point out $\frac{1}{4}$ of this figure, $\frac{1}{5}$ of it, $\frac{1}{2}$ of it, $\frac{3}{4}$ of it, $\frac{2}{5}$ of it, $\frac{3}{5}$ of it, $\frac{4}{5}$ of it, $\frac{1}{10}$ of it, $\frac{7}{10}$ of it, $\frac{9}{10}$ of it. How many 20ths of the figure are there in each.

2. Point out the sum of $\frac{1}{4}$ and $\frac{1}{5}$. This sum is what fractional part of the figure?

3. Point out the sum of $\frac{1}{2}$ and $\frac{1}{5}$. This sum is what fractional part of the figure?

Read the following and give the sum of each:

4. $\frac{1}{2}+\frac{1}{4}$	19. $\frac{1}{2}+\frac{3}{16}$	34. $\frac{3}{4}+\frac{1}{16}$	49. $\frac{3}{8}+\frac{7}{16}$
5. $\frac{1}{2}+\frac{3}{4}$	20. $\frac{1}{2}+\frac{5}{16}$	35. $\frac{3}{4}+\frac{3}{16}$	50. $\frac{3}{8}+\frac{9}{16}$
6. $\frac{1}{2}+\frac{1}{8}$	21. $\frac{1}{2}+\frac{7}{16}$	36. $\frac{3}{4}+\frac{5}{16}$	51. $\frac{3}{8}+\frac{11}{16}$
7. $\frac{1}{2}+\frac{3}{8}$	22. $\frac{1}{2}+\frac{9}{16}$	37. $\frac{3}{4}+\frac{7}{16}$	52. $\frac{3}{8}+\frac{13}{16}$
8. $\frac{1}{2}+\frac{5}{8}$	23. $\frac{1}{2}+\frac{11}{16}$	38. $\frac{1}{8}+\frac{1}{16}$	53. $\frac{3}{8}+\frac{15}{16}$
9. $\frac{1}{2}+\frac{7}{8}$	24. $\frac{1}{2}+\frac{13}{16}$	39. $\frac{1}{8}+\frac{3}{16}$	54. $\frac{5}{8}+\frac{1}{16}$
10. $\frac{1}{4}+\frac{1}{8}$	25. $\frac{1}{2}+\frac{15}{16}$	40. $\frac{1}{8}+\frac{5}{16}$	55. $\frac{5}{8}+\frac{3}{16}$
11. $\frac{1}{4}+\frac{3}{8}$	26. $\frac{1}{4}+\frac{1}{16}$	41. $\frac{1}{8}+\frac{7}{16}$	56. $\frac{5}{8}+\frac{5}{16}$
12. $\frac{1}{4}+\frac{5}{8}$	27. $\frac{1}{4}+\frac{3}{16}$	42. $\frac{1}{8}+\frac{9}{16}$	57. $\frac{5}{8}+\frac{7}{16}$
13. $\frac{1}{4}+\frac{7}{8}$	28. $\frac{1}{4}+\frac{5}{16}$	43. $\frac{1}{8}+\frac{11}{16}$	58. $\frac{5}{8}+\frac{9}{16}$
14. $\frac{3}{4}+\frac{1}{8}$	29. $\frac{1}{4}+\frac{7}{16}$	44. $\frac{1}{8}+\frac{13}{16}$	59. $\frac{5}{8}+\frac{11}{16}$
15. $\frac{3}{4}+\frac{3}{8}$	30. $\frac{1}{4}+\frac{9}{16}$	45. $\frac{1}{8}+\frac{15}{16}$	60. $\frac{5}{8}+\frac{13}{16}$
16. $\frac{3}{4}+\frac{5}{8}$	31. $\frac{1}{4}+\frac{11}{16}$	46. $\frac{3}{8}+\frac{1}{16}$	61. $\frac{5}{8}+\frac{15}{16}$
17. $\frac{3}{4}+\frac{7}{8}$	32. $\frac{1}{4}+\frac{13}{16}$	47. $\frac{3}{8}+\frac{3}{16}$	62. $\frac{7}{8}+\frac{3}{16}$
18. $\frac{1}{2}+\frac{1}{16}$	33. $\frac{1}{4}+\frac{15}{16}$	48. $\frac{3}{8}+\frac{5}{16}$	63. $\frac{7}{8}+\frac{5}{16}$

64. State which of the two fractions in each of the above is the smaller, and subtract the smaller from the larger.

WRITTEN EXERCISES

Multiply:

1. $7\frac{3}{4}\times 8\frac{1}{3}$
2. $9\frac{7}{12}\times\frac{13}{24}$
3. $12\frac{4}{9}\times\frac{15}{16}$
4. $11\frac{3}{5}\times 9\frac{4}{9}$
5. $12\frac{3}{10}\times 6\frac{3}{8}$
6. $15\frac{3}{4}\times 7\frac{1}{9}$
7. $8\frac{2}{3}\times 1\frac{2}{1}$
8. $12\frac{1}{7}\times 9\frac{4}{5}$
9. $5\frac{3}{8}\times 2\frac{4}{5}$
10. $7\frac{5}{6}\times 3\frac{3}{5}$
11. $10\frac{2}{3}\times 6\frac{1}{4}$
12. $8\frac{5}{8}\times 2\frac{3}{4}$

Divide:

13. $8\frac{5}{9}\div\frac{3}{11}$
14. $12\frac{3}{5}\div\frac{9}{10}$
15. $15\frac{5}{6}\div 6\frac{1}{3}$
16. $9\frac{3}{5}\div 3\frac{1}{5}$
17. $16\frac{2}{3}\div 12\frac{1}{2}$
18. $9\frac{3}{8}\div 6\frac{1}{4}$
19. $10\frac{2}{7}\div 3\frac{3}{7}$
20. $8\frac{5}{9}\div 5\frac{1}{2}$
21. $21\frac{1}{2}\div 3\frac{3}{4}$
22. $32\frac{2}{3}\div 6\frac{4}{5}$
23. $52\frac{1}{2}\div 4\frac{1}{4}$
24. $17\frac{3}{8}\div 3\frac{1}{3}$

Find the results of the following:

25. $9\frac{3}{4}\times 7\frac{2}{13}$
26. $20\frac{1}{2}\div 5\frac{6}{7}$
27. $10\frac{5}{6}\div 6\frac{1}{2}$
28. $16\frac{2}{3}\times 4\frac{1}{5}$
29. $15\frac{3}{4}\div 4\frac{1}{2}$
30. $12\frac{3}{5}\times 5\frac{5}{9}$
31. $25\frac{3}{4}\times 12\frac{1}{2}$
32. $21\frac{3}{5}\times 4\frac{4}{9}$
33. $17\frac{3}{5}\div 5\frac{1}{2}$
34. $20\frac{1}{4}\div 3\frac{1}{9}$
35. $18\frac{4}{5}\div 12\frac{1}{2}$
36. $20\frac{1}{4}\div 27$
37. $17\frac{3}{8}\times 3\frac{1}{3}$
38. $12\frac{1}{7}\div 9\frac{4}{5}$
39. $21\frac{3}{5}\div 4\frac{4}{9}$
40. $20\frac{1}{2}\times 5\frac{7}{6}$
41. $27\frac{1}{3}\div 3\frac{1}{9}$
42. $81\frac{1}{2}\div 8\frac{2}{3}$

Since a division may be indicated by a fraction, as in $7\div 8=\frac{7}{8}$, we have $1\frac{1}{3}\div 2\frac{1}{2}=\dfrac{1\frac{1}{3}}{2\frac{1}{2}}$

Find the value of each of the following:

43. $\dfrac{4\frac{3}{4}}{2\frac{3}{8}}$
44. $\dfrac{6\frac{1}{2}}{4\frac{1}{4}}$
45. $\dfrac{4\frac{3}{8}}{9\frac{1}{4}}$
46. $\dfrac{24}{6\frac{1}{3}}$
47. $\dfrac{4\frac{1}{6}}{7\frac{1}{4}}$
48. $\dfrac{8\frac{3}{16}}{2\frac{1}{8}}$
49. $\dfrac{7\frac{2}{3}}{4\frac{1}{4}}$
50. $\dfrac{16\frac{3}{5}}{5\frac{1}{2}}$

Review pages 28, 29. Walking at an average rate of $3\frac{1}{4}$ miles an hour, how far does a man go in 4 hours?

If you know the average speed and the time, how do you find the distance passed over?

If you know the distance an object has gone and the time it required, how do you find the average speed? State these rules carefully.

WRITTEN PROBLEMS

1. At an average speed of $46\frac{1}{2}$ miles an hour, how far will a train go in 3 hours and 45 minutes? (45 minutes is what fractional part of an hour?)

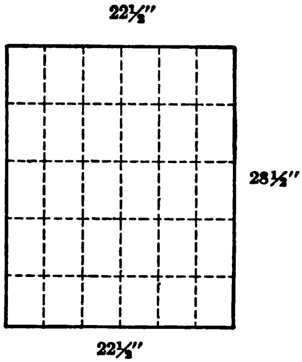
2. In July 1904 a train on the great Western Railway (England) travelled 1 hour and 24 minutes at the rate $84\frac{3}{5}$ miles per hour. How far did it go? (This is the highest speed at which a train has ever gone over 100 miles.)

Following are some fast runs of passenger trains for long distances. Find each speed in miles per hour. First reduce the time to hours and fractions of an hour.

Railroad	Terminals	Distance in miles	Time in hours and minutes
3. London & Northwestern	London Aberdeen	540	8h. 32m.
4. New York Central	New York Buffalo	$436\frac{1}{2}$	6h. 37m.
5. Lake Shore & Michigan Southern	Buffalo Chicago	525	7h. 50m.
6. Pennsylvania	Altoona Philadelphia	235	3h. 29m.

WRITTEN WORK

1. Souvenir postcards are cut from stock $22\frac{1}{2}''$ by $28\frac{1}{2}''$. If cards are cut from this stock as indicated in the figure, what will be the dimensions of the cards?



2. How many cubic feet are there in an excavation 24 feet wide, 36 feet long and 5 feet deep? How many cubic yards are there in this excavation?

3. Find the number of cubic yards in the excavation in example 2 by first reducing each dimension to yards. 5 feet are how many yards?

4. If an excavation contains 480 cubic yards, how many loads will it contain at 36 cubic feet to the load? (Note that 36 cubic feet $= 1\frac{1}{3}$ cubic yards.)

5. An excavation is 30 feet wide, 54 feet long and 7 feet deep. How many cubic yards does it contain?

6. A box is 18 inches long, 16 inches wide and 8 inches deep. How many cubic feet does it hold?

(*Suggestion:* First reduce 18 inches, 16 inches, and 8 inches to feet and fractions of feet.)

7. In a certain screw (bolt) there are $5\frac{1}{2}$ threads to the inch. What is the pitch of this thread? (*Suggestion:* Divide 1 by $5\frac{1}{2}$. See page 62.)

8. If the pitch of a certain screw is $\frac{1}{18}''$, how many times must the screw be turned around to make it enter into the wood its full length, which is $2\frac{3}{4}''$?

Drill in Fundamentals. Play game No. 3, page 30. Use examples in adding integers for each of two events.

CHAPTER II

60. Quotients to the Nearest Integer. Sometimes it is desired to get a quotient to the *nearest* integer, and to omit any reference to the remainder.

Example 1. Find the quotient of $388 \div 24$ to the nearest integer.

$$\begin{array}{r} 16 \\ 24\overline{)388} \\ \underline{24} \\ 148 \\ \underline{144} \\ 4 \end{array}$$

The exact quotient is $16\frac{4}{24} = 16\frac{1}{6}$. Since the fractional part of the quotient is less than $\frac{1}{2}$, the quotient is nearer 16 than 17. Hence 16 is the quotient to the nearest integer.

Example 2. Find the quotient of $574 \div 17$ to the nearest integer.

$$\begin{array}{r} 4 \\ 33 \\ 17\overline{)574} \\ \underline{51} \\ 64 \\ \underline{51} \\ 13 \end{array}$$

The exact quotient is $33\frac{13}{17}$. Since the fractional part of the quotient is greater than $\frac{1}{2}$, the quotient is nearer 34 than 33. Hence 34 is the quotient to the nearest integer

To find the quotient to the nearest integer, the following rule is used:

If the remainder is less than one-half the divisor, it is disregarded. If the remainder is equal to or greater than one-half the divisor one is added to the quotient.

WRITTEN EXERCISES

Find quotients to the nearest integer:

1. $346 \div 23$	6. $378 \div 8$	11. $948 \div 27$
2. $742 \div 18$	7. $946 \div 31$	12. $894 \div 62$
3. $498 \div 17$	8. $478 \div 62$	13. $798 \div 94$
4. $642 \div 13$	9. $482 \div 24$	14. $327 \div 71$
5. $798 \div 14$	10. $678 \div 35$	15. $698 \div 15$

WRITTEN EXERCISES

Divide, finding each quotient to the nearest integer:

1. $8945 \div 76$	6. $17806 \div 124$	11. $24906 \div 203$
2. $7896 \div 67$	7. $18745 \div 102$	12. $24089 \div 235$
3. $9104 \div 83$	8. $19608 \div 163$	13. $36071 \div 287$
4. $8759 \div 58$	9. $16576 \div 178$	14. $38047 \div 314$
5. $7068 \div 69$	10. $16075 \div 106$	15. $81051 \div 291$

The following rule is often useful in shortening division:

A common factor in the dividend and divisor may be cancelled without changing the quotient.

Example. Find the quotient of $6540000 \div 34000$ to the nearest integer.

$$\begin{array}{r} 192\phantom{\cancel{000}} \\ 34\cancel{000})\overline{6540\cancel{000}} \\ \underline{34}\phantom{40\cancel{000}} \\ 314\phantom{0\cancel{000}} \\ \underline{306}\phantom{0\cancel{000}} \\ 80\phantom{\cancel{000}} \\ \underline{68}\phantom{\cancel{000}} \\ 12\phantom{\cancel{000}} \end{array}$$

First divide both dividend and divisor by 1000 by striking off three zeros from each. Then divide 6540 by 34.

WRITTEN EXERCISES

Find quotient to the nearest integer:

1. $4890000 \div 371000$	7. $98400000 \div 6420000$
2. $5190000 \div 298000$	8. $89700000 \div 5960000$
3. $6780000 \div 427000$	9. $79800000 \div 6350000$
4. $5670000 \div 386000$	10. $97800000 \div 7810000$
5. $7610000 \div 597000$	11. $87500000 \div 6940000$
6. $8180000 \div 386000$	12. $97800000 \div 5980000$

61. Examples Involving Cancellation. Any problem in division may be expressed as a fraction. The fraction may then be simplified by cancellation. This is especially useful when the dividend or divisor or both are expressed as products.

Example 1. Divide 4×6 by 2.

Solution:

$$\frac{\overset{2}{\cancel{4}} \times 6}{\cancel{2}} = 12$$

Example 2. Divide $3 \times 4 \times 8$ by 18.

Solution:

$$\frac{\cancel{3} \times \overset{2}{\cancel{4}} \times 8}{\underset{3}{\cancel{\underset{\cancel{6}}{\cancel{18}}}}} = \frac{2 \times 8}{3} = \frac{16}{3} = 5\tfrac{1}{3}$$

We first divide both terms of the fraction by 3 and then by 2.

Notice that dividing one factor by a number divides the whole product by that number.

Thus, in $3 \times 4 \times 8$, dividing 3 by 3 divides the whole product by 3.

WRITTEN EXERCISES

Solve the following like Examples 1 and 2 above:

1. $\frac{5 \times 6 \times 7}{6}$
2. $\frac{7 \times 8 \times 5}{4}$
3. $\frac{3 \times 4 \times 10}{5}$
4. $\frac{3 \times 5 \times 14}{7}$
5. $\frac{5 \times 3 \times 21}{7}$
6. $\frac{16 \times 3 \times 5}{8}$
7. $\frac{5 \times 18 \times 6}{9}$
8. $\frac{3 \times 27 \times 5}{9}$
9. $\frac{32 \times 5 \times 4}{8}$
10. $\frac{4 \times 12 \times 10}{60}$
11. $\frac{6 \times 14 \times 18}{36}$
12. $\frac{9 \times 10 \times 8}{54}$
13. $\frac{3 \times 4 \times 6}{18}$
14. $\frac{14 \times 16 \times 6}{42}$
15. $\frac{4 \times 9 \times 12 \times 28}{140}$

Example. Divide $24\times35\times11$ by $6\times7\times8$.

$$\frac{\overset{4}{\cancel{24}}\times\overset{5}{\cancel{35}}\times 11}{\cancel{6}\times\cancel{7}\times\underset{2}{\cancel{8}}}=\frac{5\times11}{2}=\frac{55}{2}=27\tfrac{1}{2}$$

First cancel the common factor 6 in 24 and 6. Then cancel 4 in 8 and 4, and finally cancel 7 in 35 and 7. This leaves 5×11 in the dividend and 2 in the divisor.

Also solve without cancelling. Which method is shorter?

WRITTEN EXERCISES

In this manner obtain the results of the following by cancelling:

1. $\dfrac{32\times58\times70}{10\times14\times8}$
2. $\dfrac{42\times39\times60}{13\times7\times10}$
3. $\dfrac{36\times30\times54}{5\times12\times9}$
4. $\dfrac{54\times36\times75}{12\times6\times16}$
5. $\dfrac{72\times64\times85}{5\times24\times16}$
6. $\dfrac{81\times75\times90}{20\times9\times15}$
7. $\dfrac{25\times18\times12\times3}{27\times15\times9}$
8. $\dfrac{28\times16\times20\times7}{34\times14\times5\times21}$
9. $\dfrac{24\times20\times14\times4}{5\times12\times7\times16}$
10. $\dfrac{48\times25\times14\times9}{15\times7\times24\times6}$
11. $\dfrac{54\times22\times36\times8}{84\times18\times11\times12}$
12. $\dfrac{42\times63\times24\times11}{33\times14\times27\times8}$
13. $\dfrac{4\times7\times19\times36}{14\times38}$
14. $\dfrac{5\times9\times13\times42}{26\times21}$
15. $\dfrac{15\times6\times9\times39}{30\times52\times18}$
16. $\dfrac{9\times8\times23\times15}{45\times46\times18\times16}$
17. $\dfrac{13\times18\times40\times9}{80\times52\times45\times27}$
18. $\dfrac{17\times19\times30\times11}{57\times85\times44\times16}$
19. $\dfrac{8\times12\times18}{16\times24\times6}$
20. $\dfrac{32\times26\times17}{48\times39\times14}$
21. $\dfrac{14\times18\times20}{35\times24\times8}$

Drill in Fundamentals. Play game No. 3, page 30. Use examples in subtraction of integers for the first event, examples in multiplication of integers for the second, and examples in long division for the third.

Problem 1. How many cubic yards are there in an excavation 36 feet long, 32 feet wide and 6 feet deep?

You have solved this kind of problem before by multiplying 36 by 32, this product by 6, and then dividing by 27. We will now solve the same problem by cancellation.

We are to divide $36 \times 32 \times 6$ by 27.

$$\frac{\overset{4}{\cancel{36}} \times 32 \times \overset{2}{\cancel{6}}}{\underset{3}{\cancel{27}}} = 4 \times 32 \times 2 = 256,$$ which is the required number of cubic yards.

Problem 2. How many square feet are there in a rectangle 64 inches long and 45 inches wide?

The number of square inches, 64×45, is divided by the number of square inches in a square foot, or 12×12.

That is, $\frac{64 \times 45}{12 \times 12}$ is the required number of square feet.

Problem 3. How many cubic feet are there in a block of ice 15 inches thick, 54 inches wide, and 64 inches long.

Solution: We divide the number of cubic inches, $15 \times 54 \times 64$, in the block by the number of cubic inches in one cubic foot, or $12 \times 12 \times 12$.

That is, $\frac{15 \times 54 \times 64}{12 \times 12 \times 12}$ is the required number of cubic feet.

Complete the work in these two problems.

WRITTEN EXERCISES

1. Find the number of square feet in rectangles having the following dimensions:

$28'' \times 46''$ $\quad$ $34'' \times 56''$ $\quad$ $18'' \times 78''$ $\quad$ $21'' \times 32''$

2. Find the number of cubic feet in rectangular solids having the following dimensions:

$16'' \times 18'' \times 48''$ $\quad$ $8'' \times 15'' \times 30''$ $\quad$ $15'' \times 21'' \times 27''$

If in $2\times3\times4=24$ we erase the 4 and write $2\times3\times?=24$ we have a problem in division. That is, we divide 24 by 2×3, or 6, and thus find the missing number.

If we erase the 3 and write $2\times?\times4=24$, we again have a problem in division. That is, we divide 24 by 2×4, or 8. Similarly, if we erase the 2 and write $?\times3\times4=24$ we also have a problem in division. That is, we divide 24 by 3×4, or 12.

Example 1. Find the missing number in $3\times8\times?=768$.

Solution: The missing number is $\dfrac{\overset{\overset{32}{\cancel{96}}}{\cancel{768}}}{\cancel{3}\times\cancel{8}}=32.$

Example 2. Find the missing number in $7\times13\times?=2870$.

Solution: The missing number $=\dfrac{\overset{410}{\cancel{2870}}}{\cancel{7}\times13}=\dfrac{410}{13}=31\frac{7}{13}$

In this case we can cancel the 7, but not the the 13. So we have to divide 410 by 13.

Example 3. Find the missing number in $13\times17\times?=27490$.

Here we can not cancel at all, so we must multiply 17 by 13, and then divide 27490 by the product.

WRITTEN EXERCISES

Find the missing numbers in each of the following, using cancellation where possible:

1. $3\times9\times?=378$	6. $25\times6\times?=7980$	11. $?\times26\times2=890$
2. $?\times5\times7=230$	7. $8\times4\times?=860$	12. $?\times32\times15=1560$
3. $12\times5\times?=924$	8. $12\times9\times?=1240$	13. $9\times?\times12=1728$
4. $7\times9\times?=462$	9. $18\times24\times?=1840$	14. $6\times?\times16=986$
5. $6\times?\times9=4536$	10. $16\times?\times18=2460$	15. $8\times6\times?=2860$

ORAL EXERCISES

1. How many inch cubes can you place in a box 5 inches long, 3 inches wide and 4 inches deep?

2. If you know the length, width and depth in inches of a rectangular box, how do you find out how many inch cubes can be placed in it?

The number of cubic inches which can be placed in a box is called its *cubic contents*, or *volume*, *measured in cubic inches.*

3. If you know the length, width and height of a room in feet how do you find how many cubic feet there are in it?

4. If you know the length, width and height of a room in yards, how do you find how many cubic yards there are in it?

We now understand the meaning of the statement:

$$\textit{width} \times \textit{length} \times \textit{height} = \textit{volume}.$$

If the volume is not known we have

$$(1)\quad \textit{length} \times \textit{width} \times \textit{height} = ?$$

which is a problem in multiplication.

If the length and width are known and also the volume, but not the height, then

$$(2)\quad \textit{length} \times \textit{width} \times ? = \textit{volume}.$$

This gives a problem in division like those on page 87.

When the width or the length is unknown we have

$$(3)\quad \textit{length} \times ? \times \textit{height} = \textit{volume}.$$

$$(4)\quad ? \times \textit{width} \times \textit{height} = \textit{volume}.$$

The missing numbers are found by division, as on page 87.

We must make it clear to ourselves that the volume of a rectangular box is the *product* of the length, width, and depth. But if a product is given and one of the factors, then the other factor is found by dividing. Hence, if the volume of a box is 180 cubic inches, and if the length and width are 10 inches and 6 inches, then 60 times the depth is 180, and the depth is $180 \div 60 = 3$.

ORAL AND WRITTEN EXERCISES

(Solve orally as many as possible.)

1. State a rule for finding the volume of a rectangular box when its length, width, and depth are known.

2. How many cubic inches does a box contain if it is 10 inches long, 6 inches wide and 2 inches high?

3. How many cubic inches does a box contain if it is 10 inches long, 4 inches wide and 3 inches deep?

4. Select the dimensions of a box in different ways so that its volume shall be 120 cubic inches.

5. If you know how long and how wide a box is and also how many cubic inches it holds, how can you find out how deep the box is?

6. A box is 16 inches long and 8 inches wide. How deep must it be to contain one cubic foot?

Suggestion: $16 \times 8 \times \text{depth} = 12 \times 12 \times 12$ cubic inches.

$$\text{Hence the depth} = \frac{12 \times 12 \times 12}{16 \times 8}$$

7. A piece of timber is 36 inches long and 8 inches wide. How thick is it if it contains 864 cubic inches?

8. How high must a wheat bin be to hold 720 cubic feet if it is 10 feet long and 8 feet wide?

Compare this with the problems on areas on page 24. Also see the examples on cancellation on page 84. Try to understand that the principle in all these problems is the same. A good student learns to understand how the same principle runs through many different looking problems. The poor student sees no such connection, and therefore needs a great many separate rules. These he is sure to forget, and then he is entirely at a loss. The student who understands how problems are related has fewer rules to remember, and so gets on more easily and more rapidly.

Problem 1. At 62 cents a bushel, what is the value of a load of corn weighing 1876 pounds, one bushel of corn weighing 56 pounds?

Operations:

$$\frac{\overset{\overset{67}{\cancel{469}}}{\cancel{1876}}}{\underset{\underset{\cancel{7}}{\cancel{14}}}{\cancel{56}}} \times \overset{31}{\cancel{62}} = 67 \times 31 = 2077$$

Hence value is 2077 cents, or $20.77

Analysis:

Weight of load divided by weight of one bushel equals number of bushels.

$$\frac{1876}{56} = \text{number of bushels.}$$

Number of bushels multiplied by price per bushel equals value of load.

$$\frac{1876}{56} \times 62 = \text{value of load in cents.}$$

Problem 2. At $12.40 a ton, what is the value of a load of hay weighing 2460 pounds?

To avoid difficulty in cancellation we write the value per ton in cents. Thus:

$\frac{2464}{2000}$ represents the number of tons, and $\frac{2464}{2000} \times 1240 =$ its value in *cents*.

Cancel and complete the solution.

At the University experimental station at Urbana, Illinois, numerous experiments have been made on the effectiveness of various kinds of feeds used for farm animals. On the next page are given accounts of the feeding of four lots of horses by different methods. The accounts are copied from one of their regular bulletins. Find the missing numbers as in the problems on this page. The problems to be solved are exactly those which the makers of the bulletin had to solve.

1 bushel of corn (shelled) weighs 56 pounds 1 bushel of oats weighs 32 pounds

WRITTEN EXERCISES

Lot I. Expenditure per horse.

1912 lbs. corn (shelled)	at $0.83 a bushel	....
30 lbs. oil meal	at $28.00 a ton	....
2075 lbs. clover hay	at $13.00 a ton	....
	Total cost of feed	....

Lot II. Expenditure per horse.

1600 lbs. corn	at $0.83 a bushel	....
533 lbs. oats	at $0.45 a bushel	....
35 lbs. oil meal	at $27.00 a ton	....
2162 lbs. clover hay	at $13.00 a ton	....
	Total cost of feed	....

Lot III. Expenditure per horse.

1077 lbs. corn	at $0.83 a bushel	....
1077 lbs. oats	at $0.45 a bushel	....
34 lbs. oil meal	at $27.00 a ton	....
2194 lbs. clover hay	at $13.00 a ton	....
	Total cost of feed	....

Lot IV. Expenditure per horse.

1808 lbs. corn	at $0.83 a bushel	....
352 lbs. bran	at $20.00 a ton	....
35 lbs. oil meal	at $27.00 a ton	....
2081 lbs. clover hay	at $13.00 a ton	....
	Total cost of feed	

Drill in Fundamentals. The teacher will read simple combinations, and you will write down the answers.

62. Items of a Bill. A bill is a statement rendered by a seller to the purchaser of goods. A bill should contain the following items:

1. Name and location of seller.
2. Name and location of buyer.
3. Date of sale and price per unit of each item.
4. Total cost of each item.
5. Terms of sale, such as cash, credit for 30 days, etc.

There are also other items which usually go on bills, such as extra charges for freight, cartage deductions in the way of discounts, etc.

A BILL

Chicago, Ill., May 28, 1917.

Mr. James Wolf,
346 55th Street, Chicago.
Bought of ALBERT K. JOHNSON & Co.,
Terms: Cash the first of each month.

10 bu. potatoes	$1.10	$11	00		
5 bbl. flour	8.30	41	50		
60 doz. oranges	0.30	18	00		
35 bu. apples	0.75	26	25	$96	75

The amounts given in the first column is the cost of one unit of each article, such as one bushel of wheat, or one barrel of flour. Sometimes the letter @ is written before these amounts to show that they indicate the price per unit.

The amounts given in the next column is the cost of the total amount of each article. The single numbers given in the third column is the sum of the second column, and shows the cost of all the articles on the bill.

63. Extending and Footing Bills. Multiplying the price by the number of articles of each kind to find the cost is called *extending the bill.* Adding the cost of the items to find the total is called *footing the bill.*

WRITTEN EXERCISES

1. Copy the bill on page 92, extend the items and foot the bill.

For each of the following make out a bill, extending the items and footing the bill:

2. May 23, 1917. Brown & Co., of New York, sold to Albert Walker of Princeton, New Jersey, 42 chests of green tea at \$31.60, 20 chests of black tea at \$24.60, 10 chests of uncolored Japan tea at \$51.60, 20 boxes of lemons at \$5.30, 30 boxes of oranges at \$3.40. Terms: Cash.

3. January 7th, 1918. J. M. Lucy & Sons of Janesville, Wisconsin, sold to Geo. S. Parker of Janesville, Wisconsin, 12 dining chairs at \$22.50, one dining table at \$85.50, 4 sections of bookcases at \$5.10, one library table at \$110, 4 sitting room chairs at \$50.00. Terms: Cash to first of the month.

4. June 10, 1918. Solomender Optical Co., of Cleveland, sold to East Technical High School, 40 Federal Ruling pens at \$.45, 30 Federal Bow pencils at \$.72, 25 Federal Bow pens at \$.76, 30 compasses at \$.65, 15 compasses at \$1.60. Terms: Cash in 30 days.

5. October 8, 1917. J. T. Black & Sons of Buffalo, N. Y., sold to S. Williams of Buffalo.

6 boxes shredded wheat	260 lbs.	at	\$0.02
6 boxes tea	1360 lbs.	at	0.37
8 bbls. sugar	1740 lbs.	at	0.08½
10 bbls. flour	1840 lbs.	at	0.04⅛

6. Walter Beck & Co. of Chicago, sold to John S. Smith of Decatur, Ill., 35 suits of men's clothes at \$15.50, 45 suits at \$17.50, 60 suits at \$20.00, 25 suits at \$25.00, and 20 suits at \$30.00.

7. Get some real bills, and extend and foot them. Also make up some bills, and extend and foot them.

ORAL EXERCISES

1. What is a fraction? Name a fraction and give its numerator and also its denominator.

2. What does the denominator of a fraction tell? What does its numerator tell?

3. What are the terms of a fraction?

4. How may a fraction be changed without changing its value?

5. What is a proper fraction? an improper fraction? a mixed number? Give examples of each.

6. What are like fractions? How may like fractions be added and subtracted?

7. In adding unlike fractions what is the first operation to be performed on them?

8. What is a factor of a number? Give a number and one of its factors.

9. What is a common factor of two numbers? Give two numbers having a common factor and also the common factor.

10. What is a multiple of a number? Give a number and several of its multiples.

11. What is the least common multiple of two numbers? Give two numbers and also their L. C. M.

12. How may a common denominator of two fractions be found?

13. What is cancellation? Name a fraction which may be simplified by cancellation.

14. State the rule for multiplying two fractions. Name two fractions and give their product.

15. State the rule for dividing by a fraction. Does this rule hold whether the dividend is a fraction or an integer?

PROBLEMS WITHOUT NUMBERS

1. If the product of two numbers is given, and also one of the numbers, how may the other number be found?

2. If the speed of a train is known, and also the time of running, how may the distance be found?

3. If the time and the distance are known, how may the speed be found?

4. If the speed and the distance are known, how may the time be found?

5. If the numerator of a fraction is multiplied by a whole number, how is the value of the fraction affected?

6. If the denominator of a fraction is multiplied by a whole number, how is the value of the fraction affected?

7. If the numerator of a fraction is increased, is the value of the fraction affected?

8. If the denominator of a fraction is increased, is the value of the fraction increased or decreased?

9. If you know the sum of two fractions, and also one of the fractions, how do you find the other fraction?

10. If a number is multiplied by a proper fraction, is the product greater or less than the number itself?

11. If a number is divided by a proper fraction, is the quotient greater or less than the number itself?

12. If any number (fraction or integer) is multiplied by a number greater than one, is the product greater or less than the given number?

13. If a number is divided by an improper fraction, is the quotient greater or less than the given number?

Review.

ORAL EXERCISES

Read the following and supply the missing numbers:

1.	$\frac{1}{2}=\frac{?}{6}$	$\frac{1}{2}=\frac{?}{10}$	$\frac{1}{2}=\frac{?}{8}$	$\frac{1}{2}=\frac{?}{16}$	$\frac{3}{4}=\frac{?}{24}$
2.	$\frac{1}{3}=\frac{?}{6}$	$\frac{1}{3}=\frac{?}{9}$	$\frac{1}{3}=\frac{?}{15}$	$\frac{1}{3}=\frac{?}{27}$	$\frac{5}{6}=\frac{?}{30}$
3.	$\frac{2}{3}=\frac{?}{6}$	$\frac{2}{3}=\frac{?}{9}$	$\frac{2}{3}=\frac{?}{15}$	$\frac{2}{3}=\frac{?}{27}$	$\frac{3}{7}=\frac{?}{21}$
4.	$\frac{3}{5}=\frac{?}{15}$	$\frac{3}{5}=\frac{?}{30}$	$\frac{3}{5}=\frac{?}{45}$	$\frac{3}{5}=\frac{?}{25}$	$\frac{4}{5}=\frac{24}{?}$
5.	$\frac{5}{6}=\frac{?}{18}$	$\frac{5}{6}=\frac{15}{?}$	$\frac{5}{6}=\frac{35}{?}$	$\frac{5}{6}=\frac{45}{?}$	$\frac{3}{7}=\frac{12}{?}$
6.	$\frac{5}{8}=\frac{?}{32}$	$\frac{5}{8}=\frac{15}{?}$	$\frac{5}{8}=\frac{35}{?}$	$\frac{5}{8}=\frac{50}{?}$	$\frac{2}{3}=\frac{12}{?}$
7.	$\frac{7}{16}=\frac{14}{?}$	$\frac{7}{16}=\frac{35}{?}$	$\frac{7}{16}=\frac{21}{?}$	$\frac{7}{16}=\frac{49}{?}$	$\frac{3}{8}=\frac{18}{?}$
8.	$\frac{15}{32}=\frac{45}{?}$	$\frac{15}{16}=\frac{75}{?}$	$\frac{15}{16}=\frac{90}{?}$	$\frac{15}{16}=\frac{60}{?}$	$\frac{5}{9}=\frac{15}{?}$

Example. Reduce $3\frac{2}{3}$ to an improper fraction.

Solution: $3\frac{2}{3}=\frac{9}{3}+\frac{2}{3}=\frac{11}{3}$.

Such examples should always be solved without using pencil and paper.

Reduce each of the following to an improper fraction:

9.	$1\frac{1}{2}$	$2\frac{1}{3}$	$1\frac{2}{3}$	$2\frac{1}{4}$	$3\frac{3}{4}$	$6\frac{1}{2}$	$6\frac{1}{3}$	$5\frac{3}{7}$
10.	$3\frac{4}{5}$	$2\frac{5}{6}$	$1\frac{1}{8}$	$2\frac{3}{8}$	$3\frac{3}{8}$	$4\frac{4}{8}$	$5\frac{5}{8}$	$7\frac{7}{8}$
11.	$3\frac{5}{8}$	$3\frac{3}{6}$	$8\frac{3}{5}$	$6\frac{5}{6}$	$5\frac{3}{8}$	$7\frac{2}{3}$	$8\frac{4}{5}$	$6\frac{3}{16}$

12. Reduce each of the following numbers to 12ths: 2, 3, 5, 6, 7.

13. Reduce each of the following numbers to 16ths: 1, 2, 3, 4, 5.

Reduce each of the following to a mixed number:

14.	$\frac{7}{2}$	$\frac{9}{4}$	$\frac{13}{5}$	$\frac{15}{8}$	$\frac{18}{4}$	$\frac{17}{16}$	$\frac{24}{7}$	$\frac{35}{8}$
15.	$\frac{15}{7}$	$\frac{25}{4}$	$\frac{12}{6}$	$\frac{29}{9}$	$\frac{34}{7}$	$\frac{31}{8}$	$\frac{38}{5}$	$\frac{49}{16}$

Drill in Fundamentals. The teacher will read combinations, and you will write the answers.

ORAL EXERCISES

Give each of the following indicated sums:

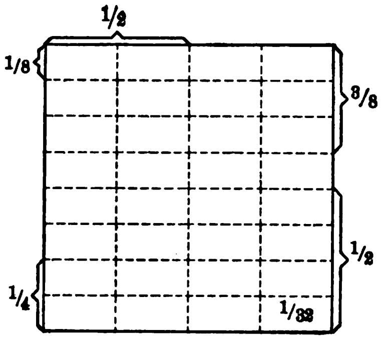
1.	$\frac{1}{3}+\frac{1}{3}$	$\frac{2}{3}+\frac{2}{3}$	$\frac{1}{2}+\frac{1}{4}$	$\frac{1}{2}+\frac{1}{8}$	$\frac{1}{2}+\frac{3}{8}$
2.	$\frac{1}{2}+\frac{5}{8}$	$\frac{1}{2}+\frac{7}{8}$	$\frac{1}{2}+\frac{3}{4}$	$\frac{1}{2}+\frac{1}{16}$	$\frac{1}{2}+\frac{3}{16}$
3.	$\frac{1}{2}+\frac{5}{16}$	$\frac{1}{2}+\frac{7}{16}$	$\frac{1}{2}+\frac{9}{16}$	$\frac{1}{2}+\frac{11}{16}$	$\frac{1}{2}+\frac{13}{16}$
4.	$\frac{1}{2}+\frac{15}{16}$	$\frac{1}{2}+\frac{1}{32}$	$\frac{1}{4}+\frac{1}{4}$	$\frac{1}{4}+\frac{1}{3}$	$\frac{1}{4}+\frac{1}{8}$
5.	$\frac{3}{4}+\frac{1}{3}$	$\frac{3}{4}+\frac{2}{3}$	$\frac{1}{4}+\frac{3}{8}$	$\frac{1}{4}+\frac{5}{8}$	$\frac{1}{4}+\frac{7}{8}$
6.	$\frac{3}{4}+\frac{1}{8}$	$\frac{3}{4}+\frac{3}{8}$	$\frac{3}{4}+\frac{5}{8}$	$\frac{3}{4}+\frac{7}{8}$	$\frac{1}{8}+\frac{3}{8}$
7.	$\frac{1}{4}+\frac{1}{16}$	$\frac{1}{4}+\frac{3}{16}$	$\frac{1}{4}+\frac{5}{16}$	$\frac{1}{4}+\frac{7}{16}$	$\frac{1}{4}+\frac{9}{16}$
8.	$\frac{1}{4}+\frac{11}{16}$	$\frac{1}{4}+\frac{13}{16}$	$\frac{1}{4}+\frac{15}{16}$	$\frac{3}{4}+\frac{1}{16}$	$\frac{3}{4}+\frac{3}{16}$
9.	$\frac{3}{4}+\frac{5}{16}$	$\frac{3}{4}+\frac{7}{16}$	$\frac{3}{4}+\frac{9}{16}$	$\frac{3}{4}+\frac{11}{16}$	$\frac{3}{4}+\frac{13}{16}$
10.	$\frac{3}{4}+\frac{15}{16}$	$\frac{3}{8}+\frac{1}{16}$	$\frac{1}{8}+\frac{1}{16}$	$\frac{1}{8}+\frac{3}{16}$	$\frac{3}{8}+\frac{3}{16}$
11.	$\frac{3}{8}+\frac{5}{16}$	$\frac{1}{8}+\frac{5}{16}$	$\frac{3}{8}+\frac{7}{16}$	$\frac{1}{8}+\frac{7}{16}$	$\frac{1}{8}+\frac{9}{16}$
12.	$\frac{3}{8}+\frac{9}{16}$	$\frac{3}{8}+\frac{11}{16}$	$\frac{1}{8}+\frac{11}{16}$	$\frac{5}{8}+\frac{3}{16}$	$\frac{7}{8}+\frac{5}{16}$
13.	$\frac{1}{3}+\frac{1}{4}$	$\frac{2}{3}+\frac{1}{4}$	$\frac{1}{3}+\frac{3}{4}$	$\frac{2}{3}+\frac{3}{4}$	$\frac{1}{3}+\frac{3}{8}$

Give each of the following indicated differences:

14.	$\frac{1}{2}-\frac{1}{4}$	$\frac{1}{2}-\frac{1}{8}$	$\frac{1}{2}-\frac{3}{8}$	$\frac{1}{2}-\frac{1}{16}$	$\frac{1}{2}-\frac{3}{16}$
15.	$\frac{1}{2}-\frac{5}{16}$	$\frac{1}{2}-\frac{7}{16}$	$\frac{1}{2}-\frac{1}{32}$	$\frac{1}{2}-\frac{3}{32}$	$\frac{1}{2}-\frac{5}{32}$
16.	$\frac{1}{2}-\frac{7}{32}$	$\frac{1}{2}-\frac{9}{32}$	$\frac{1}{2}-\frac{11}{32}$	$\frac{1}{2}-\frac{13}{32}$	$\frac{1}{2}-\frac{15}{32}$
17.	$\frac{1}{4}-\frac{1}{6}$	$\frac{1}{4}-\frac{1}{16}$	$\frac{1}{4}-\frac{3}{16}$	$\frac{3}{4}-\frac{3}{8}$	$\frac{3}{4}-\frac{5}{8}$
18.	$\frac{3}{4}-\frac{1}{16}$	$\frac{3}{4}-\frac{3}{16}$	$\frac{3}{4}-\frac{5}{16}$	$\frac{3}{4}-\frac{7}{16}$	$\frac{3}{4}-\frac{9}{16}$
19.	$\frac{3}{4}-\frac{11}{16}$	$\frac{5}{8}-\frac{1}{16}$	$\frac{5}{8}-\frac{5}{16}$	$\frac{7}{8}-\frac{13}{16}$	$\frac{7}{8}-\frac{11}{16}$

Drill in Fundamentals. Play game No. 4, page 30. Use examples in adding fractions.

ORAL EXERCISES



1. In this figure, point out $\frac{1}{2}$ of it, $\frac{1}{4}$ of it, $\frac{3}{4}$ of it, $\frac{1}{8}$ of it, $\frac{3}{8}$ of it, $\frac{5}{8}$ of it, $\frac{7}{8}$ of it, $\frac{1}{16}$ of it, $\frac{5}{16}$ of it, $\frac{9}{16}$ of it, $\frac{15}{16}$ of it. How many 32ds of the figure are there in each of these?
2. Point out $\frac{1}{2}$ of $\frac{1}{4}$, or $\frac{1}{2}\times\frac{1}{4}$. What is the product?
3. Point out $\frac{1}{2}$ of $\frac{1}{8}$. What is the product?
4. Point out $\frac{1}{2}$ of $\frac{1}{16}$. What is the product?
5. Point out $\frac{1}{2}$ of $\frac{3}{4}$. What is the product?
6. Point out $\frac{1}{2}$ of $\frac{1}{2}$. What is the product?
7. Point out $\frac{1}{2}$ of $\frac{3}{8}$. What is the product?
8. Point out $\frac{1}{4}$ of $\frac{1}{2}$. What is the product?

Read the following and give the product in each case, remembering that $\frac{1}{4}\times\frac{1}{2}$ means the same as $\frac{1}{4}$ of $\frac{1}{2}$:

9. $\frac{1}{2}\times\frac{1}{2}$	16. $\frac{3}{4}\times\frac{1}{4}$	23. $\frac{1}{2}\times\frac{3}{8}$	30. $\frac{2}{3}\times\frac{1}{4}$
10. $\frac{1}{2}\times\frac{1}{4}$	17. $\frac{3}{4}\times\frac{5}{8}$	24. $\frac{1}{2}\times\frac{5}{16}$	31. $\frac{1}{3}\times\frac{3}{4}$
11. $\frac{1}{2}\times\frac{1}{8}$	18. $\frac{3}{4}\times\frac{7}{8}$	25. $\frac{1}{2}\times\frac{13}{16}$	32. $\frac{2}{3}\times\frac{3}{4}$
12. $\frac{1}{2}\times\frac{1}{16}$	19. $\frac{1}{2}\times\frac{3}{4}$	26. $\frac{1}{2}\times\frac{15}{16}$	33. $\frac{1}{4}\times\frac{1}{3}$
13. $\frac{1}{4}\times\frac{1}{2}$	20. $\frac{1}{2}\times\frac{3}{8}$	27. $\frac{1}{2}\times\frac{1}{3}$	34. $\frac{3}{4}\times\frac{1}{2}$
14. $\frac{1}{4}\times\frac{1}{4}$	21. $\frac{1}{2}\times\frac{5}{8}$	28. $\frac{1}{3}\times\frac{2}{3}$	35. $\frac{3}{8}$ of $\frac{3}{4}$
15. $\frac{1}{4}\times\frac{1}{8}$	22. $\frac{1}{2}\times\frac{7}{8}$	29. $\frac{1}{3}\times\frac{1}{4}$	36. $\frac{5}{8}$ of $\frac{1}{4}$

ORAL AND WRITTEN EXERCISES

On this page solve all the examples you can without the aid of pencil and paper.

1.	$\frac{3}{4}\times\frac{2}{5}$	$\frac{2}{5}\times\frac{5}{8}$	$\frac{9}{16}\times\frac{10}{21}$	$\frac{8}{9}\times\frac{7}{12}$
2.	$\frac{7}{8}\times\frac{4}{5}$	$\frac{5}{12}\times\frac{3}{20}$	$\frac{3}{4}\times\frac{1}{6}$	$\frac{8}{15}\times\frac{12}{16}$
3.	$\frac{5}{16}\times\frac{3}{5}$	$\frac{8}{9}\times\frac{3}{16}$	$\frac{7}{8}\times\frac{3}{4}$	$\frac{9}{16}\times\frac{3}{8}$

Give the product of each of the following:

4.	$2\times\frac{2}{3}$	$1\times\frac{1}{4}$	$2\times\frac{3}{4}$	$2\times\frac{2}{5}$
5.	$3\times\frac{1}{3}$	$3\times\frac{2}{3}$	$3\times\frac{3}{4}$	$3\times\frac{3}{8}$
6.	$4\times\frac{1}{3}$	$4\times\frac{2}{3}$	$4\times\frac{1}{4}$	$4\times\frac{3}{4}$
7.	$4\times\frac{7}{8}$	$4\times\frac{3}{8}$	$4\times\frac{1}{8}$	$4\times\frac{5}{16}$

Multiply each of the following without first reducing the multiplicand to an improper fraction:

8.	$2\times1\frac{1}{3}$	$2\times2\frac{1}{4}$	$2\times2\frac{3}{4}$	$2\times2\frac{1}{8}$
9.	$3\times2\frac{1}{2}$	$3\times2\frac{1}{4}$	$3\times2\frac{3}{4}$	$3\times3\frac{1}{2}$
10.	$4\times1\frac{1}{2}$	$4\times2\frac{1}{3}$	$4\times3\frac{1}{4}$	$4\times2\frac{3}{4}$
11.	$8\times3\frac{2}{3}$	$7\times5\frac{4}{5}$	$9\times4\frac{7}{8}$	$8\times4\frac{3}{5}$

In making the following multiplications remember that the multiplier and multiplicand may be interchanged without changing the product. If one of these is a whole number, always use that one as the multiplier.

12.	$1\frac{1}{2}\times3$	$2\frac{1}{2}\times3$	$2\frac{3}{8}\times3$	$5\frac{1}{4}\times3$
13.	$2\frac{1}{3}\times6$	$8\frac{1}{2}\times6$	$6\frac{3}{4}\times6$	$5\frac{3}{8}\times6$
14.	$3\frac{9}{16}\times8$	$1\frac{5}{8}\times10$	$3\frac{3}{4}\times7$	$\frac{7}{16}\times9$
15.	$9\times1\frac{3}{8}$	$12\times1\frac{1}{4}$	$1\frac{5}{16}\times10$	$2\frac{1}{4}\times12$

Drill in Fundamentals. Play game No. 4 on page 30. Use examples in multiplication of fractions.

ORAL AND WRITTEN EXERCISES

(Solve as many as possible without using pencil and paper.)

Give the products of the following:

1.	$\frac{3}{4}\times\frac{3}{4}$	$\frac{3}{4}\times\frac{1}{8}$	$\frac{1}{2}\times\frac{9}{16}$	$\frac{3}{5}\times\frac{1}{5}$	$\frac{5}{6}\times\frac{3}{8}$
2.	$\frac{3}{4}\times\frac{5}{8}$	$\frac{4}{5}\times\frac{7}{8}$	$\frac{2}{3}\times\frac{3}{8}$	$\frac{2}{3}\times\frac{5}{8}$	$\frac{3}{5}\times\frac{7}{8}$
3.	$\frac{3}{4}\times\frac{4}{5}$	$\frac{2}{5}\times\frac{5}{8}$	$\frac{3}{8}\times\frac{4}{5}$	$\frac{3}{16}\times\frac{5}{8}$	$\frac{5}{8}\times\frac{3}{4}$
4.	$\frac{5}{7}\times\frac{3}{4}$	$\frac{2}{7}\times\frac{6}{7}$	$\frac{4}{5}\times\frac{15}{16}$	$\frac{7}{8}\times\frac{3}{4}$	$\frac{7}{8}\times\frac{4}{5}$

Before multiplying the following, change both multiplier and multiplicand to improper fractions or use the four-step method (see page 70):

5.	$1\frac{1}{2}\times1\frac{1}{2}$	$1\frac{1}{2}\times2\frac{1}{2}$	$1\frac{1}{2}\times3\frac{1}{2}$	$8\frac{1}{2}\times9\frac{2}{3}$
6.	$1\frac{1}{3}\times4\frac{1}{3}$	$1\frac{1}{3}\times5\frac{1}{3}$	$2\frac{3}{4}\times3\frac{1}{2}$	$16\frac{1}{4}\times8\frac{1}{2}$
7.	$6\frac{1}{2}\times8\frac{1}{2}$	$5\frac{1}{2}\times9\frac{1}{2}$	$14\frac{1}{2}\times8\frac{1}{2}$	$27\frac{3}{4}\times19\frac{1}{4}$
8.	$5\frac{1}{2}\times5\frac{1}{2}$	$12\frac{1}{2}\times14\frac{1}{2}$	$8\frac{3}{4}\times16\frac{1}{2}$	$48\frac{1}{2}\times6\frac{3}{4}$

Give the quotients of the following:

9.	$\frac{4}{5}\div2$	$\frac{6}{7}\div3$	$\frac{8}{9}\div4$	$\frac{9}{16}\div3$	$\frac{3}{4}\div3$
10.	$\frac{2}{3}\div3$	$\frac{1}{4}\div3$	$\frac{1}{8}\div3$	$\frac{5}{8}\div3$	$\frac{7}{8}\div3$
11.	$\frac{3}{4}\div4$	$\frac{5}{8}\div4$	$\frac{7}{8}\div4$	$\frac{5}{16}\div4$	$\frac{9}{16}\div4$
12.	$\frac{1}{2}\div\frac{1}{3}$	$\frac{3}{4}\div\frac{2}{5}$	$\frac{4}{5}\div\frac{2}{3}$	$\frac{5}{8}\div\frac{1}{4}$	$\frac{7}{8}\div\frac{3}{6}$

In the following do not change the dividend to an improper fraction before dividing:

13.	$4\frac{1}{2}\div2$	$6\frac{1}{2}\div2$	$8\frac{3}{4}\div2$	$10\frac{1}{2}\div2$	$12\frac{1}{3}\div2$
14.	$6\frac{1}{3}\div3$	$9\frac{2}{3}\div3$	$12\frac{1}{4}\div3$	$15\frac{3}{4}\div3$	$18\frac{5}{8}\div3$
15.	$4\frac{1}{2}\div3$	$10\frac{2}{3}\div3$	$14\frac{1}{2}\div3$	$16\frac{1}{2}\div3$	$27\frac{1}{4}\div4$
16.	$5\frac{3}{4}\div4$	$7\frac{5}{8}\div4$	$10\frac{7}{8}\div4$	$15\frac{1}{16}\div4$	$42\frac{3}{5}\div6$

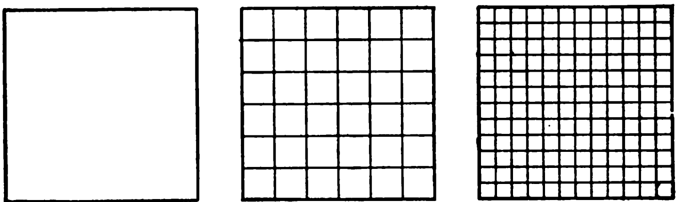
ORAL AND WRITTEN EXERCISES

Get each result in the examples below in the shortest way. See how long it takes you to do the first 21. Then see if you can do the next column in less time.

1. $8\times\frac{4}{7}$	22. $12\times15\times1\frac{1}{4}$	43. $4\frac{2}{3}\times3\frac{1}{5}\times6$
2. $6\times\frac{4}{9}$	23. $6\times4\ \times3\frac{2}{3}$	44. $\frac{7}{8}\div\frac{3}{4}$
3. $12\times1\frac{1}{2}$	24. $12\times8\ \times5\frac{3}{4}$	45. $\frac{6}{7}\div\frac{2}{3}$
4. $8\times3\frac{4}{5}$	25. $13\times\frac{3}{5}$	46. $4\frac{3}{4}\div\frac{4}{5}$
5. $9\div5\frac{5}{6}$	26. $\frac{2}{3}\times\frac{6}{7}$	47. $5\frac{1}{3}\div2\frac{2}{3}$
6. $12\div8\frac{3}{4}$	27. $\frac{4}{5}\div\frac{3}{8}$	48. $7\frac{5}{6}\div3\frac{2}{3}$
7. $16\times5\frac{7}{8}$	28. $\frac{7}{9}\div\frac{12}{21}$	49. $4\frac{2}{3}\times2\frac{4}{5}$
8. $15\div6\frac{2}{3}$	29. $\frac{5}{9}\div\frac{6}{25}$	50. $18\frac{1}{2}\times6\frac{1}{4}$
9. $2\frac{1}{3}\div6$	30. $\frac{8}{15}\div\frac{9}{16}$	51. $51\frac{1}{2}\div6\frac{1}{2}$
10. $5\frac{2}{3}\times9$	31. $\frac{7}{32}\div\frac{8}{21}$	52. $46\frac{1}{4}\times16\frac{1}{2}$
11. $8\frac{4}{5}\times15$	32. $\frac{5}{9}\div\frac{3}{8}$	53. $27\frac{1}{3}\div8\frac{1}{2}$
12. $24\frac{2}{3}\times3$	33. $\frac{7}{12}\div\frac{15}{16}$	54. $46\frac{1}{4}\div8\frac{1}{3}$
13. $19\frac{3}{5}\times10$	34. $\frac{4}{7}\div\frac{7}{8}$	55. $41\frac{1}{8}\times81\frac{1}{4}$
14. $17\frac{3}{4}\times8$	35. $3\frac{1}{2}\div2\frac{1}{2}$	56. $59\frac{1}{3}\div8\frac{2}{3}$
15. $34\frac{5}{6}\times12$	36. $1\frac{3}{4}\times2\frac{1}{3}$	57. $108\frac{1}{4}\times12\frac{1}{2}$
16. $106\frac{2}{3}\div24$	37. $4\frac{1}{2}\times2\frac{2}{3}$	58. $42\frac{3}{7}\div16\frac{1}{2}$
17. $2\times4\times6\frac{1}{2}$	38. $6\times3\frac{1}{2}\times1\frac{1}{2}$	59. $27\frac{5}{8}\div19\frac{2}{3}$
18. $6\times8\times2\frac{1}{3}$	39. $5\times2\frac{2}{3}\times1\frac{2}{3}$	60. $580\frac{1}{4}\div7$
19. $24\times5\times1\frac{2}{3}$	40. $1\frac{1}{2}\times2\frac{1}{3}\times3\frac{2}{3}$	61. $364\frac{5}{8}\div14$
20. $36\times8\times4\frac{1}{3}$	41. $5\times4\frac{2}{3}\times3\frac{2}{5}$	62. $63\frac{1}{2}\times41\frac{1}{3}$
21. $35\times8\times4\frac{1}{3}$	42. $6\frac{1}{2}\times3\frac{3}{4}\times2\frac{1}{5}$	63. $81\frac{3}{8}\times31\frac{1}{4}$

64. Drawing to Scale. In making drawings of large objects the drawings must be smaller than the objects themselves.

If the drawing is half as long and half as wide as the thing it represents it is said to be "to the scale 1: 2" or "$\frac{1}{2}$." If one inch in the drawing represents 10 feet in the object the scale is said to be "1 inch to 10 feet" or $\frac{1}{120}$.



Scale $\frac{1}{1}$ Scale $\frac{1}{6}$ Scale $\frac{1}{12}$

In the figure the square to the left represents one square inch, and the scale is 1: 1 or $\frac{1}{1}$. The next square represents a square 6 inches on the side, and the scale is 1 : 6 or $\frac{1}{6}$. The last square represents a square 12 inches on the side, and the scale is 1:12 or $\frac{1}{12}$. If this last square is made to represent a square 12 feet on the side the scale would be 1 inch to 12 feet or 1 : 144 ($\frac{1}{144}$).

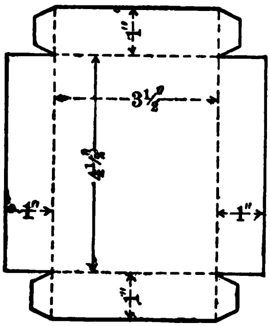
In order to make a drawing to scale of an object we must first measure it, and then decide the scale to be used. The larger the object to be represented the smaller the scale should be.

1. What scale would you use in representing each of the following:
   (a) The top of a desk 30″ x 36″.
   (b) A blackboard 3′ x 18′.
   (c) A schoolroom floor 30′ x 36′.
   (d) A schoolyard 160′ x 180′.
   (e) A city block 300′ x 480′.
   (f) A farm 160 rods by 240 rods.

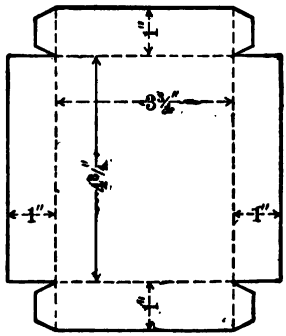
2. Make a drawing to scale of each of the things named under Example 1.

65. **Working Drawings.** People who build houses, make furniture, machinery, and many other things first make careful drawings. These drawings show the sizes of all the parts, and by following the drawings carefully the parts will be of the right size, and will fit together properly.

The following are drawings for a small candy box which some fifth-grade children made. The cardboard is bent up along the dotted lines and the corners fastened.



Box



Cover

ORAL AND WRITTEN EXERCISES

1. How long and how wide is the bottom of the box?
2. How deep will the box be?
3. How long and how wide will the cover be? Why is the cover made larger than the box?
4. Make drawings on cardboard of this box and also of the cover, using the scale 1 : 1. That is, make the drawing just the size of the box. Then cut out to make the box and the cover. After finishing the box measure its dimensions with care to see that you get it right.
5. Measure the length and width of a piece of street and draw to scale.

Scale: 1 inch represents 1000 miles.

1. On this map what distance is represented by 2 inches? by 3 inches? by $2\frac{3}{4}$ inches? by $1\frac{7}{8}$ inches? by $2\frac{7}{16}$ inches?

2. Measure accurately within $\frac{1}{16}$ of an inch the distance on this map between Boston and Chicago. According to this measurement, what is the straight-line distance between Boston and Chicago? First decide exactly where each city is located.

3. In the same manner find the straight-line distances from the City of New York to Chicago, from New York to St. Louis, to Denver, to San Francisco, to Seattle.

4. Find the distances between Chicago and New Orleans, Minneapolis and Seattle, Denver and San Francisco, Boston and St. Louis.

5. Locate your own home as nearly as you can on this map. Find the distances from your home to New York, to Chicago, to Buffalo, to Cleveland, to Denver, to San Francisco.

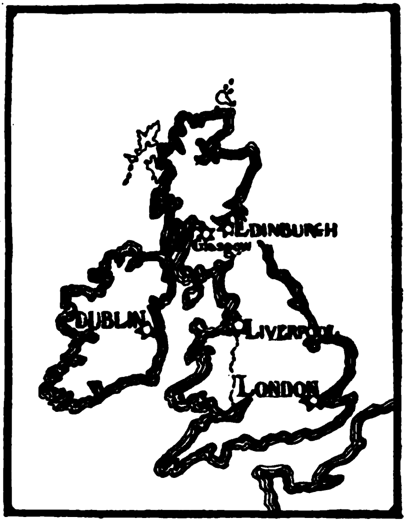
6. On a large wall map of the United States measure these same distances and find the actual distances.

ORAL AND WRITTEN EXERCISES

Answer as many as you can without using pencil and paper.

1. On this map what is represented by 1 inch? $1\frac{5}{16}$ inches? $2\frac{7}{8}$ inches?

2. By means of a ruler measure accurately within $\frac{1}{16}$ of an inch the distance on the map between London and Liverpool. What is the distance between London and Liverpool?



Scale: 1 inch represents 300 miles.

3. Find the distances from London to each of the following cities: Glasgow, Dublin, Edinburgh.

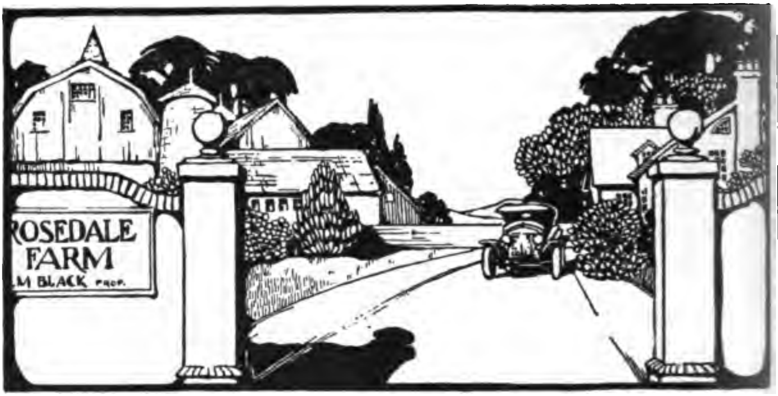
4. What is the straight-line distance from the most southerly point in England to the most northerly point in Scotland?

5. What is the distance from the most westerly point of Ireland to the most easterly point of England?

6. What is the extreme length of Ireland?

7. Get a map of your county, and find to what scale it is drawn. Then find the distances from the county seat to several places in the county. Also find the distances between other important points.

8. Get a map of your own State, and find to what scale it is drawn. Then find the distances from your home to several cities or other interesting places in the State.



1. At $14\frac{5}{8}$ cents a pound, what is the value of a bale of cotton weighing 496 pounds?

2. At $1\frac{4}{5}$ tons of hay to the acre, how many tons will $17\frac{1}{2}$ acres yield?

3. If one cow eats $2\frac{3}{8}$ tons of hay a year, how many tons will 35 cows eat?

4. If a meadow yields on an average $2\frac{1}{4}$ tons of hay to the acre, how many acres are required to furnish 83 tons of hay?

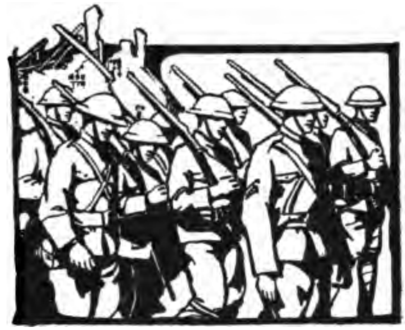
5. At $17.50 a ton, what is the value of a load of hay weighing 2380 pounds?

6. At 95 cents a bushel, what is the value of a load of corn weighing 2420 pounds? (One bushel of corn on the cob weighs 70 pounds.)

7. A field containing $42\frac{3}{4}$ acres yields 2790 bushels of corn. What is the yield per acre?

8. At $23.50 per ton, what is the cost of 1355 pounds of bran?

9. At $.75 a bushel what is the value of a load weighing 3150 lbs? (One bushel of potatoes weighs 60 lbs.)



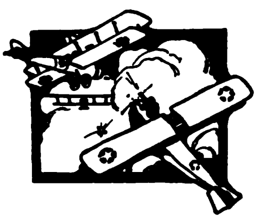
1. A military step is $2\frac{1}{2}$ feet. How many such steps are there in a mile?

2. If a regiment marches at the rate of $3\frac{3}{4}$ miles per hour, how long will it take to march $17\frac{1}{2}$ miles?

3. On a forced march a regiment marched $21\frac{3}{8}$ miles in $5\frac{1}{4}$ hours. How many miles per hour did they march?

4. A fast military aeroplane has a speed of 160 miles per hour. What portion of an hour will it take this aeroplane to cover a distance of 48 miles? How many minutes is this?

Gunpowder is made of sulphur, charcoal and saltpetre. While the composition of different powders varies slightly, it is substantially $\frac{1}{10}$ sulphur and $\frac{3}{4}$ saltpetre, and the rest charcoal.



5. What fraction of gunpowder is charcoal?

6. How many pounds of sulphur, saltpetre and charcoal are there in 1000 pounds of gunpowder?

7. 25 pounds of sulphur is sufficient for making how many pounds of gunpowder? (*Suggestion:* 25 is $\frac{1}{10}$ of what number?)

8. 75 pounds of saltpetre is sufficient for making how many pounds of gunpowder? (*Suggestion:* 75 is $\frac{3}{4}$ of what number? See page 77.)

9. 50 pounds of charcoal is sufficient for making how many pounds of gunpowder? (*Suggestion:* 50 is $\frac{3}{20}$ of what number?)

10. Make and solve other problems on the composition of gunpowder. See who can make the most interesting problems.

United States Coins.

Following are the combinations of metal used for making coins in the United States:

Coin:	Weight:	Composition:
Gold coin	$25\frac{4}{5}$ grains per dollar	$\frac{9}{10}$ gold, $\frac{1}{10}$ copper
Silver dollar	$412\frac{1}{2}$ grains	$\frac{9}{10}$ silver, $\frac{1}{10}$ copper
Half-dollar	$192\frac{9}{10}$ grains	$\frac{9}{10}$ silver, $\frac{1}{10}$ copper
5-cent piece	$77\frac{4}{35}$ grains	$\frac{3}{4}$ copper, $\frac{1}{4}$ nickel
1-cent piece	48 grains	$\frac{19}{20}$ copper, $\frac{1}{20}$ tin and zinc

Note that *one pound = 7000 grains.*

1. How much do two half-dollars weigh? Compare this with the weight of one silver dollar? What is the difference?

2. Quarters and dimes are made of the same composition of metals as the dollar and half-dollar, and weigh $\frac{1}{2}$ and $\frac{1}{5}$ as much as a half-dollar respectively. What is the weight of a quarter? of a dime?

3. How many grains of copper are there in one 5-cent piece?

4. What is the weight in pounds of $1000 in gold coin?

5. What is the weight of the pure gold in $5000 in gold coin?

6. How many grains of gold coin can be made from one pound of pure gold? (*Suggestion:* 7000 is $\frac{9}{10}$ of what number?)

7. How many dollars of gold coin can be made from one pound of pure gold?

LIGHTING A BUILDING

A standard amount of window space for a living-room is $\frac{1}{6}$ the floor space, for a bedroom $\frac{1}{10}$ the floor space, and for a schoolroom $\frac{1}{5}$ the floor space.

1. A living-room 13 feet wide and 17 feet long has 2 windows each $3\frac{1}{2}$ feet wide and $5\frac{3}{4}$ feet high. Is this window space sufficient according to the above standard? (*Suggestion:* Divide the total area of the windows by the floor space area. Is the quotient greater or less than $\frac{1}{6}$?)

2. A schoolroom 27 feet wide and 33 feet long has 7 windows each 3 feet wide and $8\frac{3}{4}$ feet high. Is this window space sufficient according to the above standard?

3. A bedroom 12 feet wide and 14 feet long has one window $3\frac{1}{4}$ feet wide and $5\frac{3}{4}$ feet high. Is this window space sufficient according to the above standards?

4. Measure the dimensions of your schoolroom and also the windows. Is your room sufficiently lighted according to the above standard?

5. Measure the dimensions of your bedroom and also the windows in it. Is the room sufficiently lighted according to the standard given above?

1. The sum of two numbers is $84\frac{5}{8}$, and one of the numbers is $36\frac{1}{4}$. What is the other number?

2. The product of two numbers is $4\frac{3}{4}$, and one of the numbers is $3\frac{1}{2}$. What is the other number?

3. A contractor is to dig 1 mile of sewer. He averages $4\frac{5}{8}$ rods a day. How many days will it take him to dig the sewer?

4. A screw $1\frac{7}{8}$ inches long goes into the wood $\frac{3}{16}$ of an inch each time it is turned around. How many times must this screw be turned around to go into the wood its whole length?

5. A man finds that his automobile averages $12\frac{1}{2}$ miles on a gallon of gasoline. How many miles can he go on $17\frac{3}{4}$ gallons?

6. A steel rod will stand a pull of 40,000 pounds per square inch of cross-section. How much of a pull will a rod stand which is $1\frac{1}{8}$ inches thick and $1\frac{3}{4}$ inches wide?

7. Good building brick can stand a pressure of 4500 pounds to the square inch. At this rate, how much pressure can a brick stand which is $8\frac{1}{4}$ inches long and $4\frac{1}{8}$ inches wide?

8. The circumference of a certain wheel is $16\frac{3}{8}$ feet. How many times will this wheel revolve in going 1 mile?

9. It is estimated that $\frac{2}{5}$ of the total population of a certain city are registered voters. According to this, what is the population of a city which has 12,685 registered voters?

10. A boy who lives $\frac{7}{8}$ of a mile from school walks this distance four times each school day. How far does he walk in one school month of 20 days?

11. A boy buys papers for $1\frac{3}{8}$ cents apiece, and sells them at 2 cents apiece. How much does he make in one week if he sells 346 papers?

66. **Decimal Fractions.** In \$4.75 the 4 stands for 4 dollars, the 7 stands for 7 dimes or $\frac{7}{10}$ of a dollar, and the 5 stands for 5 cents or $\frac{5}{100}$ of a dollar. Twenty-five cents is $\frac{25}{100}$ of a dollar, and is written \$.25; 5 cents is $\frac{5}{100}$ of a dollar, and is written \$.05. This way of writing fractions of a dollar is an example of a general way of writing certain fractions, which we now proceed to study.

Fractions whose denominators are 10, 100, 1000, etc., are called *decimal fractions*, and are usually written in a form different from that of other fractions. Thus, $\frac{7}{10}$ is written .7, and $\frac{3}{100}$ is written .03. In each case only the numerator is written, and the denominator is indicated by the location of the period, which is called the *decimal point.*

An integer in the first place to the right of the decimal point represents tenths, an integer in the second place represents hundredths, and so on.

$\frac{34}{100}$ is written .34. That is, $.34=\frac{3}{10}+\frac{4}{100}$.

The integer in the third place represents thousandths.

Thus, $.007=\frac{7}{1000}$, and $.049=\frac{4}{100}+\frac{9}{1000}$, or $\frac{49}{1000}$

67. **The Decimal Form.** Fractions written in the form .7, .34, .049 are said to be in the decimal form. Mixed numbers, such as 3.8, 76.089, are also said to be in the decimal form. Numbers in the decimal form are usually referred to simply as decimals.

68. **One-Place, Two-Place, Three-Place Decimals.** The decimal .7 is a one-place decimal, .34 and .70 are two-place decimals, .080, .487, .094 are three-place decimals, and so on.

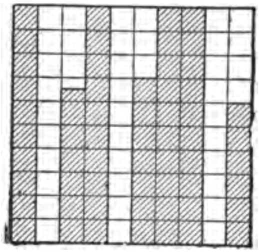
A zero is often placed to the left of the decimal point to make certain that the decimal point will be noticed. Thus, .7 may be written 0.7.

While decimal fractions are exactly the same kind of fractions as those we have studied before, we shall find that when written in the decimal form they are like whole numbers in many respects.

ORAL EXERCISES

1. If the large square represents 1, then one of the small squares represents what?

2. How many hundredths does the shaded column to the left represent? How many tenths?



3. How many hundredths are represented by the shaded part to the right?

4. How many hundredths are represented by the second shaded part from the right? This part shows that $.2+.07=.27$.

5. Show that the next shaded part represents .165. Note that one-half of .01 is .005.

6. How many hundredths are represented by each of the unshaded areas?

7. Draw a figure to represent each of the following: .6, .08, .21, .64.

8. Read the numbers 400, 40, 4, .4, .04, .004.

The number 444.444 is a mixed decimal consisting of the whole number 444 and the decimal fraction .444.

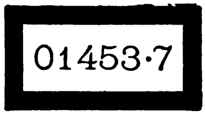
9. In the number 444.444 what is represented by each figure?

69. **Place Value in Decimals.** We now notice that we can say of a mixed decimal what we have said of a whole number, namely: *Any figure represents just ten times as much as is represented by the same figure in the next place to the right of it.*

This is a very important fact, since it enables us to treat decimals in much the same way that we treat ordinary whole numbers.

Complete this statement: In a decimal any figure represents as much as is represented by the same figure in the next place to the left of it.

On the cyclometer dial shown here how many miles are represented by the figure 1? By the 4? By the 5? By the 3? By the 7?



70. **Reading Decimals.** The fraction .064 represents thousandths, and is read "Sixty-four thousandths." The number .1846 represents ten-thousandths, and is read, "One thousand eight hundred forty-six ten-thousandths."

In reading a decimal the word "*and*" is used between the integral and the fractional parts. Thus, 347.981 is read, "Three hundred forty-seven *and* nine hundred eighty-one thousandths."

A one-place decimal is read *tenths*, a two-place decimal is read *hundredths*, a three-place decimal is read *thousandths*, a four-place decimal is read *ten thousandths*, and so on.

ORAL EXERCISES

Read the following numbers:

59.93, 491.076, 187.407, 81.034, 0.076, 1.0564, 0.064, 9.1560, 428.91, 20.048, 0.009, 182.046.

Persons who do a great deal of computing read a number like 3641.0047: thirty-six, forty-one, *point* double 0, forty-seven. 4007.2901 is read: four, double 0, seven, *point*, twenty-nine, 0, one.

In this manner read the following numbers:

4217.2156, 1800.2406, 5060.3060, 7000.0028,4900.3008,4021.4026.

WRITTEN EXERCISES

Write the following numbers, in the decimal form:

1. One hundred seventy-five and twenty-four hundredths.
2. Seventy-six and four hundred thirteen thousandths.
3. Two and sixty-nine hundredths.
4. Sixty-three and two hundred forty-one thousandths.
5. Five hundred forty-seven and ninety-eight thousandths.
6. Five thousand four hundred seventy-nine and eight tenths.

71. **Addition of Decimals.** Since $\frac{8}{10}=\frac{80}{100}=\frac{800}{1000}$, we see that we can annex as many zeros as we wish to the right of a decimal fraction without changing its value.

Example: Add, 43.096, 2.864, 517.04, 210.8, 784.915.

Solution; First write the numbers so that the decimal points stand in a straight column.

43.096	It is customary to annex zeros so there will be the same number of decimal places in all the numbers.	
2.864		
517.040	*Thousandths:*	Adding we get 15. Write 5, carry 1 (hundredth).
210.800	*Hundredths:*	Adding we get 21. Write 1, carry 2 (tenths).
784.915	*Tenths:*	Adding we get 27. Write 7, carry 2 (ones).
1558.715	*Ones:*	Adding we get 18. Write 8, carry 1 (ten).
		The remaining columns are added as usual.

Put a decimal point in the sum directly below the decimal points in the numbers added.

In practice we simply add the first column, getting 15. Then we say "Write 5, carry 1," and so on, as if there were no decimal point.

From this example we see that decimal numbers are added exactly like whole numbers. This is one of the great advantages of decimal fractions over other fractions.

WRITTEN EXERCISES

Copy and add the following.

1. 2194.0491+73.81+2.091+8,468+.0412.

2. 43.805+125.1704+3104.1509+1.0478.

3. .00478+35.419+749.041+875.0149.

4. 3146.1508+.04912+89.17+7.46+8.05.

5. 142.823+92.764+39.87+810.094+38.971.

Drill in Fundamentals. Play game No. 3 on page 30. Add common fractions for the first event, multiply fractions for the second event, and divide fractions for the third.

We have already written a number like 25 dollars and 75 cents in the form $25.75. This is a real decimal.

WRITTEN EXERCISES

Add the following:

1.	2.	3.
$81.09	$124.36	$240.23
243.74	48.19	75.14
61.35	74.06	89.12
24.76	135.24	195.35
146.51	73.41	47.56

4.	5.	6.
$720.14	$583.21	$29.41
86.03	96.15	36.23
94.25	83.52	147.28
147.50	146.31	633.91
89.40	284.19	339.17

Example: From 49.06 subtract 25.324.

```
49.060
25.324
------
23.736
```

Write the numbers so that the decimal points stand in a column. If necessary annex zeros in the minuend until it has as many decimal places as the subtrahend. Subtract exactly as if the numbers were whole numbers. Put a decimal point in the remainder directly under the decimal points in the subtrahend and minuend.

Subtract the following:

7.	8.	9.
24.49	123.01	239.12
17.28	84.30	184.40

10.	11.	12.
196.035	384.067	186.07
83.12	168.07	116.181

13. Following are readings of an automobile cyclometer. Find how far the machine went each day:

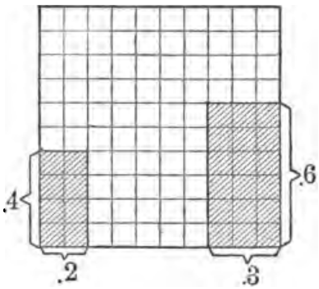
Monday morning	1489.4	Wednesday night	1947.8
Monday night	1643.8	Thursday night	2102.0
Tuesday night	1793.1	Friday night	2284.6

72. Multiplication of Decimals. Notice that in the large square below one vertical column represents .1 of the whole figure.

ORAL EXERCISES

1. The shaded area on the left represents .4 of .2. How many hundredths of the whole figure is this?

2. The shaded area on the right represents .6 of .3. How many hundredths of the whole figure is this?



3. In this same figure point out .7 of .6. How many hundredths of the whole figure is this:

4. Draw a figure like this to represent .7 of .2. How many hundredths is this?

5. In this same figure show .1 of .3. How many hundredths is this?

6. In this figure point out .4 of .8. How many hundredths of the whole figure is this?

Since .4 of .2 is the same as $.4 \times .2$, and .6 of .3 is the same as $6 \times .3$, we have just found these products.

7. From the examples already solved find the following products: $.7 \times .2$, $.1 \times .3$, $.7 \times .6$, $.4 \times .8$.

8. Give each of the following products, finding as many as you can without drawing a figure:

$.3 \times .7$	$.5 \times .8$	$.1 \times .5$	$.3 \times .3$	$.4 \times .4$
$.5 \times .5$	$.6 \times .6$	$.7 \times .7$	$.8 \times .8$	$.9 \times .9$
$.4 \times .8$	$.3 \times .5$	$.5 \times .3$	$.6 \times .3$	$.7 \times .3$
$.7 \times .6$	$.3 \times .9$	$.5 \times .2$	$.6 \times .9$	$.7 \times .5$
$.8 \times .9$	$.4 \times .7$	$.5 \times .7$	$.6 \times .7$	$.7 \times .8$
$.4 \times .8$	$.4 \times .2$	$.5 \times .9$	$.6 \times .8$	$.7 \times .9$

We will now study the multiplication of decimals systematically and without using the figure.

Example: Multiply .3 by .4.

Since .3 and .4 may be written $.3=\frac{3}{10}$ and $.4=\frac{4}{10}$, we simply need to find the product of $\frac{3}{10}\times\frac{4}{10}=\frac{12}{100}=.12$. Hence $3.\times.4=.12$. Similarly $.24\times.8=\frac{24}{100}\times\frac{8}{100}=\frac{192}{1000}=.192$.

WRITTEN EXERCISES

In this manner find the following indicated products:

1. $.15\times.3$	2. $.41\times.5$	3. $.03\times.12$
4. $.04\times.73$	5. $.14\times.21$	6. $.64\times.02$

73. **Placing the Decimal Point in the Product.** In each of these the product could be obtained more easily by multiplying as if the numbers were whole numbers, and then properly placing the decimal point in the product.

The examples $.5\times.3=.15$, $.41\times.5=.205$, and $.03\times.12=.0036$ illustrate the following rule:

> *To find the product of two numbers, one or both of which are decimals, multiply as if they were whole numbers, and then point off as many decimal places in the product as there are decimal places in both factors together.*

ORAL EXERCISES

According to this rule, what is the number of decimal places in the products of the following:

1. $.04\times.6$	5. $.01\times.04$	9. $.25\times.75$
2. $.26\times.04$	6. $.391\times.214$	10. $.24\times.04$
3. $.39\times.6$	7. $.702\times.4$	11. $.62\times.081$
4. $.35\times.6$	8. $.104\times.12$	12. $.81\times.142$

Find the products in these examples, and make sure you put the decimal point in the right place.

ORAL EXERCISES

Find the products of the following, using the rule on page 117.

1. .08×.07	4. .09×.05	7. .9×.08
2. .09×.04	5. .90×.8	8. .8×.02
3. .70×.4	6. .07×.6	9. .06×.04

Example. Multiply 3.4 by 6.

$$\begin{array}{r} 3.4 \\ 6 \\ \hline 20.4 \end{array}$$

By the rule on page 117 we get 20.4 as the product. Test this by writing $3.4 = 3\frac{4}{10}$ and then taking the product, $6 \times 3\frac{4}{10}$.

WRITTEN EXERCISES

Multiply:

1. 54×.9	7. 16×.28	13. 4×.024
2. 8×.42	8. 5×.162	14. 8×.08
3. 9×.092	9. 7×.25	15. 9×.422
4. 3×.64	10. 41×.64	16. 5×.62
5. 5×6.3	11. 8×12.5	17. 6×48.4
6. 9×4.5	12. 7×24.3	18. 4×38.45

Example. Multiply 2.463 by 45.

$$\begin{array}{r} 2.463 \\ 45 \\ \hline 12315 \\ 9852 \\ \hline 110.835 \end{array}$$

Multiply exactly as if the numbers were integers, and then place the decimal point in the product according to the rule on page 117. The decimal points are omitted in the partial products.

In this manner multiply the following:

19. 64×14.36	22. 29×197.013	25. 17×96.048
20. 78×85.019	23. 124× 69.103	26. 59×19.340
21. 1×1566.207	24. 245× 61.01	27. 132×243.01

Example. Multiply 3.7 by 2.4.

3.7 2.4 148 74 8.88	**Again multiply exactly as if the numbers were ordinary integers, and then place the decimal point in accordance with the rule on page 117.**

To verify that this gives the correct result, we write 3.7 and 2.4 in forms $3\frac{7}{10}$ and $2\frac{4}{10}$, and then multiply them as ordinary mixed numbers. In which form are these numbers multiplied more easily?

74. **Products to the Nearest Tenth, Nearest Hundredth, etc.** The method of finding products to the nearest tenth, etc., is shown in the following examples:

Examples. Find the product of 8.7×19.7 to the nearest tenth, and find the product of $.78 \times 39.42$ to the nearest hundredth.

19.7 8.7 1379 1576 171.39	**The product is nearer 171.4 than 171.3.** **Hence, 171.4 is the required product.**	39.42 .78 31536 27594 30.7476	**The product is nearer 30.75 than 30.74.** **Hence 30.75 is the required product.** **The product to the nearest thousandth is 30.748.**

In case the exact product is, for instance, 17.85, it is customary to take 17.9 as the nearest tenth, though 17.8 is equally near the exact result.

WRITTEN EXERCISES

Find the exact product of the first five, the products of the next five to the nearest hundredth, and of the last five to the nearest thousandth.

1. 3.9×6.2	6. 4.8×1.64	11. $3.14 \times .86$
2. 1.3×5.9	7. 3.52×5.72	12. 4.93×3.61
3. 82.5×2.4	8. 5.23×7.52	13. 7.46×9.23
4. 41.6×3.8	9. 39.8×26.7	14. 4.9×2.785
5. 1.36×45.2	10. 41×4.79	15. 8.6×1.426

Gold coins weigh $25\frac{4}{5}$, or 25.8 grains per dollar. The silver dollar weighs $412\frac{1}{2}$ or 412.5 grains. The half-dollar weighs $192\frac{9}{10}$, or 192.9 grains. The metal in the gold and silver coins is $\frac{9}{10}$, or .9 pure, the remaining .1 being copper. (See page 108.)

WRITTEN EXERCISES

In the following problems use the decimal form of the fraction:

1. How many grains of pure gold per dollar are there in a gold coin? How many grains of copper?

2. How many grains of pure silver are there in a silver dollar? How many grains of copper?

3. How many grains of pure silver are there in 200 half-dollars? How many grains of copper?

4. How many grains of pure silver are there in 100 silver dollars? How many grains of copper?

On page 108 similar problems were solved, using the common fractions. Which are the more convenient, the common fractions or the decimal fractions?

The composition of gunpowder (see page 107) may be expressed in decimals: $\frac{1}{10}$ or .1 is sulphur, $\frac{3}{4}$ or $\frac{75}{100}$ or .75, is saltpetre, and $\frac{3}{20}$ or $\frac{15}{100}$ or .15, is charcoal.

5. How many pounds of each of these ingredients are needed to make 1000 pounds of gunpowder?

The usual measure of distance used at sea is the knot, or nautical mile. This is 6080 feet.

6. How much does a nautical mile differ from 1.151 statute miles? (*Suggestion:* First multiply 1.151 by 5280.)

7. On her trial trip the Mauretania (the fastest merchant steamship afloat) made 27.3 knots per hour. How many ordinary miles is this, counting 1.151 miles as one knot.

1. The fastest journey ever made across the Atlantic Ocean was made at an average speed of 26.02 knots per hour. How many miles per hour was this (1 knot = 1.151 miles)?

2. One rod is $5\frac{1}{2}$ (5.5) yards. How many square yards are there in one square rod? How many square yards are there in 160 square rods, or one acre?

3. One rod is $16\frac{1}{2}$ (16.5) feet. How many square feet are there in one square rod? In one acre?

4. One bushel contains approximately 2150.43 cubic inches. Multiply this number of cubic inches by .8. By how much does the result differ from the number of cubic inches in one cubic foot? (1728 cu. in. = cu. ft.)

5. Assuming that one cubic foot is .8 of a bushel, how many bushels are there in 10 cubic feet? in 100 cubic feet?

6. A grain bin is $8\frac{1}{2}$ (8.5) feet long and $6\frac{3}{4}$ (6.75) feet wide. How many bushels of grain does it hold if the grain is 5 feet deep in the bin? Regard one cubic foot as .8 of a bushel.

7. One cubic foot of water weighs about 62.5 pounds. Ice weighs .92 times as much as water. What is the weight of one cubic foot of ice?

8. By how much does 35 cubic feet of ice differ in weight from one ton?

9. The weight of cork is .24 that of water. What is the weight of one cubic foot of cork? By how much does this differ from the weight of one cubic foot of water?

10. The weight of the water in the Dead Sea is 1.24 times that of ordinary fresh water. What is the weight of one cubic foot of water from the Dead Sea?

11. Pure gold weighs 19.36 times as much as water. What is the weight of one cubic foot of gold? (See example 6.)

75. Multiplying by 10, 100, etc. To multiply a decimal by 10 move the decimal point one place to the right, to multiply by 100 move the decimal point two places to the right, and to multiply by 1000 move the decimal point three places to the right. If necessary annex zeros.

Thus, $100 \times .4 = 40.$ and $100 \times 4. = 400.$

This follows directly from the place value in the decimal notation.

ORAL EXERCISES

Give the products in the following, annexing zeros if necessary:

1. $100 \times .3$	5. $100 \times .03$	9. $100 \times .003$
2. 100×2.4	6. 100×24.01	10. $100 \times .247$
3. $100 \times 4.$	7. $100 \times 70.$	11. $100 \times .510$
4. $1000 \times .04$	8. $1000 \times .3202$	12. 1000×5.2

13. How may a number be multiplied by 10,000?

76. Dividing by 10, 100, 1000, etc. To divide by 10, 100, 1000 move the decimal point one, two, or three places to the left.

This may be seen directly from the definition of division.

Thus, $10 \times 4.151 = 41.51$, *therefore* $41.51 \div 10 = 4.151$.
$100 \times 4.151 = 415.1$, *therefore* $415.1 \div 100 = 4.151$.
$1000 \times 4.151 = 4151$, *therefore* $4151 \div 1000 = 4.151$.

ORAL EXERCISES

Perform the following indicated operations, if necessary prefixing zeros before placing the decimal point:

1. $.21 \div 10$	6. $84.21 \div 10$	11. $.08 \div 100$
2. $.5 \div 100$	7. $6.93 \div 100$	12. $.001 \div 1000$
3. $.28 \div 1000$	8. $421. \div 1000$	13. $.096 \div 100$
4. $17. \div 100$	9. $27.5 \div 10000$	14. $.02 \div 1000$
5. $.24 \div 100$	10. $.03 \div 10$	15. $0.71 \div 1000$

Example. Divide 871.490 by 37.

$$\begin{array}{r} 4 \\ 23.55\cancel{3} \\ 37\overline{)871.490} \\ \underline{74} \\ 131 \\ \underline{111} \\ 204 \\ \underline{185} \\ 199 \\ \underline{185} \\ 140 \\ \underline{111} \\ 29 \end{array}$$

Divide exactly as if there were no decimal point in the dividend. The first number in the quotient is 2 (tens), which is placed directly above the tens in the dividend. Place a decimal point in the quotient directly above the decimal point in the dividend.

To test, multiply the quotient by 37 and add the remainder.

If the last remainder is less than half the divisor it is disregarded. If it is equal to or greater than half the divisor, 1 is added to the last figure in the quotient. In the above example the quotient is 23.554, which is correct to the nearest thousandth.

$$\begin{array}{r} 23.553 \\ \underline{37} \\ 164871 \\ \underline{70659} \\ 871.461 \\ \underline{29} \\ 871.490 \end{array}$$

WRITTEN EXERCISES

In each of the following find the quotient to the nearest thousandth.

1. 29.374 ÷ 19	13. 46.94 ÷ 29
2. 134.048 ÷ 87	14. 789.0013 ÷ 497
3. 218.309 ÷ 122	15. 849.0147 ÷ 830
4. 197.016 ÷ 88	16. 973.0504 ÷ 97
5. 246.009 ÷ 129	17. 841.9062 ÷ 196
6. 213.017 ÷ 301	18. 9470.841 ÷ 856
7. 715.290 ÷ 285	19. 3746.240 ÷ 563
8. 649.290 ÷ 386	20. 2913.572 ÷ 365
9. 947.0341 ÷ 278	21. 1923.275 ÷ 1240
10. 649.021 ÷ 486	22. 4936.216 ÷ 365
11. 785.463 ÷ 594	23. 746.539 ÷ 416
12. 973.058 ÷ 786	24. 1342.741 ÷ 894

Example 1. Divide 1.2042 by 87.

$$\begin{array}{r} .0138 \\ 87\overline{)1.2042} \\ \underline{87} \\ 334 \\ \underline{261} \\ 732 \\ \underline{696} \\ 36 \end{array}$$

The first dividend is 1.20, or 120 hundredths, and the first quotient figure is placed directly above the last figure in the first dividend (in hundredth's place). The decimal point in the quotient is placed directly above the decimal point in the dividend.

WRITTEN EXERCISES

Find quotients to the nearest ten-thousandth.

1. 2.3604 ÷ 198
2. 3.0421 ÷ 297
3. 7.0809 ÷ 346
4. 8.4012 ÷ 427
5. 9.4708 ÷ 597
6. 96.058 ÷ 3970
7. 84.096 ÷ 298
8. 78.009 ÷ 3090
9. .0781 ÷ 317
10. .0091 ÷ 248
11. 0.4729 ÷ 67
12. 1.7164 ÷ 24
13. 46.2091 ÷ 73
14. 19.0246 ÷ 94
15. 5.9007 ÷ 31

Example 2. Divide 34.1415 by 2.36.

$$\begin{array}{r} 7 \\ 14.4\cancel{6} \\ 236\overline{)3414.15} \\ \underline{236} \\ 1054 \\ \underline{944} \\ 1101 \\ \underline{944} \\ 1575 \\ \underline{1416} \\ 159 \end{array}$$

First multiply both divisor and dividend by 100 to get rid of the decimal in the divisor.

Then divide as before. Since the remainder is greater than half the divisor, add 1 to the last figure in the quotient. The result is 14.47, which is correct to the nearest hundredth.

Find quotients to the nearest tenth.

16. 8.937 ÷ 1.45
17. 9.403 ÷ 2.57
18. 7.516 ÷ 3.08
19. 12.046 ÷ 4.03
20. 16.471 ÷ 5.43
21. 23.057 ÷ 6.24
22. 34.079 ÷ 8.03
23. 25.573 ÷ 9.24
24. 30.057 ÷ 8.76
25. 52.91 ÷ 1.41
26. 7.943 ÷ 2.78
27. 3.298 ÷ .781

Example 1. Divide 2479.3 by 3.76. Find the quotient to the nearest thousandth.

```
             4
         6.593
3.76)2479.3
     2256
      2233
      1880
       3530
       3384
        1460
        1128
         332
```

To get the quotient beyond tenths, annex zeros to the remainders. This has exactly the same effect as if the dividend were written 2479.300, which we know from page 114 equals 2479.3.

The quotient to the nearest thousandth is 6.594.

Example 2. Divide 164.7 by 2.051.

Multiplying *dividend* and divisor by 1000, we have 2051)164700. Notice that we must annex zeros in the dividend before moving the decimal point three places to the right.

WRITTEN EXERCISES

Find the quotients to the nearest thousandth:

1. $.947 \div 8.76$	11. $.04 \div .007$	21. $1.16 \div .217$
2. $.89 \div 3.47$	12. $.00471 \div .81$	22. $2.34 \div .214$
3. $.975 \div .431$	13. $1.1 \div 1.65$	23. $5.26 \div .043$
4. $.202 \div 6.9$	14. $2.1 \div 1.84$	24. $.783 \div .471$
5. $.7471 \div 9.8$	15. $3.674 \div 2.39$	25. $.519 \div .125$
6. $.9743 \div 8.7$	16. $75.7 \div 4.32$	26. $6.509 \div .375$
7. $.0891 \div .064$	17. $9.047 \div 4.06$	27. $39.28 \div 5.84$
8. $12.07 \div .089$	18. $18.048 \div 5.19$	28. $67.14 \div 51.6$
9. $.008 \div .075$	19. $9.391 \div .462$	29. $41.94 \div 65.1$
10. $.09 \div .199$	20. $3.409 \div .062$	30. $1205.4 \div .0142$

Drill in Fundamentals. Play game No. 1, page 30. Use examples in dividing fractions.

PROBLEMS OF THE DAIRY

1. A certain cow yields 9840 pounds of milk in one year. Of this milk .041 is butter fat. What is the value of the butter fat at 43 cents a pound? Find result to the nearest cent.

(Review table of liquid measure. See page 288.)

2. A farmer feeds on an average 2.16 tons of hay a year to each cow. How much hay does he feed 17 cows? What is the value of this hay at $14.50 a ton?

3. A quart of milk weighs 2.15 pounds. How many quarts of milk does the cow in problem 1 yield in one year? At an average price of $.042 per quart, what is the value of this milk? Get result to the nearest cent.

4. An empty milk can weighs 16.8 pounds. Filled with milk it weighs 81.4 pounds. How many pounds of milk are there in the can? At 8.6 pounds to the gallon, how many gallons does this can hold?

5. A farmer sells on an average 14.3 gallons of milk each day, during the month of January. If he gets 18.6 cents a gallon, how much does he get for his milk this month? Find result to the nearest cent.

MISCELLANEOUS PROBLEMS

1. A load of coal weighs 7580 pounds after deducting the weight of the wagon. How many tons of coal are there in the load?

Solution: Since one ton weighs 2000 pounds, the number of tons is $\frac{7580}{2000}=\frac{7.58}{2.00}=3.79$.

2. An automobile runs 165 miles on 14 gallons of gasoline. How far does it run on one gallon? Get result to the nearest tenth of a mile.

3. How many acres are there in a field 24.6 rods wide and 61.3 rods long?

In the following problems find all results to the nearest cent:

4. At \$6.75 a ton, what is the cost of a load of coal weighing 6840 pounds?

Solution: The number of tons is $\frac{6840}{2000}$. Hence the cost in dollars is $\frac{6840}{2000}\times 6.75=3.42\times 6.75=23.085$. Hence, \$23.09 is the cost.

5. At \$1.86 a bushel, what is the value of a load of wheat weighing 2360 pounds? (1 bushel of wheat weighs 60 pounds.)

6. At \$0.94 a bushel, what is the value of a load of corn in the ear weighing 3240 pounds? (1 bushel of corn in the ear weighs 70 pounds.)

7. Hay is selling at \$18.45 a ton. What is the value of a load weighing 2480 pounds?

8. At \$0.28 per square foot, what is the value of a lot 22.7 feet wide and 87.3 feet deep?

9. In the first non-stop aeroplane flight across the Atlantic, made by Alcock and Brown, on June 15th, 1919, the distance covered was 1650 nautical miles. If one nautical mile is 1.151 ordinary miles, find this distance in ordinary miles.

77. Reducing Common Fractions to Decimals. Since decimal fractions are more convenient in many ways than common fractions, the latter are often reduced to decimals.

Example. Reduce $\frac{3}{16}$ to a decimal fraction.

```
     .1875
16)3.0000
   1 6
   140
   128
    120
    112
     80
```

Since $\frac{3}{16}$ means $3 \div 16$, we need only to divide 3 by 16. We write a decimal point after the 3 and add as many zeros as we wish. The result of dividing is .1875.

In practice we use short division when the divisor is not too large. Thus, to reduce $\frac{3}{8}$ to a decimal we write

$$8)\overline{3.000}\,.375 \quad \text{or simply} \quad 8)\overline{3.}\,.375$$

The great majority of fractions in practical use have 2, 4, 8, 16, 32, or 64 as denominators. We will now reduce such fractions to decimals.

We know at sight that $\frac{1}{2} = .5$, $\frac{1}{4} = .25$. That is, halves may be reduced to 10ths, and 4ths to 100ths. Similarly, $\frac{1}{8} = .125$.

WRITTEN EXERCISES

1. Reduce each of the following to a decimal: $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$. From these we see that 8ths are reducible to 1000ths.

2. Reduce $\frac{1}{16}$, $\frac{3}{16}$, $\frac{5}{16}$, $\frac{7}{16}$, $\frac{9}{16}$, $\frac{11}{16}$, $\frac{13}{16}$, $\frac{15}{16}$, to decimals. From these we see that 16ths are reducible to 10000ths.

3. Write all proper fractions which are in their lowest terms and whose denominators are 32. Reduce five of these to decimals. What do you conclude about the reducibility of 32ds?

4. Write all proper fractions which are in their lowest terms and whose denominators are 64. Reduce five of these to decimals. What do you conclude about the reducibility of 64ths?

Example 1. Reduce $\frac{1}{3}$ to a decimal fraction, giving the result correct to the nearest thousandth.

$$3\overline{)1.000}\quad .333$$

Dividing, we obtain a quotient, .333 and a remainder, 1. Since the remainder is less than one-half of 3, 333 is the required quotient.

Example 2. Reduce $\frac{2}{3}$ to a decimal fraction, giving the result to the nearest thousandth.

$$3\overline{)2.000}\quad .667$$

Dividing, we get .666 with a remainder of 2, which is more than $\frac{1}{2}$ of 3. Hence, the result to the nearest thousandth is .667.

WRITTEN EXERCISES

Reduce the following fractions to decimals and give results to the nearest ten-thousandths.

1. $\frac{7}{8}$	6. $\frac{3}{16}$	11. $\frac{4}{11}$	16. $\frac{5}{16}$	21. $\frac{9}{14}$
2. $\frac{3}{5}$	7. $\frac{5}{12}$	12. $\frac{7}{9}$	17. $\frac{7}{15}$	22. $\frac{11}{16}$
3. $\frac{4}{7}$	8. $\frac{3}{8}$	13. $\frac{5}{8}$	18. $\frac{5}{14}$	23. $\frac{11}{18}$
4. $\frac{4}{9}$	9. $\frac{9}{11}$	14. $\frac{6}{11}$	19. $\frac{9}{16}$	24. $\frac{11}{12}$
5. $\frac{6}{7}$	10. $\frac{8}{9}$	15. $\frac{7}{12}$	20. $\frac{5}{18}$	25. $\frac{7}{16}$

Sometimes a common fraction is reduced to part decimal and part common fraction.

Thus, $\frac{1}{3}=.33\frac{1}{3}$. This is read "$33\frac{1}{3}$ hundredths." Similarly, $\frac{2}{3}=.66\frac{2}{3}$.

WRITTEN EXERCISES

Reduce each of the following to a two-place decimal and a fraction (if necessary):

1. $\frac{5}{6}$	6. $\frac{7}{12}$	11. $\frac{9}{16}$	16. $\frac{7}{20}$
2. $\frac{3}{8}$	7. $\frac{11}{12}$	12. $\frac{11}{16}$	17. $\frac{9}{20}$
3. $\frac{5}{8}$	8. $\frac{3}{16}$	13. $\frac{13}{16}$	18. $\frac{11}{20}$
4. $\frac{7}{8}$	9. $\frac{5}{16}$	14. $\frac{15}{16}$	19. $\frac{13}{20}$
5. $\frac{5}{12}$	10. $\frac{7}{16}$	15. $\frac{3}{20}$	20. $\frac{17}{20}$

78. Fractions Reducible to Decimals. The fraction $\frac{1}{2}$ can be reduced to 10ths because 10 is a multiple of 2. Likewise $\frac{1}{4}$ can be reduced to 100ths because 100 is a multiple of 4.

On the other hand, $\frac{1}{3}$ or $\frac{2}{3}$ cannot be reduced exactly to a decimal, because none of the numbers, 10, 100, 1000, etc., is a multiple of 3.

Since the numbers 10, 100, 1000, etc., have no prime factor except 2 and 5, it follows that:

> *A fraction may be reduced exactly to a decimal if its denominator has no factor except 2 and 5; otherwise not.*

That is, fractions with denominators, 2, 4, 5, 8, 16, 20, 25, 32, 50, can be reduced exactly to decimals, while fractions whose denominators are 3, 7, 12, 15, 24 cannot be so reduced.

ORAL EXERCISES

State which of the following can be reduced exactly to decimals: $\frac{3}{5}$, $\frac{5}{8}$, $\frac{4}{9}$, $\frac{5}{12}$, $\frac{9}{16}$ $\frac{7}{20}$, $\frac{9}{25}$, $\frac{9}{32}$, $\frac{1}{120}$, $\frac{1}{75}$.

79. Fractions and Decimals to be Memorized. There are certain fractions whose decimal equivalents should be memorized. These are:

$\frac{1}{2}=.50$	$\frac{1}{5}=.20$	$\frac{1}{8}=.12\frac{1}{2}$
$\frac{1}{3}=.33\frac{1}{3}$	$\frac{2}{5}=.40$	$\frac{1}{12}=.08\frac{1}{3}$
$\frac{2}{3}=.66\frac{2}{3}$	$\frac{3}{5}=.60$	$\frac{1}{16}=.06\frac{1}{4}$
$\frac{1}{4}=.25$	$\frac{4}{5}=.80$	$\frac{1}{20}=.05$
$\frac{3}{4}=.75$	$\frac{1}{6}=.16\frac{2}{3}$	$\frac{1}{25}=.04$

For the purpose of comparison, common fractions are sometimes reduced to mixed decimal and common fractions, even when they can be reduced exactly to decimals having a larger number of places.

Thus, the fractions $\frac{3}{8}$, $\frac{2}{5}$, $\frac{7}{20}$, $\frac{5}{14}$ equal respectively $.37\frac{1}{2}$, .40, .35, $.35\frac{5}{7}$. These may now readily be arranged in order to their magnitude.

1. How may a decimal number be multiplied by 10? by 100? by 1000?

2. How may a decimal number be divided by 10? by 100? by 1000?

3. Tell how to multiply decimals, also how to divide them.

4. How do you find the value of a load of wheat if you know the price per bushel and the weight of the wheat?

5. If you know the number of cubic yards to be removed from an excavation, and also the number which can be removed in one day, how can you tell how long it will take to complete the work?

6. If you know the weight of a load of coal when weighed on the wagon, and also the weight of the empty wagon, how can you tell how many tons there are in the load?

7. If you know the dimensions in feet of a grain bin, and the number of cubic feet in one bushel, how do you find the number of bushels which the bin will hold?

8. If you know the dimensions of a basement in feet, how do you find how many cubic yards it contains?

9. If you know the dimensions of a box in inches, how do you find how many cubic feet it contains?

10. If you know the area of a rectangle and also its length, how do you find its width?

11. If you know the volume of a box and two of its dimensions, how do you find the third dimension?

12. A farmer knows the area of two hay fields and the number of tons of hay in one of them. How can he find the number of tons in the other field if the two fields run about the same number of tons to the acre?

ORAL EXERCISES

1. In a certain class of 39 pupils there are 18 boys. What fractional part of the class are boys? Clearly $^{18}/_{39}$ or $^{6}/_{13}$ of the class are boys. What fractional part of the class are girls?

2. Count the number of boys and girls in your class. What fractional part of the class are girls? What fractional part of the class are boys?

3. The baseball team of the Jones School played 17 games and won 11. What fraction of the games played did they win?

4. The baseball team of the Franklin School played 23 games and won 15. What fraction of the games played did they win?

Example. Which of the two teams in examples 3 and 4 won a larger fraction of the games played?

Solution: Reducing the fractions $\frac{11}{17}$ and $\frac{15}{23}$ to a common denominator, we have $\frac{11}{17}=\frac{253}{391}$ and $\frac{15}{23}=\frac{255}{391}$, which can be compared easily.

It is frequently necessary to arrange in the order of their magnitude a large number of fractions. This is done most easily by reducing them to decimals, and then comparing the decimals.

WRITTEN WORK

1. Reduce to a decimal the fraction won of the games played by each of the following schools:

School	*Games played*	*Won*	*Lost*	*Standing*
Crane..................	17	8	9	....
Marshall..............	18	10	8	....
Lake View.............	19	9	10	....
Carter Harrison........	18	9	9	....
Wendell Phillips.......	17	10	7	....
Hyde Park...........	16	7	9	....
Nicholas Seun........	19	9	10	....

WRITTEN WORK

The following were the number of games played and the number of games won and lost by each team of the National and American Leagues during the season 1918:

National League			American League		
Club	Games Played	Games Won	Club	Games Played	Games Won
Boston..........	123	53	Boston.........	126	75
Brooklyn........	126	57	Chicago........	134	57
Chicago.........	129	84	Cleveland.......	129	73
Cincinnati.......	128	68	Detroit.........	126	55
New York.......	124	71	New York......	123	60
Philadelphia.....	123	55	Philadelphia....	128	52
Pittsburgh.......	125	65	St. Louis.......	124	60
St. Louis........	129	51	Washington.....	128	72

1. What fraction of the number of games played was won by each team? Reduce each of these fractions to a decimal, correct to the nearest thousandth. Make a list of the teams in each league, showing their standing at the end of the season. Arrange them in the order of their standing.

This is exactly how the standing of these teams is computed from day to day during the whole baseball season.

2. Find the number of games won and lost by some baseball teams in your neighborhood during the last season. What fraction of the games played did they win?

3. Find the number of games lost and won by each team in one of the big leagues during the season just passed, and make a list showing their standing for the season.

4. Find the number of boys and the number of girls in each room in your school building, and arrange them in order so that one having the largest fractional part of girls shall be first, the one with the next largest fractional part of girls second, and so on. Do the same for the boys. If your school is very small the teacher will suggest numbers of boys and girls for an eight-room school.

80. Percentage. Many fractions are regularly expressed as hundredths. Indeed this is done so frequently that a special name is given to hundredths. Instead of saying so many *hundredths,* we say so many *per cent.* The words *per cent* come from the Latin *per centum* (in the hundred). The symbol % stands for per cent. Thus, $.06 = 6\%$, and $.15 = 15\%$.

Example. How many per cent are there in .048?

Solution: To find how many per cent. or hundredths there are in a number multiply it by 100. Hence $100 \times .048 = 4.8$ is the number of per cent in .048.

We can also see this directly. The 4 stands in hundredths' place, and hence represents 4 hundredths, or 4%. The 8 is in thousandths' place, and hence represents 8 tenths of one hundredth, or .8%.

You should make the above perfectly clear to yourself, as it is fundamental in all work involving per cent.

How many per cent are there in each of the following decimals?

1. .043	4. .125	7. .142	10. .392
2. .254	5. .093	8. .286	11. .005
3. .035	6. .085	9. .491	12. .017

Example. Express $\frac{1}{3}$ as per cent, giving result to the nearest tenth of one per cent.

Solution: Reduce $\frac{1}{3}$ to a decimal correct to the nearest thousandth and multiply by 100.

That is, $\frac{1}{3} = .333 = 33.3\%$.

WRITTEN EXERCISES

Reduce each of the following fractions to the nearest thousandth, and read the results as per cent: $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{3}{5}$, $\frac{1}{6}$, $\frac{5}{6}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$.

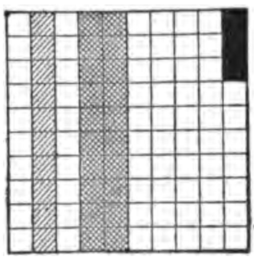
WRITTEN EXERCISES

Reduce each of the fractions given below to thousandths. Write and read each as per cent to the nearest tenth of one per cent.

1. $\frac{1}{2}$	6. $\frac{5}{8}$	11. $\frac{7}{8}$	16. $\frac{9}{16}$
2. $\frac{1}{4}$	7. $\frac{11}{12}$	12. $\frac{5}{12}$	17. $\frac{11}{16}$
3. $\frac{3}{4}$	8. $\frac{1}{5}$	13. $\frac{3}{16}$	18. $\frac{13}{16}$
4. $\frac{1}{3}$	9. $\frac{3}{7}$	14. $\frac{5}{16}$	19. $\frac{15}{16}$
5. $\frac{2}{3}$	10. $\frac{3}{8}$	15. $\frac{7}{16}$	20. $\frac{11}{12}$

81. **Special Study of Per Cent.** The use of per cent (hundredths) is so important that we will study such fractions more fully:

ORAL EXERCISES



1. How many small squares are there in this figure?
2. How many per cent of the large square are there in one small square?
3. How many per cent of the large square are there in 5 small squares? In 10 small squares?
4. How many per cent of the large square are black?
5. How many per cent of the large square are shaded so ?
6. How many per cent of it are shaded so ?
7. How many per cent of the large square are there in 25 small squares? in 50 small squares? in 100 small squares?
8. If a number is in the form of a decimal, how do you find the number of per cent to which it is equal? Show by examples.
9. How do you find the number of per cent. equal to a given common fraction? Show by an example.

ORAL EXERCISES

1. Find .06 of 200. Find 6% of 200.
2. Find 5% or .05 of each of the following: 100, 200, 300, 400, 500, 600, 700, 800, 900, 1000.
3. Find 10% or .10 of each of the numbers in example 2.
4. Find 20% of each of the following: 200, 400, 600. Notice that 20% is the same as 1/5.
5. Find 25% of 400, of 600, of 800, of 1000. Notice that 25% is the same as 1/4.
6. 50% is equal to what fraction? Find 50% of each of the following: 240, 684, 860.

REDUCTION OF 40% ON ALL SUITS AND OVERCOATS

This is from an advertisement announcing a special sale of clothing. It means that the prices are reduced 40% of the usual price. Thus, a suit marked $25 will be reduced .40×$25=$10.00, and hence will sell for $15.00.

ORAL AND WRITTEN EXERCISES

At a reduction of 40%, what are the prices of these articles? Give as many as you can orally.

1. A suit................marked $30.00
2. A raincoat............marked $18.00
3. An overcoat..........marked $25.00
4. A suit................marked $40.00
5. An overcoat..........marked $36.00
6. Boy's suit............marked $12.00

Look in your paper to see if there are any advertisements like these. Bring one to school and make problems from it.

82. **Discounts.** Many articles, such as books, are listed at certain prices in catalogues, and then sold at a *discount*. If a book is listed at $1.50 and sold at a discount of 20%, this means that the price is reduced by 20% of the list price.

Following is a bill for such a book:

THE WEBSTER FORD COMPANY,
Chicago, Illinois.

August 14, 1918.

Sold to Robert J. Brown,
Des Moines, Iowa.

August 13.	Successful Houses and How to Build Them...	$1.80
	Less 20%	.36
	Net..................................	$1.44

WRITTEN EXERCISES

Make out bills for the following, using the name of a company you know as the seller, and your own name as the buyer:

1. One book at $2.50, discount 10%.
2. One book at $3.50, discount 25%.
3. A pair of shoes marked $5.00, discount 30%.
4. A picture marked $15, discount 20%.

Sometimes discount is allowed for immediate cash payments, or for other reasons. Make out bills for the following:

5. One Century Dictionary at $85, discount 10% for cash.
6. One set of Encyclopedia Britannica, cloth binding, $150, discount 5% for cash.
7. One set of the same Encyclopedia, suède binding, $225, discount 5% for cash.
8. One piano $450, discount 10% for cash.

WRITTEN EXERCISES

1. Copy and fill out the following bill. Insert the name of a store you know as the seller and use your own name as the purchaser. Use the date on which you are doing this work.

CRAWFORD & COMPANY,
Cleveland, Ohio.

December 30, 1918.

Sold to Stephen Langmaid,
489 E. 103rd Street, City.

Dec. 4.	7 pairs of curtains at $3.75 per pair	
	1 picture $15.00	
	1 picture $7.50	
	1 picture $10.75	
	6 small pictures at $1.40 apiece	
	1 clock $18.00	
	Less 10%	
	Net	

Make a bill for each of the following:

2. 6 dining room chairs at $4.75 apiece, 3 parlor chairs at $7.80 apiece, 2 rugs at $45.00 apiece, 3 small rugs at $7.50 apiece, 1 center-table $30, one sideboard $60, one dining table $45. Discount, 10%.

3. One set dishes $35.00, one fireless cooker $15.00, 3 pans at 75c., one bread-box $1.25, other kitchen utensils $12.40. Discount, 5%.

4. 16 sheets at 45c., 24 pillow-cases at 25c., 5 dozen towels at $3.40 per dozen, 8 bath towels at 32c. apiece, 18 dish-towels at 12c. apiece. Discount 15%.

5. One suit $27.50, one pair of shoes $6.50, one pair of shoes, $8.00, one overcoat $35.00, 6 shirts at $2.25 apiece, 4 suits of underwear at $1.75 a suit, one hat $5.00, 8 pairs of socks at 45c. a pair. Discount 20%.

CHAPTER III.

THE DECIMAL NUMBER SYSTEM

ORAL REVIEW

1. In the number 22, what does each 2 represent?
2. In the number 4444, what does each 4 represent?
3. In the number 3.3, what does each 3 represent?
4. In the number 55.55, what does each 5 represent?
5. In the number 66,666.6666 what does each 6 represent?
6. Which is *ones'* place? *tens'* place? *hundreds'* place? *thousands'* place?
7. Which is *tenths'* place? *hundredths'* place? *thousandths'* place? *ten thousandths'* place?

83. **Place Value in the Decimal System.** The value of the figures in each place is shown by the following:

billions	hundred millions ten millions millions	hundred thousands ten thousands thousands	hundreds tens ones	tenths hundredths thousandths ten thousandths
2	895	325	674	.6492
Billions	Millions	Thousands	Ones	Decimal Fractions.

This number is read, two billion, eight hundred ninety-five million, three hundred twenty-five thousand, six hundred seventy-four, and six thousand four hundred ninety-two ten thousandths

A decimal fraction is read as a whole number, and the name of the last figure is given to it. Thus if the last figure is in thousandths' place the fraction is read as thousandths.

84. Place Value in the Decimal Notation. The most important fact about the decimal notation is:

> *A figure in any one place represents ten times as much as the same figure in the next place to the right.*

This gives rise to the rule for multiplying and dividing a number by 10, 100 and so on.

That is, to multiply a number like 6 by 10 we need only to place the 6 in tens' place, which we do by annexing one zero to the right, or by moving the decimal point one place to the right.

ORAL EXERCISES

1. How may a number be multiplied by 10? By 100?
2. If in 74.26 the decimal point is moved one place to the right, how is the value of each figure changed?

Notice that by moving the decimal point one place to the right, each figure is made to represent ten times as much as it did before.

3. How may a number like 34.78 be multiplied by 10? By 100? By 1000?
4. If in 419.74 the decimal point is moved one place to the left, how is the value of each figure changed.

Notice that by moving the decimal point one place to the left each figure is made to represent one-tenth as much as it did before.

5. How may a number like 1983.71 be divided by 10? By 100? By 1000?
6. How may a number like .064 be divided by 10? By 100? By 1000?

It is difficult to overestimate the importance of the decimal notation in our modern life, in which a great deal of computing is required. To convince yourself of this write two numbers like 286 and 1892 in the Roman notation and try to get their products.

85. Adding by Groups. In adding columns greater speed may be made by *grouping* some of the figures.

6
5
4
7
9
2
8
7
3
51

Beginning at the top we notice when we come to 2 that 2 and 8 make 10, and also that 3 and 7 make 10. Adding, we say, 6, 11, 15, 22, 31, 41, 51. If we add upward, we say 10, 20, 29, 36, 40, 45, 51.

Numbers other than those which make 10 are sometimes grouped, but that is more difficult, and much more practice is required to make it worth while. If the sum of two or more numbers is 10, they should always be grouped.

ORAL EXERCISES

In this manner add the following:

1.	2.	3.	4.	5.	6.	7.	8.
4	9	5	5	8	5	4	3
9	1	3	7	9	4	8	9
8	4	9	9	7	3	2	2
6	6	6	8	8	2	4	6
4	7	2	2	2	1	6	8
9	4	7	4	4	8	7	2
1	3	6	3	7	9	3	4
6	8	4	6	6	1	5	3
4	7	8	9	3	5	8	7

WRITTEN EXERCISES

Copy and add the following:

9.	10.	11.	12.
5973	43006	1234	4331
1209	1487	9876	6675
3400	97500	379	890
9146	2469	340	3943
256	891	8926	86598
47	5643	317	1543
8091	796	3894	246
4237	84	327	9400
3107	651	1650	5920
1376	5390	8728	9347

WRITTEN EXERCISES

See how long it takes you to find the sums of the following:

1.	2.	3.	4.
8540	65200	9428	9839
719	1648	256	4206
39570	924	3847	31936
672	19467	4600	2476
1349	3928	3902	3930
8558	7276	25607	6042
3400	13400	19008	54320
12560	6731	2480	1719
7780	8764	5937	8943
1242	8193	6527	8370
5491	2789	1012	1494

To add decimals write them in a column, so that the decimal points are in a straight column. Then add them exactly the same as whole numbers.

Copy and add the following:

5. 4.08+390.2+568.+.091+8.34+43.67+1.24.

6. 268.009+13.031+7.032+2.901+.0084.+8.734+3.19.

7. 67480+.2913+4.721+.0216+3.491+1.742+.0291.

8. 0917+2.947+4.496+4.246+8.428+.098+.72.

9. 1.098+7.854+.0321+0246+5.40+2.190+5.741.

10. 81.09+4.657+1.32+2.624+5.40+2.190+5.741.

11. 67.81.09+12+.736+58.24+16.67+0219+.192.

12. .1804+.129+4.91+2.502+10.84+6.18+.49.

13. 2.790+3.094+.076+1.914+.004+0.191+14.68.

14. .894+76.4+1.426+.094+9.473+864.+29.9.

15. 0.496+3.98+43.6+743.1+0.084+0.91+1.87.

Example 1. From 1843 subtract 587.

1843	
587	Explain each step in this subtraction.
1256	

WRITTEN EXERCISES

Subtract:

1. 5432	2. 1956	3. 672	4. 785
2205	1448	439	357
5. 3491	6. 1205	7. 2742	8. 5034
2378	987	1894	2870
9. 6007	10. 27450	11. 3700	12. 65391
3442	3719	1394	31628
13. 31620	14. 19300	15. 4302	16. 65283
19374	8234	2397	37809

Example 2. From 192.4 subtract 24,651.

192.400	First annex enough zeros in the minuend to make number of decimal
24.651	places the same as in the subtrahend. Subtract as if there were
167.749	no decimal points present.

To subtract one decimal number from another write them so that the decimal points stand in a column.

Subtract the following:

17. 27.09 − 12.38	24. 1.064 − 0.8704
18. 5.928 − .064	25. 81.90 − 35.086
19. 29.4 − 3.78	26. 12.1 − 9.872
20. 326.04 − 237.208	27. 0.947 − 0.1796
21. 675 − 279.046	28. 1.304 − 0.847
22. 56.2 − 13.489	29. 24.76 − 3.942
23. 4.98 − 1.094	30. .0194 − .0083

	Boys		Girls	
Age	Height	Weight	Height	Weight
2	2.60	25.01	2.56	23.53
3	2.82	28.30	2.78	26.2
4	3.04	31.38	3.00	28.67
5	3.26	34.64	3.21	32.10
6	3.44	38.80	3.33	35.29
7	3.58	40.14	3.50	38.65
8	3.76	43.08	3.68	42.82
9	4.00	49.95	3.92	47.10
10	4.18	55.03	4.08	51.95
11	4.36	59.77	4.26	56.57
12	4.57	67.34	4.41	64.35
13	4.72	75.81	4.60	72.65
14	4.93	88.75	4.76	80.45
15	5.07	96.40	4.92	92.04
16	5.18	107.50	5.02	94.02
17	5.36	116.56	5.10	104.43
18	5.44	127.59	5.13	112.55

The above table gives the average height and weight of boys and girls from 2 to 18 years of age. The heights are given in feet and decimal fractions of feet, and the weights in pounds and decimal fractions of pounds.

WRITTEN EXERCISES

1. Make a table like the above, showing the gain in height of the boys each year.

2. Make a table like the above, showing the gain in weight of the boys each year.

3. Make tables like these for the girls.

A table such as the above can be only a rather rough approximation. It would be different for different nationalities and also for different groups of the same nationality.

86. Multiplying by 50, 25, 125. A number may be multiplied by 50 by multiplying it by 100 and dividing the product by 2. Hence to multiply by 50 annex two zeros and divide by 2.

That is, $50\times786=2\overline{)78600}=39300$.

Such products may be written down at once as $50\times786=39300$.

A number may be multiplied by 25 by multiplying it by 100 and dividing the product by 4. Hence to multiply by 25 annex two zeros and divide by 4.

That is, $25\times826=4\overline{)82600}=20650$.

Since 125 is $\frac{1}{8}$ of 1000, a number may be multiplied by 125 by multiplying by 1000 and then dividing by 8. Hence, to multiply by 125 annex three zeros and divide by 8.

That is, $125\times942=8\overline{)942000}=117750$.

WRITTEN EXERCISES

1. Write the products of 50×748, 50×910, 50×3487.

2. Write the products of 25×731, 25×398, 25×448, 25×13900, 25×437.

3. Write the products of 125×640, 125×872, 125×448, 250×384, 500×434. 500×192.

Multiply:

4. 183	5. 914	6. 4021	7. 5970
241	286	607	635
8. 3491	9. 1205	10. 2700	11. 5034
237	386	189	860
12. 6070	13. 2450	14. 4270	15. 3910
423	371	394	163
16. 3021	17. 2390	18. 4302	19. 5283
473	824	267	809

WRITTEN EXERCISES

1. If you did not know how to multiply .1 by .2 show how you could find the product by writing these as common fractions.

2. In the same way show how you could find the products of .03×.08, .09×.7, .13×.10.

3. State the rule for placing the decimal point in the product of two decimal fractions.

Find the products of the following:

4. .097×8.56	5. 2.91×.0031	6. 7.42×.421
7. 137×.062	8. 5.34×.571	9. .0056×841
10. .095×3.47	11. .026×.079	12. 1.49×.084
13. .184×265.1	14. 5.9×.0734	15. 1.024×.0075

16. A load of coal contains 2.45 tons. How much does it cost at $5.75 a ton? Express the result to the nearest cent.

17. A farmer hauls $3\frac{1}{4}$ (3.25) cords of wood in a load? What is the value of 9 such loads at $4.25 a cord?

18. At $0.85 per thousand cubic feet, what is the bill for 5860 cubic feet of gas? Express 5860 cubic feet as 5.86 thousands.

19. An oriental rug is 6 feet 3 inches (6.25 feet) wide, and 9 feet 9 inches (9.75 feet) long. At $1.60 a square foot, what is its value? Express the result to the nearest cent.

20. A lot in the city of New York was sold at $1.45 a square foot. What was its value if it was 24.37 feet wide and 81.64 feet deep? Express the result to the nearest dollar.

21. At 57.5 pounds to the cubic foot, find the weight in tons of the ice on a pond of 75,000 square feet if it is 1 foot thick?

22. At $8.25 a ton what is the cost of a year's supply of coal consisting of 18.5 tons?

ORAL EXERCISES

Give the steps in long division, and illustrate with an example.

WRITTEN EXERCISES

1. Divide 492.76 by 28.

Explain how the first figure in the quotient is placed. Where is the decimal point in the quotient placed?

2. Divide 5.9176 by 231.

To place the decimal point in the quotient we must prefix a zero. Complete the division.

3. Divide 1.3467 by 2.145.

Explain how the divisor is changed into a whole number.

4. Find quotient in 8462÷35 to the nearest integer.

Explain why it is 242 and not 241.

5. Find quotient in 394.2÷58.7 to nearest hundredth.

Explain why it is 6.72 and not 6.71.

Find the quotients in the following to the nearest integers:

6. 32)76500	7. 192)364000	8. 286)484000
9. 81)62000	10. 394)84300	11. 392)64800

In the following find the quotients to the nearest tenth:

12. 4.7)679.24	13. 18.6)37.94	14. 49)876.20
15. 189)68.60	16. 75.4)19470.8	17. 38.6)3678.93

Find the quotients in the following to the nearest hundredth:

18. 63.9)49.28	19. 2.91)874.97	20. 31.9).0749
21. 14.7)524.36	22. 13.9)49.75	23. 4.76)0.4213
24. 67.8)24.93	25. 0.87)47.916	26. 0.94).3124
27. 42.4)324.98	28. 1.46)61.97	29. 0.018)5.9762

1. State the principle of place value in the decimal notation. Does this principle hold for both integers and decimals?

2. How may a whole number be multiplied by 10? by 100? by 1000? How may a decimal fraction be multiplied by these numbers?

3. How may a whole number or a decimal fraction be divided by 10? by 100? by 1000?

4. Compare addition of whole numbers and of decimal fractions. Are there any differences, and if so, what?

5. Compare subtraction of whole numbers and of decimal fractions. Are there any differences, and if so, what?

6. State the rule for multiplying decimal fractions.

7. State the rule for placing the decimal point in the quotient when the divisor is a whole number.

8. A certain divisor is in the form of a decimal fraction. How may it be changed to a whole number?

9. Give a short method for multiplying by 25, by 125.

10. Give a short method for dividing by 25, by 125.

11. How is a quotient found to the nearest unit? To the nearest tenth? To the nearest hundredth?

12. Give a definition of subtraction which shows how subtraction is related to addition.

13. Give a definition of division which shows how division is related to multiplication.

14. If the age of each pupil in a class is given, how may the average age be found?

15. If the dimensions of a box are measured in inches how may its volume in cubic feet be found?

1. In a certain American city having a population of 567,000 there are 69,600 telephones. How many telephones are there for each 1000 inhabitants?

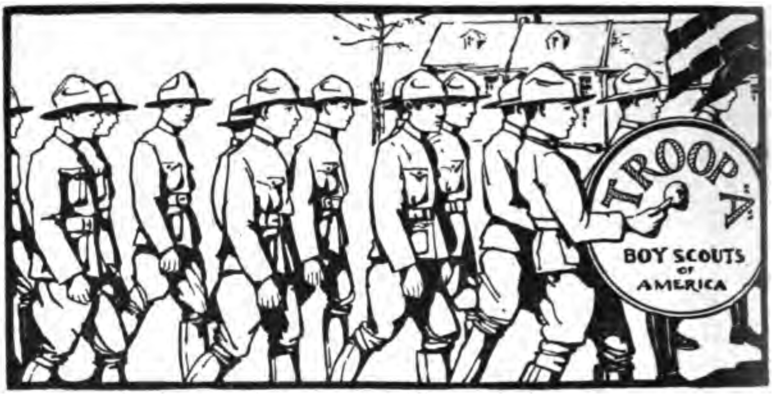
2. In the city of Tokio with a population of 2,170,000 there were 27,800 telephones in a recent year. How many telephones were there for each 1000 inhabitants. Compare with the result in example 1.

3. It has been found that it takes on an average 5.6 pounds of corn to produce one pound of pork when feeding it to hogs. A farmer feeds 4680 pounds of corn to his hogs. How many pounds of pork should this produce?

4. During 11 years a certain cattle company averaged feeding 13.3 pounds of grain and 9.4 pounds of hay for each pound of gain in beef cattle. How many tons of grain and hay did they feed a herd which gained 7890 pounds?

5. On a certain farm the average yield of hay per acre is $2\frac{3}{5}$ tons. At this rate what is the value of the yield from 27 acres if hay is worth $17.40 per ton? (Reduce $2\frac{3}{5}$ to a decimal.)

6. The American bushel contains 2150.42 cubic inches, and the English Imperial bushel 2218.192 cubic inches. What is the difference in cubic inches between 1000 American and 1000 English bushels? How many American bushels would this difference amount to?



In a troop of boy scouts each boy buys an outfit as follows:

Hat	$1.15	Shirt	$1.50	Hatchet	$1.50
Coat	2.15	Gaiters	1.10	Staff	.25
Belt	.60	Breeches	2.50	Knife	1.25

WRITTEN EXERCISES

1. Find the cost of each boy's outfit.

2. If there are 32 boys in the troop, find how much the outfits of all the boys cost.

3. In the month of July John worked 17 days on the golf links as a caddie. He earned 75 cents a day for 6 days, $1 a day for 7 days, and $1.25 a day for 4 days. How much did he earn? Did he earn enough to pay for his boy scout outfit? How much did he have left after buying the outfit?

4. On a camping trip the boy scouts went fishing and swimming. What articles would they bring for these purposes? Find what these articles would cost, and make problems about them.

A troop of 32 boys with three scoutmasters went on a two-weeks' camping trip. They borrowed tents and other necessaries from the U. S. Forestry Service.

PROBLEMS

1. If the camp was 37 miles from the boys' homes, how far did each boy travel going to the camp and returning?

2. Food cost each boy $3 a week. How much did he pay for food for the two weeks?

3. Beside the regular outfit John bought a fishing rod for $1.75, hooks and flies for $2.25, and a reel for $2.50. How much did these articles cost him?

4. Harry bought a camera for $5.00, films for 75 cents, and paid $1.25 for developing pictures. How much did he pay for these altogether?

5. If transportation cost each boy one cent a mile, how much did the whole troop pay for transportation, including the transportation of the scoutmasters?

6. At $3 a week for food, how much did the food cost the whole troop, including the scoutmasters, for the two weeks of camping?

7. Make up a list of the total expenditure of each boy for outfit, transportation, and food. What are these expenses? What are the total expenses of the troop for all these items, including the three scoutmasters?

87. Meaning of a Fraction. If a line one inch long is divided into 4 equal parts, and 3 of them are taken, we have a line $\frac{3}{4}$ of an inch long.

Hence, $\frac{3}{4}$ may be regarded as 3 of the 4 equal parts of anything. This illustrates the following statement:

I. *The denominator of a fraction tells into how many equal parts a whole has been divided, and the numerator tells how many of these parts are taken.*

If a line 3 inches long is divided into 4 equal parts, then each part is $\frac{3}{4}$ of an inch long.

Hence, $\frac{3}{4}$ may also be regarded as 3 divided by 4. This illustrates the following statement:

II. *A fraction is the indicated quotient in which the numerator is divided by the denominator.*

These two ways of regarding a fraction should be clearly understood, since they are both in practical use.

ORAL EXERCISES

Give the meaning of each of the following fractions in the two ways just discussed: $\frac{7}{8}$, $\frac{5}{7}$, $\frac{13}{9}$, $\frac{13}{16}$, $\frac{25}{4}$, $\frac{11}{32}$, $\frac{47}{9}$.

88. Fractional Units. Such fractions as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, which have a numerator 1, are called *fractional units*, or *unit fractions*.

The fraction $\frac{2}{3}$ represents 2 of the fractional units $\frac{1}{3}$. In $\frac{4}{5}$ the fractional unit is $\frac{1}{5}$, and $\frac{4}{5}$ represents 4 of these.

ORAL EXERCISES

1. What is the fractional unit in each of the following: $\frac{3}{4}$, $\frac{7}{8}$, $\frac{5}{6}$, $\frac{5}{9}$, $\frac{13}{16}$, $\frac{7}{16}$, $\frac{7}{12}$, $\frac{3}{10}$? How many of the fractional units of each fraction are represented by each of these?

2. What is the fractional unit in each of the following, and how many of them are represented: $\frac{3}{7}$, $\frac{4}{3}$, $\frac{9}{8}$, $\frac{14}{9}$, $\frac{3}{16}$, $\frac{21}{5}$?

ORAL EXERCISES

1. How many 3ds are there in 1? how many 4ths? how many 10ths? how many 65ths?

2. How many 8ths are there in 1? in 2? in 4? in 7? in 9?

3. How many 6ths are there in 5? in 6? in 8? in 10?

4. How many ones are there in $\frac{3}{3}$? in $\frac{6}{3}$? in $\frac{9}{3}$? in $\frac{15}{3}$? in $\frac{36}{3}$? in $\frac{48}{3}$? in $\frac{60}{3}$?

5. What is a mixed number? Give an example.

6. How many ones are there in $\frac{8}{3}$? How many 3ds are left over?

Reduce the following to mixed numbers:

7. $\frac{9}{4}$, $\frac{15}{4}$, $\frac{12}{5}$, $\frac{17}{5}$, $\frac{17}{4}$, $\frac{21}{8}$, $\frac{27}{8}$, $\frac{33}{8}$.

8. $\frac{21}{4}$, $\frac{49}{8}$, $\frac{78}{4}$, $\frac{34}{3}$, $\frac{47}{5}$, $\frac{89}{9}$, $\frac{94}{12}$, $\frac{73}{8}$.

9. $\frac{25}{6}$, $\frac{43}{7}$, $\frac{58}{9}$, $\frac{67}{8}$, $\frac{35}{4}$, $\frac{59}{7}$, $\frac{84}{9}$, $\frac{53}{6}$.

10. Reduce each of the numbers 4, 3, 5, 6, 7, 8, 2, 6, 9, 12 to 3ds.

11. Reduce each of the numbers, 5, 2, 6, 9, 4, 3, 8, 7, 1 to 7ths.

12. What is a proper fraction? An improper fraction? Give an example of each. Is $\frac{5}{5}$ a proper or an improper fraction?

Reduce the following to improper fractions:

13. $1\frac{1}{2}$, $3\frac{1}{4}$, $5\frac{1}{3}$, $6\frac{2}{3}$, $7\frac{3}{4}$, $9\frac{5}{8}$, $7\frac{7}{8}$, $3\frac{9}{16}$.

14. $2\frac{7}{8}$, $5\frac{3}{16}$, $3\frac{1}{32}$, $1\frac{5}{32}$, $4\frac{3}{16}$, $2\frac{5}{16}$, $3\frac{7}{16}$, $5\frac{9}{16}$.

15. $4\frac{5}{12}$, $8\frac{5}{8}$, $9\frac{3}{8}$, $1\frac{5}{64}$, $1\frac{9}{64}$, $2\frac{11}{32}$, $3\frac{7}{32}$, $2\frac{15}{32}$.

16. $\frac{2}{2}$ of a number is what part of it?

17. $\frac{3}{3}$ of 15 is what? $\frac{9}{9}$ of 18 is what?

18. $\frac{5}{5}$ of a number is what part of the number?

19. What is a fractional unit? What number in a fraction tells the size of the fractional unit? What number tells how many of these units are represented?

ORAL EXERCISES

1. Compare $\frac{1}{2}$ and $\frac{1}{4}$, $\frac{1}{4}$ and $\frac{1}{8}$, $\frac{1}{3}$ and $\frac{1}{15}$, $\frac{1}{5}$ and $\frac{1}{20}$.
2. How is the size of a fractional unit affected by multiplying the denominator by 2? by 3? by 4?
3. If the denominator of a fraction is multiplied by any number, what must be done with the numerator to avoid changing the value of the fraction?

Read and supply the missing numbers:

4. $\frac{3}{4}=\frac{?}{8}$ $\quad\frac{3}{5}=\frac{?}{15}$ $\quad\frac{3}{6}=\frac{?}{12}$ $\quad\frac{?}{3}=\frac{2}{6}$
5. $\frac{3}{7}=\frac{?}{14}=\frac{?}{21}=\frac{?}{28}=\frac{?}{35}=\frac{?}{42}=\frac{?}{49}=\frac{?}{56}$
6. $\frac{5}{8}=\frac{?}{16}=\frac{?}{24}=\frac{?}{32}=\frac{?}{40}=\frac{?}{48}=\frac{?}{56}=\frac{?}{64}$
7. $\frac{7}{9}=\frac{?}{18}=\frac{?}{27}=\frac{?}{36}=\frac{?}{45}=\frac{?}{54}=\frac{?}{63}=\frac{?}{72}$

Give the sums of the following:

8. $\frac{1}{2}+\frac{1}{4}$	18. $\frac{1}{2}+\frac{1}{8}$	28. $\frac{1}{4}+\frac{5}{16}$
9. $\frac{1}{2}+\frac{3}{4}$	19. $\frac{1}{2}+\frac{3}{8}$	29. $\frac{1}{4}+\frac{7}{16}$
10. $\frac{1}{4}+\frac{1}{8}$	20. $\frac{1}{8}+\frac{1}{16}$	30. $\frac{1}{4}+\frac{9}{16}$
11. $\frac{1}{4}+\frac{3}{8}$	21. $\frac{1}{8}+\frac{3}{16}$	31. $\frac{1}{2}+\frac{1}{8}$
12. $\frac{1}{4}+\frac{5}{8}$	22. $\frac{3}{8}+\frac{1}{16}$	32. $\frac{1}{2}+\frac{3}{8}$
13. $\frac{1}{4}+\frac{7}{8}$	23. $\frac{5}{8}+\frac{1}{16}$	33. $\frac{1}{2}+\frac{1}{16}$
14. $\frac{3}{4}+\frac{1}{8}$	24. $\frac{3}{8}+\frac{3}{16}$	34. $\frac{1}{2}+\frac{3}{16}$
15. $\frac{3}{4}+\frac{3}{8}$	25. $\frac{3}{8}+\frac{5}{16}$	35. $\frac{1}{2}+\frac{5}{16}$
16. $\frac{3}{4}+\frac{6}{8}$	26. $\frac{1}{4}+\frac{1}{16}$	36. $\frac{1}{2}+\frac{1}{5}$
17. $\frac{3}{4}+\frac{7}{8}$	27. $\frac{1}{4}+\frac{3}{16}$	37. $\frac{1}{2}+\frac{2}{5}$

38. In each of examples 8–37 subtract the smaller fraction from the larger.

ORAL EXERCISES

1. If $\frac{2}{3}$ of a number is subtracted from it, what fractional part of it remains?

2. Two men own a mill. One man owns $\frac{3}{5}$ of it. What fraction of the mill does the other man own?

3. In a certain school $\frac{4}{7}$ of the pupils are girls. What fraction of the pupils are boys?

4. A farmer plants $\frac{1}{5}$ of his farm in corn, while $\frac{2}{5}$ of it is in pasture and meadow. What fraction of the farm is left for other purposes?

5. In a closely built city $\frac{9}{16}$ of the space is covered with buildings. The remainder is in streets, alleys, and lanes. What fraction of the city is covered by streets, alleys and lanes?

Give the remainders in the following:

6. $1-\frac{2}{3}$	10. $2-\frac{3}{8}$	14. $4-\frac{11}{16}$	18. $9-\frac{3}{5}$
7. $1-\frac{3}{5}$	11. $2-\frac{7}{16}$	15. $4-\frac{15}{32}$	19. $12-\frac{5}{7}$
8. $1-\frac{3}{7}$	12. $2-\frac{5}{8}$	16. $4-\frac{7}{15}$	20. $6-\frac{9}{32}$
9. $1-\frac{3}{8}$	13. $2-\frac{9}{16}$	17. $8-\frac{3}{8}$	21. $4-\frac{7}{9}$

22. A school baseball team wins 7 games and loses 5. What fraction of the games played do they win? What fraction do they lose?

WRITTEN EXERCISES

In each of the following pairs of fractions which is the greater, and how much?

1. $\frac{3}{4}, \frac{7}{8}$	5. $\frac{5}{8}, \frac{11}{16}$	9. $\frac{7}{9}, \frac{13}{18}$
2. $\frac{3}{8}, \frac{7}{15}$	6. $\frac{7}{8}, \frac{13}{16}$	10. $\frac{7}{8}, \frac{19}{32}$
3. $\frac{2}{3}, \frac{5}{8}$	7. $\frac{5}{12}, \frac{11}{24}$	11. $\frac{5}{12}, \frac{7}{16}$
4. $\frac{3}{4}, \frac{13}{16}$	8. $\frac{7}{12}, \frac{13}{24}$	12. $\frac{9}{16}, \frac{13}{24}$

89. Factors. If a whole number is the product of two other whole numbers, these are called *factors* of the given number.

Thus, 2 and 3 are factors of 6; 4 and 3 are factors of 12; 1 and 6 are also factors of 6.

90. Primes and Composite Numbers. A number which has no factors except 1 and itself is called a *prime* number. Thus, 1, 2, 3, 5, 7, 11 are prime numbers.

Numbers which are not prime are called *composite numbers.*

ORAL EXERCISES

1. Give two factors of each composite number below 30.

The prime factors of 8 are 2, 2, 2, because 2 is a prime number and because $2 \times 2 \times 2 = 8$. Similarly, 2, 2, 3 are the prime factors of 12.

2. Give the prime factors of 9, 16, 18, 20, 24, 27, 30, 32.

3. What is the first step in adding or subtracting fractions not having a common denominator?

WRITTEN EXERCISES

Reduce the following to like fractions. (See pages 42 and 49.)

1. $\frac{1}{2}, \frac{3}{4}, \frac{5}{8}$	4. $\frac{3}{16}, \frac{5}{14}, \frac{7}{30}$	7. $\frac{3}{8}, \frac{15}{18}, \frac{5}{30}$
2. $\frac{1}{6}, \frac{3}{8}, \frac{5}{12}$	5. $\frac{3}{8}, \frac{5}{12}, \frac{4}{15}$	8. $\frac{3}{16}, \frac{7}{24}, \frac{7}{30}$
3. $\frac{4}{5}, \frac{3}{4}, \frac{1}{3}$	6. $\frac{1}{7}, \frac{1}{9}, \frac{4}{15}$	9. $\frac{5}{12}, \frac{11}{18}, \frac{7}{30}$

Perform the following indicated operations:

10. $\frac{1}{4}+48+\frac{5}{12}$	15. $\frac{9}{10}+\frac{1}{12}+\frac{5}{60}$	20. $\frac{5}{6}+\frac{7}{8}+\frac{9}{10}$
11. $\frac{5}{6}-\frac{3}{4}$	16. $\frac{3}{4}+\frac{5}{6}+\frac{7}{8}$	21. $\frac{7}{12}+\frac{5}{9}+\frac{5}{6}$
12. $\frac{9}{16}-\frac{7}{12}$	17. $\frac{8}{9}+\frac{5}{6}+\frac{3}{4}$	22. $\frac{3}{8}+\frac{5}{16}+\frac{7}{24}$
13. $\frac{7}{12}+\frac{8}{12}+\frac{5}{18}$	18. $\frac{4}{5}+\frac{3}{7}+\frac{8}{21}$	23. $\frac{1}{3}+\frac{1}{12}+\frac{1}{15}$
14. $\frac{7}{8}+\frac{3}{5}+\frac{9}{10}$	19. $\frac{1}{12}+\frac{1}{15}+\frac{1}{16}$	24. $\frac{4}{9}+\frac{5}{18}+\frac{7}{27}$

ORAL AND WRITTEN EXERCISES

1. How do you multiply a fraction by an integer?

Give the products of the following, being careful to perform all possible cancellations before multiplying:

2. $\frac{3}{4}\times 6$, $\frac{2}{3}\times 7$, $\frac{4}{5}\times 15$, $\frac{7}{10}\times 25$.

3. $\frac{5}{6}\times 16$, $\frac{3}{4}\times 18$, $\frac{2}{5}\times 12$, $\frac{3}{7}\times 21$.

4. Give the rule for multiplying one fraction by another.

Give the products indicated below. Do as many as you can without using pencil and paper.

5. $\frac{5}{7}\times\frac{4}{5}$, $\frac{3}{5}\times\frac{2}{9}$, $\frac{6}{7}\times\frac{3}{4}$, $\frac{1}{4}\times\frac{2}{5}$

6. $\frac{3}{10}\times\frac{5}{9}$, $\frac{7}{8}\times\frac{3}{4}$, $\frac{7}{12}\times\frac{5}{8}$, $\frac{9}{16}\times\frac{4}{15}$.

7. What is the best method for multiplying a mixed number by a whole number, as $2\frac{3}{4}$ by 6?

8. Does it make any difference whether you multiply $2\frac{3}{4}$ by 6 or 6 by $2\frac{3}{4}$?

9. Give the products of $1\frac{1}{2}\times 2$, $2\frac{1}{3}\times 6$, $4\times 1\frac{4}{5}$, $5\frac{2}{3}\times 6$.

10. Find the product of $1\frac{1}{3}\times 2\frac{3}{4}$ by reducing each mixed number to an improper fraction.

11. Find the product of $34\frac{1}{4}\times 18\frac{3}{8}$ by the four-step method. (See page 70.)

The method of example 10 is usually shorter when the numbers are small, and the four-step method when they are large.

Find the products of the following:

12. $2\frac{1}{4}\times 4\frac{1}{5}$	13. $1\frac{1}{2}\times 5\frac{1}{3}$	14. $8\frac{1}{4}\times 3\frac{2}{3}$
15. $6\frac{4}{5}\times 5\frac{3}{4}$	16. $4\frac{1}{2}\times 5\frac{1}{3}$	17. $12\frac{1}{3}\times 2\frac{1}{3}$
18. $8\frac{1}{2}\times 5\frac{2}{3}$	19. $10\frac{4}{5}\times 4\frac{4}{9}$	20. $6\frac{3}{5}\times 10\frac{2}{3}$
21. $16\frac{1}{3}\times 7\frac{1}{2}$	22. $41\frac{1}{2}\times 56\frac{3}{4}$	23. $38\frac{1}{4}\times 16\frac{2}{3}$

ORAL EXERCISES

1. How do you divide a fraction by an integer?

2. Give the quotients: $\frac{2}{3}\div 2$, $\frac{2}{3}\div 3$, $\frac{3}{5}\div 2$, $\frac{6}{7}\div 12$.

3. How do you divide any number by a fraction?

Find the quotients of the following:

4. $\frac{3}{5}\div 6$	6. $\frac{4}{5}\div 4$	8. $\frac{8}{9}\div 4$	10. $\frac{5}{8}\div 15$
5. $\frac{4}{7}\div 2$	7. $\frac{7}{9}\div 3$	9. $\frac{5}{12}\div 10$	11. $\frac{9}{16}\div 6$

WRITTEN EXERCISES

1. $\frac{2}{3}\div\frac{1}{2}$	5. $\frac{3}{5}\div\frac{2}{3}$	9. $\frac{5}{6}\div\frac{3}{5}$	13. $6\div\frac{3}{5}$
2. $\frac{3}{4}\div\frac{2}{3}$	6. $\frac{5}{6}\div\frac{3}{4}$	10. $\frac{7}{8}\div\frac{5}{16}$	14. $3\div\frac{5}{7}$
3. $\frac{3}{8}\div\frac{3}{4}$	7. $\frac{5}{6}\div\frac{3}{8}$	11. $\frac{9}{16}\div\frac{3}{4}$	15. $4\div\frac{7}{8}$
4. $\frac{5}{8}\div\frac{3}{4}$	8. $\frac{4}{5}\div\frac{7}{12}$	12. $\frac{7}{9}\div\frac{14}{16}$	16. $9\div\frac{4}{5}$

Example 1. Divide $8\frac{3}{4}$ by 3.

$$8\tfrac{3}{4}\div 3=\tfrac{35}{4}\div 3=\tfrac{35}{12}=2\tfrac{11}{12}.$$

Divide the following:

17. $4\frac{1}{3}\div 2$	20. $9\frac{4}{5}\div 3$	23. $8\frac{1}{2}\div 3$	26. $8\frac{3}{4}\div 5$
18. $7\frac{2}{3}\div 3$	21. $12\frac{3}{4}\div 3$	24. $4\frac{5}{6}\div 2$	27. $17\frac{2}{3}\div 4$
19. $4\frac{3}{8}\div 6$	22. $18\frac{4}{5}\div 5$	25. $19\frac{3}{7}\div 6$	28. $24\frac{1}{2}\div 3$

Example 2. Divide $1\frac{2}{3}$ by $2\frac{4}{5}$.

$$1\tfrac{2}{3}\div 2\tfrac{4}{5}=\tfrac{5}{3}\div\tfrac{14}{5}=\tfrac{5}{3}\times\tfrac{5}{14}=\tfrac{25}{42}$$

Divide the following:

29. $3\frac{1}{4}\div 2\frac{2}{3}$	33. $7\frac{1}{8}\div 5\frac{3}{4}$	37. $6\frac{2}{3}\div 8\frac{1}{2}$
30. $7\frac{1}{5}\div 9\frac{2}{3}$	34. $12\frac{1}{4}\div 2\frac{3}{5}$	38. $16\frac{7}{8}\div 2\frac{3}{4}$
31. $8\frac{1}{3}\div 3\frac{1}{4}$	35. $6\frac{3}{4}\div 2\frac{1}{4}$	39. $4\frac{5}{16}\div 2\frac{1}{2}$
32. $3\frac{4}{5}\div 1\frac{1}{3}$	36. $24\frac{3}{4}\div 6\frac{1}{3}$	40. $12\frac{5}{8}\div 4\frac{7}{8}$

91. **Reducing Fractions to Decimals.** Since a common fraction may be regarded as an indicated division (see page 152), it may be reduced to a decimal by carrying out the division.

Example 1. Reduce $\frac{3}{8}$ to a deoimal. $\quad 8\overline{)3.000}$ = .375

Example 2. Reduce $\frac{3}{7}$ to a decimal. $\quad 7\overline{)3.0000}$ = .4285, remainder 5.

On dividing we find .4285 with a remainder 5. Hence, .4286 is the decimal value of $\frac{3}{7}$ carried to 4 places of decimals. (At this point read again page 82.)

WRITTEN EXERCISES

Reduce the following to three-place decimals:

$$\frac{7}{9},\ \frac{4}{7},\ \frac{8}{13},\ \frac{5}{11},\ \frac{8}{19},\ \frac{5}{31},\ \frac{7}{23},\ \frac{9}{29},\ \frac{14}{23}.$$

ORAL EXERCISES

Find the products in the following:

1. $2\times\frac{2}{5}$	7. $2\times\frac{3}{4}$	13. $3\times\frac{5}{8}$	19. $2\times\frac{3}{8}$
2. $2\times\frac{2}{7}$	8. $4\times\frac{3}{4}$	14. $4\times\frac{3}{8}$	20. $4\times\frac{5}{8}$
3. $2\times1\frac{1}{2}$	9. $2\times1\frac{1}{3}$	15. $2\times1\frac{2}{3}$	21. $2\times2\frac{1}{3}$
4. $6\times2\frac{1}{4}$	10. $6\times2\frac{3}{4}$	16. $6\times3\frac{1}{8}$	22. $6\times3\frac{5}{8}$
5. $3\times2\frac{1}{4}$	11. $3\times2\frac{3}{4}$	17. $3\times5\frac{3}{16}$	23. $3\times5\frac{9}{16}$
6. $\frac{1}{2}\times\frac{1}{2}$	12. $\frac{1}{2}\times\frac{1}{3}$	18. $\frac{1}{2}\times\frac{2}{3}$	24. $\frac{1}{2}\times\frac{1}{4}$

WRITTEN EXERCISES

Perform the indicated operations in the following:

1. $\frac{1}{2}+\frac{1}{2}+\frac{3}{4}$	4. $\frac{7}{8}-\frac{6}{7}+\frac{5}{12}$	7. $10\frac{1}{2}+4\frac{3}{4}$
2. $\frac{4}{5}+\frac{6}{7}+\frac{6}{15}$	5. $2\frac{1}{2}+4\frac{1}{3}+1\frac{5}{8}$	8. $\frac{5}{8}+\frac{3}{4}\times\frac{5}{6}$
3. $\frac{3}{5}-\frac{2}{7}+\frac{3}{4}$	6. $12\frac{1}{3}+2\frac{1}{2}$	9. $\frac{5}{6}+\frac{7}{10}+\frac{8}{15}$

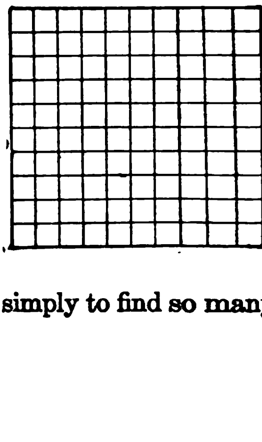
92. Percentage. In very many of the ordinary affairs of life fractions are expressed as *hundredth* or *per cent.* "Per cent" means *in the hundred,* or *hundredths.* The sign of per cent is %. Thus, 1% = .01, or 1/100.

That part of arithmetic which deals with per cents is called *percentage.*

Per cents may always be expressed as decimals. Thus, 2% = .02, 7% = .07, 25% = .25.

ORAL EXERCISES

1. Read the following per cents as decimals: 8%, 4%, 12%, 35%, 40%, 75%.



2. How many small squares are there in this figure?
3. Point out 1% of this figure, 2%, 5%, 10%.
4. Draw a figure like this on the board and point out 20% of it, 30%, 40%, 60%, 80%, 100%.

To find a certain per cent of a number is simply to find so many hundredths of it.

Thus, 5% of 400 = .05 × 400 = 20.

And 9% of 360 = .09 × 360 = 32.40.

5. Find 2% of 200, of 300, of 400.
6. Find 3% of 200, of 500, of 600, of 400.
7. Find 4% of 600, of 800, of 300, of 400.
8. Find 5% of 200, of 600, of 300, of 400.

It is easily seen that percentage involves no new principles of arithmetic. It is simply another name given to what is already known from the study of common fractions and decimals.

93. **Importance of Percentage.** It is not easy to appreciate fully the importance of the use of percentage. Thus, when a farmer takes the milk of two cows to the creamery, he is told that one sample contains 3.8% of butter fat, and the other 4.5%. Suppose that he had been told that one contained $\frac{19}{500}$ and the other $\frac{9}{200}$ of butter fat!

The miner has ore tested, and is told it contains so many % of metal. We say that 33% of the population of Massachusetts are foreign born, that 50.8% of the total population in that state are women, and so on. Through constant use we have become so accustomed to per cent (hundredths) that 8%, for example, has a more direct meaning to us than the equal fraction, $\frac{2}{25}$.

94. **Work in Percentage.** A large share of the work in arithmetic from now on will consist of the solution of problems involving percentage. Your success will depend mainly on two things:

95. **Conditions of the Problem.** The first condition on which your success will depend is a clear understanding of the problems to be solved and of the business practices that give rise to them.

Every effort must be made to learn just what the problem is. You should be on the constant lookout to learn the methods used in business in your own community. Try to visit places of business to see how things are done. The text book will give you descriptions of each topic as it is taken up. You will obtain information from your teacher, your father, and from others who have had practical experience.

96. **No New Principles of Arithmetic.** The second condition on which your success will depend, is a full appreciation of the fact that no new processes of arithmetic are involved.

If percentage and its applications puzzle you, remember that you are only dealing with decimal fractions. Never forget that all you need in solving business problems is the arithmetic you have already studied, some common sense, and a clear understanding of the conditions which give rise to your problems.

97. Base, Rate, Percentage. In the statement "5% of 600 is 30," 600 is called the *base*, 5% or .05 the *rate per cent*, or merely the *rate*, and 30 the *percentage*.

The number of which a certain per cent is taken is called the *base;* the number of per cent is called the *rate per cent*, or simply the *rate;* the result is called the *percentage*.

ORAL EXERCISES

1. Name the base, the rate and the percentage in each of the following: 7% of 100 is 7, 7% of 300 is 21, 15 is 5% of 300.

2. What is 10% of 100? of 200? of 300? of 400? of 500? of 600? of 700? of 800? of 900?

3. What is 20% of each of the numbers in example 2?

4. What is 30% of each of the numbers in exercise 2?

5. What is 50% of each of the numbers in exercise 2?

6. What is 100% of each of the numbers in exercise 2?

Give the following:

7. 8% of 4000, 6% of 6000, 4% of 5000.

8. 3% of 4000, 7% of 2000, 12% of 3000.

9. 4% of 7000, 10% of 1800, 6% of 8000.

10. 6% of 250, 4% of 450, 8% of 650?

11. 3% of 50, of 150, of 450, of 5000.

12. 2% of 100, of 400, of 600, of 1000.

13. 1% of 200, of 300, of 400, of 500.

14. 7% of 300, of 500, of 600, of 800.

15. 40% of 200, of 400, of 500, of 700.

16. 5% of 100, of 500, of 700, of 800.

Example. If the base is 340, and the rate is 15%, what is the percentage?

$$\begin{array}{l} 340 \text{ (base)} \\ \underline{.15} \text{ (rate)} \\ 1700 \\ \underline{340} \\ 51.00 \text{ (percentage)} \end{array}$$

This requires us to find 15% or .15 of 340.

WRITTEN EXERCISES

Find the percentage in the following:

	Base	Rate	Percentage		Base	Rate	Percentage
1.	280	12%	......	18.	86425	40%	......
2.	6700	16%	......	19.	29640	25%	......
3.	1245	25%	......	20.	12400	36%	......
4.	3775	20%	......	21.	38000	4%	......
5.	194	5%	......	22.	5400	18%	......
6.	486	6%	......	23.	1140	60%	......
7.	395	9%	......	24.	690	80%	......
8.	1516	8%	......	25.	428	52%	......
9.	2840	7%	......	26.	3460	67%	......
10.	3650	12%	......	27.	4900	14%	......
11.	2490	16%	......	28.	5400	9%	......
12.	5480	25%	......	29.	6420	12%	......
13.	6870	45%	......	30.	1940	15%	......
14.	3918	13%	......	31.	6780	35%	......
15.	2400	42%	......	32.	9500	48%	......
16.	6600	64%	......	33.	8450	55%	......
17.	27400	35%	......	34.	1250	62%	......

98. Suggestions for Solving Problems. Read the problem with care. If the statement consists of more than one sentence, read each sentence by itself, and insist that you understand it. You can not answer a question unless you know what the question is, and you can not solve a problem unless you know just what the problem is.

In a problem involving percentage, decide which number is the rate, which the base, and which the percentage. In such a problem two of these will be given, and you will be required to find the third.

WRITTEN WORK

1. During one year a certain cow yields 8640 pounds of milk, containing 4% butter fat. How many pounds of butter fat are there in this milk?

2. If 44% of the population are voters, how many voters are there in a state having a population of 2,640,000?

3. The weight of the flour obtained from a certain kind of wheat is 72% of the weight of the wheat. How many pounds of flour will be obtained from one bushel (60 pounds) of wheat?

4. A fruit dealer is obliged to sell a certain lot of fruit for 80% of what it cost him. For how much does he sell it if the cost was $20?

5. A bankrupt firm find they can pay 65% of their debts. How much can they pay if their total debt is $48,600?

6. A piece of land worth $5600 increased 15% in value. How much did the land increase in value?

7. If bread made from a certain brand of flour weighs 35% more than the flour, how much bread can be made from a sack containing 96 pounds of flour?

8. If 76% of a certain kind of potato is water, how many pounds of water are there in a bushel of potatoes weighing 60 pounds.

WRITTEN WORK

1. If 32% of a certain ore is copper, how many pounds of copper are there in one ton (2000 pounds)? What is the value of this copper at 19 cents a pound?

2. Corn left in the crib 4 months shrinks 7% in weight. How much will 1680 bushels shrink in this time?

3. In a certain school there were 260 children in the 4th grade, 245 in the 5th grade, 230 in the 6th grade. In the 4th grade of this school 85% passed in arithmetic. How many passed in arithmetic in this grade?

4. In the 5th grade 80% passed in arithmetic. What per cent did not pass? How many did not pass in this grade?

5. In the 6th grade 90% passed in arithmetic. How many per cent did not pass? How many children passed?

6. A family living in a city calculate to spend about 15% of their income for rent. How much can they pay for rent if their income is $1800? About how much rent per month can they pay?

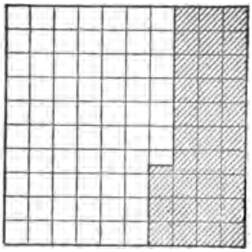
7. This family spends 24% of their income for food. How much a month is this?

8. A certain grade of automobile is supposed to depreciate 40% in value the first year. How much does such a car depreciate in one year if it was worth $2150 when new? How much is it worth at the end of one year?

9. In one year a certain cow yields 10480 pounds of milk, 4% of which is butter fat. At 38 cents a pound, how much is this butter fat worth?

10. In a school with 1200 pupils 19% of them are in the first grade, 17% in the second 15% in the third, 14% in the fourth, 12% in the fifth 10% in the sixth, 7% in the seventh, and 6% in the eighth grade. How many are there in each grade?

ORAL EXERCISES



1. Point out one per cent of the large square. Also point out 10% of it.
2. How many per cent. of the large square are shaded? ($\frac{1}{3}$ of the small square is shaded.)
3. What common fraction of the large square is shaded?
4. How many per cent. of the large square are not shaded?
5. What common fraction of the large square is not shaded?

In reducing $\frac{1}{3}$ to a decimal we get $3\overline{)1.00}^{\,.33\frac{1}{3}}$, which also shows that $\frac{1}{3}=33\frac{1}{3}\%$.

Similarly, $\frac{2}{3}=66\frac{2}{3}\%$.

To reduce a common fraction to per cents, reduce it to a decimal to the nearest hundredth and write as per cent.

$\frac{1}{2}=.50=50\%$	$\frac{1}{12}=.08\frac{1}{3}=8\frac{1}{3}\%$
$\frac{1}{3}=.33\frac{1}{3}=33\frac{1}{3}\%$	$\frac{1}{15}=.06\frac{2}{3}=6\frac{2}{3}\%$
$\frac{2}{3}=.66\frac{2}{3}=66\frac{2}{3}\%$	$\frac{1}{16}=.06\frac{1}{4}=6\frac{1}{4}\%$
$\frac{1}{4}=.25=25\%$	$\frac{1}{20}=.05=5\%$
$\frac{1}{5}=.20=20\%$	$\frac{1}{25}=.04=4\%$
$\frac{1}{6}=.16\frac{2}{3}=16\frac{2}{3}\%$	$\frac{1}{40}=.02=2\frac{1}{2}\%$
$\frac{1}{8}=.12\frac{1}{2}=12\frac{1}{2}\%$	$\frac{1}{50}=.02=2\%$

Also notice that:

$$\frac{3}{8}=3\times 12\tfrac{1}{2}\%=37\tfrac{1}{2}\%$$
$$\frac{2}{15}=2\times 6\tfrac{2}{3}\%=13\tfrac{1}{3}\%$$
$$\frac{3}{16}=3\times 6\tfrac{1}{4}\%=18\tfrac{3}{4}\%$$
$$\frac{5}{16}=5\times 6\tfrac{1}{4}\%=31\tfrac{1}{4}\%$$

These instances suggest a method for finding the per cent equivalents of many fractions.

Example 1. $\frac{7}{12}$ equals how many per cents?

Since $\frac{1}{12}=8\frac{1}{3}\%$, $\frac{7}{12}=7\times8\frac{1}{3}\%=58\frac{1}{3}\%$.

ORAL EXERCISES

Find the missing numbers in the following:

1. $\frac{3}{4}=?\%$	6. $\frac{3}{8}=?\%$	11. $\frac{2}{15}=?\%$	16. $\frac{7}{16}=?\%$
2. $\frac{2}{5}=?\%$	7. $\frac{5}{8}=?\%$	12. $\frac{4}{15}=?\%$	17. $\frac{9}{16}=?\%$
3. $\frac{3}{5}=?\%$	8. $\frac{7}{8}=?\%$	13. $\frac{7}{15}=?\%$	18. $\frac{11}{16}=?\%$
4. $\frac{4}{5}=?\%$	9. $\frac{5}{13}=?\%$	14. $\frac{3}{16}=?\%$	19. $\frac{3}{20}=?\%$
5. $\frac{5}{6}=?\%$	10. $\frac{7}{13}=?\%$	15. $\frac{5}{16}=?\%$	20. $\frac{6}{25}=?\%$

Example 2. Find $6\frac{1}{4}\%$ of 6400.

Since $6\frac{1}{4}\%=\frac{1}{16}$, we take $\frac{1}{16}$ of 6400, which equals 400.

Example 3. Find $37\frac{1}{2}\%$ of 56.

Since $37\frac{1}{2}\%=\frac{3}{8}$, we find $\frac{3}{8}$ of 56, which equals 21.

Find the percentage in each of the following:

	Base	Rate		Base	Rate
21.	240	50%	31.	48	$6\frac{1}{4}\%$
22.	96	$33\frac{1}{3}\%$	32.	72	$37\frac{1}{2}\%$
23.	24	$66\frac{2}{3}\%$	33.	48	$62\frac{1}{2}\%$
24.	400	25%	34.	60	$13\frac{1}{3}\%$
25.	800	75%	35.	128	$12\frac{1}{2}\%$
26.	480	20%	36.	32	$18\frac{3}{4}\%$
27.	360	$16\frac{2}{3}\%$	37.	160	$2\frac{1}{2}\%$
28.	640	$12\frac{1}{2}\%$	38.	80	$12\frac{1}{2}\%$
29.	2400	$8\frac{1}{3}\%$	39.	48	$8\frac{1}{3}\%$
30.	3000	$6\frac{2}{3}\%$	40.	450	$6\frac{2}{3}\%$

99. Uses of Fractional Per Cents. In many practical applications fractional per cents other than those given on the preceding pages are used. Such per cents are usually expressed as decimal fractions. (For some practical examples see page 169.)

Example. Find 14.7% of 840.

$$\begin{array}{r} 840 \\ \underline{.147} \\ 5880 \\ 3360 \\ \underline{840} \\ 123.480 \end{array}$$

Since 14.7% = .147, the required percentage may be found by multiplying the base by .147.

WRITTEN EXERCISES

1. Find 7.3% of 390.
2. Find 8.9% of 7140.
3. Find 13.1% of 3140.
4. Find 21.7% of 9360.
5. Find 84.4% of 2240.
6. Find 41.7% of 6120.
7. Find 52.3% of 735.8
8. Find 62.8% of 9272.
9. Find 49.4% of 417.40.
10. Find 76.3% of 26.31.

ORAL DRILL

Find the following by using common fractions:

1. Find $33\frac{1}{3}$% of 60. Of 120. Of 300. Of 600.
2. Find 25% of 16. Of 40. Of 200. Of 400.
3. Find 20% of 50. Of 80. Of 200. Of 500.
4. Find $16\frac{2}{3}$% of 60. Of 300. Of 600. Of 720.
5. Find $12\frac{1}{2}$% of 48. Of 72. Of 120. Of 600.
6. Find $8\frac{1}{3}$% of 48. Of 72. Of 120. Of 600.
7. Find $6\frac{1}{3}$% of 60. Of 90. Of 300. Of 600.
8. Find $6\frac{1}{4}$% of 64. Of 128. Of 800. Of 1600.

WRITTEN WORK

1. During one year a certain cow yields 9420 pounds of milk averaging 4.1% of butter fat. How many pounds of butter fat is this? At 42 cents a pound, how much is it worth?

On dairy farms the milk from each cow is tested, and the amount of milk weighed. The table below shows the results for one year for a herd of 12 cows.

No. of cow	Milk, lbs.	% fat	Fat, lbs.	No. of cow	Milk, lbs.	% fat	Fat, lbs.
1	7930	4.3%		7	10700	3.9%	
2	8370	4.2%		8	9874	4.3%	
3	9470	4.1%		9	8960	4.1%	
4	6390	4.4%		10	7530	3.6%	
5	7690	3.9%		11	6590	4.6%	
6	8760	4.0%		12	11800	3.8%	

2. Find the number of pounds of butter fat yielded by each cow.

Rule a sheet of paper properly, copy the table given above, and fill in the columns giving the number of pounds of butter fat.

Such a table enables the farmer to decide which ones of his cows are the more valuable.

3. Ordinary butter contains about $85\frac{1}{2}\%$ of butter fat. How many pounds of butter fat are there in 685 pounds of butter. (*Suggestion:* $85\frac{1}{2}\% = 85.5\% = .855$.)

4. The United States government pays $4\frac{3}{4}\%$ each year in interest on the last war loan. How much interest does a man receive on $8550 of this loan.

5. An article valued at $60.00 is sold at a loss of $16\frac{2}{3}\%$. How much was the loss?

6. A certain ore contains 4.7% gold. How many pounds of gold are there in 10 tons of such ore?

Example 1. .42 is equal to how many per cent?

Since *per cent* means hundredths, .42 = 42%.

Example 2. .378 is equal to how many per cent?

.378 contains 37 hundredths and .8 of one hundredth. Hence, .378 = 37.8%.

Rule. To reduce a decimal fraction to per cent, move the decimal point two places to the right.

That is, 1.96 = 196%, and .0745 = 7.45%.

ORAL EXERCISES

Read the following as per cents:

1. .29	5. .52	9. .024	13. .82
2. .06	6. .124	10. .075	14. 1.081
3. .191	7. .242	11. .0094	15. .078
4. .314	8. .006	12. 1.04	16. .942

Example. Express $\frac{3}{7}$ as per cent, correct to the nearest tenth of one per cent.

$$\begin{array}{r} .429 = 42.9\% \\ 7\overline{)3.000} \end{array}$$

Since $\frac{1}{10}$ of 1% is equal to one-thousandth, we reduce $\frac{3}{7}$ to a decimal to the nearest thousandth.

WRITTEN EXERCISES

Reduce the fractions in the following to per cents, giving each to the nearest tenth of 1%.

1. $\frac{3}{8}$	5. $\frac{3}{13}$	9. $\frac{9}{16}$	13. $\frac{8}{19}$	17. $\frac{8}{75}$
2. $\frac{4}{7}$	6. $\frac{7}{13}$	10. $\frac{9}{11}$	14. $\frac{11}{21}$	18. $\frac{9}{65}$
3. $\frac{7}{8}$	7. $\frac{9}{13}$	11. $\frac{8}{15}$	15. $\frac{5}{34}$	19. $\frac{12}{89}$
4. $\frac{5}{11}$	8. $\frac{5}{16}$	12. $\frac{9}{17}$	16. $\frac{6}{43}$	20. $\frac{15}{98}$

Example 1. 1 is how many per cent of 4?

First Solution: Since 1 is $\frac{1}{4}$ of 4, reduce $\frac{1}{4}$ to .25, which equals 25%.

Second Solution: 1% of 4 = .04, and .04 is contained 25 times in 1.

These solutions are not essentially different, but the first seems to be more direct. Choose one, and use that only.

Example 2. 3 is how many per cent of 8?

Solution: $3/8 = .375 = 37.5\%$, or .08 is 1% of 8, and .08 is contained 37.5 times in 3.

ORAL EXERCISES

1. 5 is how many per cent of 20?
2. 6 is how many per cent of 30?
3. 12 is how many per cent of 24?
4. 8 is how many per cent of 40?
5. 7 is how many per cent of 28?
6. $\frac{1}{2}$ is how many per cent of 5?

Example. 17 is how many per cent of 24? (*Suggestion*: Express $\frac{17}{24}$ as per cent, or divide 17 by 1% of 24.)

WRITTEN EXERCISES

In the following find the rate to the nearest per cent:

1. 35 is how many per cent of 47?
2. 47 is how many per cent of 82?
3. 64 is how many per cent of 689?
4. 43 is how many per cent of 37?
5. 140 is how many per cent of 468?
6. 365 is how many per cent of 468?
7. 160 is how many per cent of 280?

WRITTEN WORK

In the problems on this page in which the rate is required find the rate to the nearest tenth of one per cent.

1. A roast of beef weighed 6 pounds before roasting, and $4\frac{3}{4}$ pounds after roasting. How many pounds in weight were lost in roasting? How many per cent. of the first weight was this? (*Suggestion:* $1\frac{1}{4}$ or 1.25 is how many per cent of 6?)

2. In testing a certain ore, it was found that 250 pounds of ore contained 23 pounds of metal. What per cent of the ore was metal?

3. In a school containing 760 pupils 114 were graduated. How many per cent of the whole school were graduated?

4. In a town having a population of 15,640, there were 1960 pupils in the public schools. How many per cent of the total population were in the school?

5. A bankrupt firm owes $18,630, and has $15,850 with which to pay its debts. What per cent of its debts can this firm pay?

6. An automobile worth $3450, when new, decreases in value 30 per cent the first year. How much is it worth at the end of the year?

7. If you spend $4\frac{1}{2}$ hours (270 minutes) in school each day, and if you spend 25 minutes of this time on arithmetic, what per cent of the whole time do you spend on arithmetic?

8. During one week a boy solved 64 examples, of which 57 were right. What per cent of the examples did he get right?

9. In a school there are 124 pupils in the first grade, 112 in the second, 106 in the third, 98 in the fourth, 92 in the fifth, 86 in the sixth, 72 in the seventh, and 58 in the eighth. What per cent of the total number are in each grade? What is the sum of these per cents? (*Suggestion:* First find the total number in the school.)

WRITTEN EXERCISES

Reduce the following fractions to per cent, correct to the nearest tenth of one per cent:

1. $\frac{3}{7}$	8. $\frac{14}{17}$	15. $\frac{15}{19}$	22. $\frac{5}{32}$
2. $\frac{8}{9}$	9. $\frac{7}{9}$	16. $\frac{4}{13}$	23. $\frac{7}{16}$
3. $\frac{3}{5}$	10. $\frac{8}{3}$	17. $\frac{13}{21}$	24. $\frac{9}{64}$
4. $\frac{6}{7}$	11. $\frac{6}{13}$	18. $\frac{41}{64}$	25. $\frac{19}{40}$
5. $\frac{3}{8}$	12. $\frac{5}{16}$	19. $\frac{38}{93}$	26. $\frac{47}{56}$
6. $\frac{5}{9}$	13. $\frac{8}{17}$	20. $\frac{64}{87}$	27. $\frac{29}{47}$
7. $\frac{5}{11}$	14. $\frac{5}{18}$	21. $\frac{12}{17}$	28. $\frac{38}{57}$

Find the missing numbers in the following, giving the required rate correct to the nearest tenth of one per cent:

	Base	Rate	Percentage		Base	Rate	Percentage
29.	47	600%	......	41.	$4\frac{1}{2}$	.....	2
30.	680	$4\frac{1}{2}$%	......	42.	12	.....	$3\frac{1}{4}$
31.	560	.....	280	43.	43	.....	17
32.	4960	.....	840	44.	670	$16\frac{2}{3}$%	......
33.	350	$12\frac{1}{2}$%	......	45.	590	$12\frac{1}{2}$%	......
34.	6240	.....	140	46.	12900	$6\frac{1}{4}$%	......
35.	7900	15%	......	47.	4960	17%	......
36.	14800	$33\frac{1}{3}$%	......	48.	8400	.....	2100
37.	9400	$12\frac{1}{2}$%	......	49.	17640	$5\frac{1}{4}$%	......
38.	6400	.....	800	50.	9400	9%	......
39.	4500	.....	1500	51.	18400	.....	1650
40.	560	.....	70	52.	5960	.....	680

WRITTEN WORK

1. A merchant estimates that of the accounts owing to him 15% cannot be collected. How much does he expect to lose of $8500?

2. If 15% of a merchant's accounts cannot be collected, how many per cent are collectible? How much of $8500 does the merchant expect to collect?

3. A church subscription amounts to $1245.60. If it is estimated that 95% of this can be collected, how much is it supposed can be collected? What per cent of the total subscription is supposed to not be collectible?

4. A merchant pays his lawyer 10% for collecting a note of $1500. How much does the lawyer receive? What per cent of the value of the note does the merchant receive?

5. A farmer puts 1450 bushels of corn in his crib in the fall. How many bushels does he expect to sell if he expects the corn to shrink 9% before selling?

6. The corn which a farmer places in his crib shrinks 12% before it is sold. What per cent of the amount put into the crib does he sell? How many bushels will he have to sell from 1200 bushels put into the crib in the fall? (Corn shrinks more the longer it is left in the crib.)

7. Only 35% of the electric current supplied to an electric toaster is used in toasting the bread; the remainder is lost. What per cent of the electric current supplied the toaster is lost? How much of $1.50 worth of electric current is lost?

8. Water expands 9% when it freezes into ice. How much will 10 gallons of water expand in freezing into ice?

9. A factory valued at $14,000 is insured for 65% of its value, what would the owner lose if the factory were completely destroyed by fire?

ORAL WORK

1. What is the meaning of the words, "per cent"?

2. Give an example in percentage, stating what number is the base and what number is the rate.

3. How may a given rate per cent be expressed as a decimal?

4. If the base and the rate are given how may the percentage be found?

5. If the per cent spoiled of a consignment of fruit is given, how may the per cent not spoiled be found?

6. If you know a man's income and what per cent of it he spends, how do you find the amount which he spends? How do you find the per cent of his income which he saves? How do you find how much he saves?

7. If you know the cost of a new automobile and the per cent decrease in value in one year, how do you find its value at the end of one year?

8. How may a decimal fraction be reduced to per cents?

9. How may a common fraction be reduced to per cents?

10. If you know the number of pupils in a grade, and also the number that passed, how do you find how many per cent passed? How do you find how many per cent failed?

11. If you know how many problems a boy had to solve in one month, and also how many he really did solve, how do you find how many per cent of his problems he solved?

12. How can you find your spelling grade in per cent for a month? A perfect grade is how many per cent?

13. If you know the base and the percentage, how may the rate be found?

100. The Elements of Percentage. There are three numbers involved in every problem in percentage, viz: *Base, Rate,* and *Percentage.* In any such problem, two of these numbers are given, directly or indirectly, and we are to find the third.

We have learned that when the base and rate are given, the percentage is found by multiplying the base by the rate expressed as a decimal. That is:

$$\textit{rate} \times \textit{base} = \textit{percentage} \qquad \text{(P)}$$

We shall refer to this equation as equation (P), meaning the equation of percentage.

The first step in solving a problem in percentage is to find which two of the three elements, base, rate and percentage, are given. The remaining one can then always be found by referring to equation (P).

If the rate and the base are given, we need to find the missing member in

$$\text{rate} \times \text{base} = ?$$

That is, we have a problem in multiplication.

If the base and percentage are given, we need to find the missing members in

$$? \times \text{base} = \text{percentage}$$

That is, we have a problem in division.

If the rate and percentage are given, we need to find the missing number in

$$\text{rate} \times ? = \text{percentage}$$

That is, we again have a problem in division.

ORAL EXERCISES

1. State in words the problem represented by the equation: rate × base = ? How is it solved?

2. State in words the problems represented by the equations: ? × base = percentage, rate × ? = percentage. How are they solved?

Compare the three problems in percentage with finding the missing numbers in:

$$3\times4=?\ ,\ 3\times?=12,\ ?\times4=12$$

Also compare these problems with the problems on price, cost and number of articles sold, or with those in length, width, and area. (See pages 24, 26, 28.)

You should now be able to see that the same idea runs through all these problems. If you understand this you will have less trouble with any of them than you otherwise would.

ORAL EXERCISES

In each of the following state which is the base, which is the rate, and which is the percentage.

1. 5% of 800 is 40
2. 5% of 900 is 45
3. 75 is 5% of 1500
4. 12½% of 800 is 100
5. 1650 is 20% of 8250
6. 15% of 6500 is 975.
7. 25% of 64 is 16.
8. 50 is 5% of 1000
9. 48 is 5% of 960
10. 360 is 6% of 6000
11. 18% of 780 is 140.40
12. 33⅓% of 1560 is 520
13. 24% of 45 is 10.80
14. 224 is 40% of 560
15. 768 is 80% of 960
16. 35% of 1250 is 437.5
17. Give a rule for finding the percentage when the rate and the base are given.
18. Give a rule for finding the rate when the base and the percentage are given.

The problem of finding the base when the rate and percentage are given occurs less frequently than the other two. While finding the base is not more difficult than finding the rate, the problems which naturally involve this case come up in the seventh grade, and exercises of this sort will be deferred until then.

101. **List Price and Marking Price.** Goods sold at wholesale (and some goods sold at retail) are described in catalogues in which prices are given. Such prices are called *list prices.* Goods for sale in retail stores usually have price tags attached. The price shown on these is called *marking price.*

102. **Discount.** Any reduction from list or marking price is *commercial discount,* or simply *discount.* Discounts are allowed for many reasons. The list price is always high enough to cover any rise in the market price which is likely to occur for some time. Hence goods are regularly sold at a discount. Special discounts are also given on large sales, for prompt payment and for other reasons. Discount is figured as *so many per cent of the list price or marking price.*

ORAL EXERCISES

1. Try to find out the reasons why stores in your city or neighborhood offer goods at a discount at certain times of the year. Also try to find out whether goods are sold at a discount for other reasons.

2. Give equivalent in common fractions of the following discounts: 50%, 25%, 20%, 10%, $33\frac{1}{3}$%. (See page 165.)

3. A chair marked $18, is reduced 25%. By how much is the price of the chair reduced?

4. If the price of a chair is reduced by $\frac{1}{4}$, for how many per cent of its original price does it sell?

5. What per cent of the list price is paid for goods sold at a discount of 10%? of 20%? of 30%? of 40%? of 15%? of 25%?

6. What per cent of the marking price is paid for goods sold at a discount of 51%? of 35%? of 45%? of 55%?

7. Find advertisements in the papers offering discounts, and make problems from them.

Problem. What is the selling price of goods listed at $125 if sold at a discount of 15%?

$125	Since the price is discounted 15%, the selling price is 85%, or .85 of
.85	the list price.
625	
1000	
$106.25 selling price.	

This problem may also be solved by finding 15% of $125 (the list price) and subtracting this from the list price. The method given here is shorter, however.

ORAL EXERCISES

1. If goods are reduced 12%, how do you find the selling price?
2. If goods are reduced 35%, how do you find the selling price?

WRITTEN EXERCISES

Find the selling price of each of the following:

1. Silk listed at $2.25 a yard; discount 25%.
2. Woolen goods listed at $1.80 a yard; discount 15%.
3. Men's suitings listed at $2.80 a yard; discount 45%.
4. Men's shirts listed at $18.00 a dozen; discount 40%.
5. Collars listed at $1.50 a dozen; discount 35%.
6. Knives and forks listed at $16.00 a dozen; discount 45%.
7. Spoons listed at $12.00 a dozen; discount 18%.

8.	1 sideboard	$70.00
	A china closet	85.00
	Discount 12%	

9.	1 rug	$145.00
	6 pairs of curtains	48.00
	2 pieces of tapestry	34.00
	4 pictures	120.00
	Discount 8%	

103. **Short Methods.** The shortest method for finding the selling price is not the same for all cases. The following examples illustrate this:

Example 1. Discount 25%. Deduct $\frac{1}{4}$ of the list price.

Example 2. Discount 20%. Deduct $\frac{1}{5}$ of the list price.

Example 3. Discount 10%. Deduct $\frac{1}{10}$ of the list price.

Example 4. Discount $12\frac{1}{2}$%. Deduct $\frac{1}{8}$ of the list price.

Example 5. Discount $16\frac{2}{3}$%. Deduct $\frac{1}{6}$ of the list price.

Example 6. Discount $33\frac{1}{3}$%. Deduct $\frac{1}{3}$ of the list price.

Example 7. Discount 15%. Find 85% of the list price.

Example 8. Discount 45%. Find 55% of the list price.

ORAL EXERCISES

Give the shortest method for finding the selling price for each of the following discounts: 30%, 40%, 35%, $66\frac{2}{3}$%, 18%.

WRITTEN EXERCISES

Find the selling price in each of the following:

1.	5,000 envelopes at $1.55 per thousand.............	$
	10,000 envelopes at $1.35 per thousand............	
	5,000 sheets of stationery at $1.75 per thousand.....	______
	Discount 15%	
2.	6 office chairs at $12..........................	$
	3 filing cases at $45...........................	
	4 office desks at $35..........................	______
	Discount $16\frac{2}{3}$%	
3.	1 set of account books.........................	$30.00
	10,000 reams of copy paper......................	55.00
	1 doz. cases carbon paper......................	18.00
	Discount 20%	

WRITTEN EXERCISES

1. Goods marked \$70 were sold at a discount of 5%. What was the selling price?

2. Goods listed at \$45 were sold at a discount of 60%. What was the selling price?

3. Goods listed at \$250 were sold at a discount of 45%. What was the selling price?

4. Goods listed at \$100 were sold for \$62. What was the discount per cent?

5. Goods marked \$40 were sold for \$30. What was the discount per cent?

6. Goods listed at \$60 were sold for \$40. What was the discount per cent?

7. Goods listed at \$2.50 were sold at a discount of 60%. What was the selling price?

Find the selling prices in the following, using short methods:

	List price	Rate discount		List price	Rate discount
8.	\$260.00	35%	18.	\$90.00	35%
9.	1.50	70%	19.	75.00	45%
10.	4.00	65%	20.	60.00	15%
11.	9.50	55%	21.	1250.00	10%
12.	45.00	35%	22.	4.50	40%
13.	7.50	75%	23.	3.00	65%
14.	90.00	15%	24.	35.00	30%
15.	45.00	$12\frac{1}{2}$%	25.	40.00	$33\frac{1}{3}$%
16.	30.00	$16\frac{2}{3}$%	26.	60.00	$18\frac{3}{4}$%
17.	120.00	$6\frac{1}{4}$%	27.	150.00	$22\frac{1}{2}$%

104. Profits. People engage in business for the purpose of making money. Merchants make money by selling goods at a higher price than they pay for them. The difference between the buying and the selling price is called *gross profit,* or simply *profit.* The profit is usually computed as a certain per cent of the dealer's buying price. Thus, if a dealer buys a table for \$20.00 and sells it at \$30.00, his profit is \$10.00. The \$10.00 profit is 50% of the buying price.

ORAL EXERCISES

1. An article is bought for \$40 and sold for \$60. What is the gain?
2. An article is bought for \$50 and sold at a gain of \$25. What is the selling price?
3. An article is sold for \$25, which is a gain of \$8.00. Find the buying price?
4. If you know the buying and the selling price, how do you find the gross profit?
5. If you know the buying price and the gain, how do you find the selling price?
6. If you know the selling price and the gain, how do you find the buying price?
7. An article is bought for \$20 and sold at a gain of 25%. Find the selling price.

Find the selling price of each of the following:

8. Buying price \$60. Gain 20%.
9. Buying price \$20. Gain 50%.
10. Buying price \$300. Gain 60%.
11. Buying price \$1.50. Gain 50%.
12. Buying price 40 cents. Gain 100%.

If an article is sold at a gain of 25%, the selling price is 125% of the buying price, since the buying price is 100% of itself.

If an article is sold at a loss of 10%, the selling price is 90% of the buying price.

ORAL EXERCISES

1. If goods are sold at a gain of 30%, how many per cent of the buying price is the selling price?

2. If goods are sold at a gain of 45%, how many per cent of the buying price is the selling price?

3. If goods are sold at a loss of 15%, how many per cent of the buying price is the selling price?

4. If goods are sold at a loss of 5%, how many per cent of the buying price is the selling price?

5. If goods are sold at a gain, is the selling price more or less than 100% of the buying price?

6. If goods are sold for the price at which they were bought, is the selling price more or less than 100% of the buying price?

7. If goods are sold at a loss, is the selling price more or less than 100% of the buying price?

In the following, how many per cent of the buying price is the selling price?

Gain	Gain	Loss	Loss
8. 20%	15. 20%	22. 12%	29. 20%
9. 15%	16. 55%	23. 18%	30. $33\frac{1}{3}$%
10. 25%	17. 45%	24. $12\frac{1}{2}$%	31. 45%
11. 45%	18. 60%	25. $8\frac{1}{3}$%	32. 50%
12. $12\frac{1}{2}$%	19. 15%	26. $16\frac{1}{3}$%	33. 75%
13. $16\frac{1}{3}$%	20. 50%	27. 15%	34. 80%
14. 27%	21. 75%	28. 25%	35. 41%

Problem 1. A man buys a farm for $14,800, and sells it at a gain of 8%. What is the selling price?

$14,800 1.08 118400 14800 $15984.00 = selling price.	The selling price is 108% of the buying price. Hence the selling price is found by multiplying the buying price by 1.08.

Problem 2. A man buys an automobile for $1600 and sells it at a loss of 35%. What is the selling price?

$1600 = buying price. .65 8000 9600 $1040.00 = selling price.	The selling price is 65% of the buying price; hence the selling price is found by multiplying the buying price by .65.

WRITTEN EXERCISES

1. A house and lot are bought for $14,500, and are sold at a loss of 5%. Find the selling price.

2. A man buys an automobile for $2800, and sells it at a loss of 15%. Find the selling price.

3. Find the selling price in each of the following:

	Buying price	Rate of gain		Buying price	Rate of gain
4.	$1450.00	25%	11.	$1250.00	35%
5.	$1900.00	15%	12.	$412.00	18%
6.	$3700.00	28%	13.	$380.00	12%
7.	$4900.00	12%	14.	$4200.00	27%
8.	$800.00	55%	15.	$8960.00	5%
9.	$24,600.00	4%	16.	$12,000.00	14%
10.	$1860.00	8%	17.	$7400.00	7%

105. The Rate of Gain or Loss. The problem of finding the *rate of gain or loss,* when the buying price and the selling price are given, occurs frequently.

Problem 1. A man bought a house for \$3400, and sold it for \$3700. What was the gain per cent?

$$3400\overline{)300.00} = 34\overline{)3.00}\quad \begin{array}{r} .088 = 8.8\% \\ \underline{2\ 72} \\ 280 \end{array}$$

The gain was \$300, and this we find to be 8.8% of \$3400, the buying price. This result is correct to the nearest tenth of one per cent.

In practice the same number of zeros in the dividend and divisor are omitted before starting the dividing

Problem 2. A man bought a steamboat for \$180,000, and sold it two years later for \$145,000. What was the loss per cent on this transaction?

$$\begin{array}{r} .194 = 19.4\% \text{ rate of loss.} \\ 18\overline{)3.5} \\ \underline{1.8} \\ 1.70 \\ \underline{1.62} \\ .080 \end{array}$$

The loss was \$35,000, and this we find to be 19.4% of \$180,000, the buying price. The result is correct to the nearest tenth of one per cent. Divide both dividend and divisor by 10000 before starting the dividing.

> *To find the rate per cent, gain or loss, find what per cent of the buying price the gain or loss is.*

WRITTEN EXERCISES

Find the rate, gain or loss, in each of the following, to the nearest tenth of one per cent:

	Buying price	Selling price		Buying price	Selling price
1.	\$1900.00	\$1400.00	5.	\$1.50	\$2.75
2.	\$1200.00	\$1500.00	6.	\$3.00	\$5.00
3.	\$35.00	\$65.00	7.	\$0.15	\$0.25
4.	\$45.00	\$75.00	8.	\$0.60	\$1.00

106. Commission. An agent or employee sometimes receives as his compensation a certain fraction or per cent of the amount involved in a transaction. This compensation is called a *commission.*

107. Business Transacted on Commission. There is a large variety of business transactions on which a commission is paid.

A real estate agent sells a piece of ground or a building for a customer and receives a certain per cent of the selling price as his commission. An architect draws the plans for a building, and superintends its erection, and receives for his work a certain per cent of the amount paid for the building. A commission merchant in a city receives eggs, butter, fresh veal, etc., from the farmers and sells these to local dealers. The pay for his work is a certain per cent of the amount involved in the sale. All these are said to receive a certain commission.

108. Commission, How Computed. Commission is always computed as so many per cent on the amount involved in the transaction; that is,

> *An agent's commission for selling is computed as a certain per cent of the selling price. An agent's commission for buying is computed as a certain per cent of the purchase price.*

It should be clear that the subject of commission involves no new principles of arithmetic. (Read again page 161.)

ORAL EXERCISES

1. Find the agent's commission for selling a piece of land for $4,000, if the rate is 5%.

2. Find an agent's commission for selling a city building for $50,000, if the rate is 2%.

3. Find commission for selling land for $8000, if the rate is 4%.

WRITTEN WORK

1. A real estate man sells a lot for \$2400. At the rate of 5%, what is his commission?

2. A broker sells a motor boat for \$2350 and deducts a commission of 5%. How much does the owner get from the sale?

3. A house is to be built for \$8500. The architect's fee for drawing the plans is 6% of this sum. What is his fee.

4. A commission merchant sells 50 dozen eggs for 38c a dozen, and deducts 10% for his commission. How much does he transmit to the producer?

5. At the rate of 12½%, what is the commission for selling 180 pounds of veal at 13½c a pound? How much does the farmer get for the veal?

6. At 4% what is the architect's commission on a building for which the building contractor gets \$125,000? How much does the building cost when the architect's commission is added?

7. An agent sells a second-hand automobile for \$1250. How much does the owner get if the agent deducts 8% as his commission?

8. An agent buys a motor boat for \$2750. How much does the boat cost the purchaser if the agent charges a commission of 3% for buying.

9. A property is sold for \$15,000. If the commission for selling is 2½%, how much does the seller get, after the commission for selling is deducted?

10. A man is offered \$5750 for a house and lot, with a 5% commission to be deducted. He can sell the house for \$5500 without commission. Which is the better offer?

11. Make and solve other problems on commission. Find out what work is done on commission in your neighborhood.

109. Interest. If you loan a man $100 for one year, then at the end of the year he owes you not only the $100 which you loaned him, but a few dollars besides. That is, the man will pay you for the use of the money during the year.

Such payment for the use of money is called *interest.* It is rent paid for the use of money, just as rent is paid for the use of a house or a farm. If the man pays $5 for the use of $100 for one year, it is said that the *rate of interest* is 5%, and if he pays $6 for the use of $100 for one year, the rate of interest is 6%.

If a business man borrows money from a bank he is obliged to pay the bank a certain number of dollars per year for every hundred dollars he borrows; that is, he pays a certain % of the amount borrowed.

These are instances of a universal custom. Anyone who borrows money must pay interest for the use of it, and anyone who loans money receives interest on it. Interest is always reckoned as a certain number of dollars per year for each $100 borrowed—that is, at a certain rate of per cent per year.

Problem. Find the interest for one year on $600 at 5%.

(*Suggestion*: Find 5% of $600.)

ORAL EXERCISES

1. Find 5% of 100, 200, 300. What is the interest on $100 for one year at 5%? on $200? on $300?

2. Find the interest on $300 for one year at 6%.

Find the interest for one year on each of the following:

3. $800 at 5%	7. $1500 at 5%	11. $10,000 at 4%
4. $600 at 4%	8. $2000 at 4%	12. $8000 at 6%
5. $300 at 7%	9. $5000 at 5%	13. $12,000 at 7%
6. $1000 at 6%	10. $2500 at 6%	14. $20,000 at 5%

Problem 1. What is the interest on \$6840 at 6% for one year?

\$6840
.06
\$410.40

Find 6% of \$6840, which is \$410.40, the required interest.

Frequently the interest charged involves a fraction of a per cent. Thus, $4\frac{1}{2}\%$ and $5\frac{1}{2}\%$ are common rates of interest.

Problem 2. What is the interest on \$4580 for one year at $5\frac{1}{2}\%$?

\$4580
.055
22900
22900
\$251.900

You should notice that the solution of the problem consists in multiplying 4580 by $.05\frac{1}{2}$ or .055.

\$45.80	1%
22.90	$\frac{1}{2}\%$
229.00	5%
\$251.90	$5\frac{1}{2}\%$

Example 2 may also be solved by taking 1% of 4580, then $\frac{1}{2}\%$, and then 5%, and adding the $\frac{1}{2}\%$ and 5%.

WRITTEN EXERCISES

Find the interest for one year on each of the following:

1. \$645 at 5%
2. \$4260 at 6%
3. \$7600 at 5%
4. \$375 at 7%
5. \$960 at 8%
6. \$12250 at 6%
7. \$500 at 9%
8. \$9240 at 4%
9. \$25400 at 5%
10. \$35650 at 4%
11. \$3600 at $4\frac{1}{2}\%$
12. \$5700 at $5\frac{1}{2}\%$
13. \$15000 at $5\frac{1}{2}\%$
14. \$84200 at $4\frac{1}{2}\%$
15. \$670 at $6\frac{1}{2}\%$
16. \$7500 at $4\frac{1}{2}\%$
17. \$42000 at $5\frac{1}{2}\%$
18. \$260000 at $5\frac{1}{4}\%$
19. \$35600 at $5\frac{1}{2}\%$
20. \$13800 at $4\frac{3}{4}\%$
21. \$4980 at $5\frac{1}{2}\%$
22. \$16500 at $6\frac{1}{2}\%$
23. \$12800 at $5\frac{1}{2}\%$
24. \$2500 at 7%

WRITTEN EXERCISES

1. What is the interest for one year on $1600 at 6%?

2. What is the interest for one year on $2500 at $5\frac{1}{2}$%?

3. What is the interest for one year on $5000 at $4\frac{3}{4}$%?

4. A man borrows $14,000 to invest in a farm. What is the yearly interest if the rate is $5\frac{1}{2}$%?

5. A city borrows $145,000 at $5\frac{1}{2}$% to build a bridge. What is the yearly interest on this loan?

6. A capitalist finds that during one year his investments net him $5\frac{3}{4}$% interest. At this rate, what is his income from an investment of $480,000?

7. A schoolhouse in a large city costs $860,000. At $5\frac{1}{4}$%, what is the yearly interest on this sum? If this building holds 2400 pupils how much does the interest amount to for each pupil?

In deciding whether it pays to buy a house or to rent one, we must figure interest on the investment.

8. A family paying $60 a month rent find that they can build an equally good house for $8000. If the rate of interest is 6%, and if taxes and repairs on the house amount to $260 yearly, which is cheaper, to rent or to build?

9. A man buys an automobile for $1650. At the end of one year it is worth $1200. If he allows interest on the investment at 7%, and $470 for gasoline, oil, tires, and repairs, how much does it cost him for the use of the machine for one year, including the depreciation?

10. A man buys a farm of 240 acres at $110 an acre. At 6% what is the yearly interest on the buying price? If taxes are $180, and repairs on buildings $250, each year, what is the cost per year of running this farm?

11. Make and solve other problems in reckoning interest.

ORAL EXERCISES

Find the selling price of the following:

	List price	Rate discount		List price	Rate discount
1.	$100	25%	6.	$25	10%
2.	$5	20%	7.	$40	8%
3.	$12	$33\frac{1}{3}$%	8.	$1.50	$33\frac{1}{3}$%
4.	$30	20%	9.	$8.00	30%
5.	$2.50	50%	10.	$75	10%

Find the selling price of the following:

	Cost	Rate gain		Cost	Rate gain
11.	$10.00	40%	16.	$0.25	100%
12.	$35.00	50%	17.	$80.00	25%
13.	$0.50	100%	18.	$300.00	10%
14.	$3.00	25%	19.	$5000	20%
15.	$30.00	50%	20.	$1.25	20%

Find the agent's commission on the following:

	Selling price	Rate com.		Selling price	Rate com
21.	$200	10%	26.	$25	10%
22.	$4000	5%	27.	$60	5%
23.	$800	5%	28.	$700	10%
24.	$50,000	2%	29.	$480	$12\frac{1}{2}$%
25.	$1200	20%	30.	$6000	5%

Find the yearly interest on the following:

	Principal	Rate		Principal	Rate
31.	$200	6%	34.	$3000	7%
32.	$4000	5%	35.	$5000	8%
33.	$600	6%	36.	$4500	6%

FINDING THE RATE OF INTEREST

110. **Principal, Rate, Interest.** The sum on which interest is computed is called the *principal.* The number of dollars paid for the use of $100 for one year is called the rate of interest, or simply the *rate,* and the sum paid for the use of the principal is called the *interest.* The principal is the amount loaned or borrowed.

111. **Finding the Rate.** Problems occur frequently in which the principal and the interest are given and the rate is required.

Problem 1. What is the rate if the interest for one year on $600 is $30?

Solution: As on pages 176, 177, we find that 30 is 5% of 600. Hence the required rate is 5%.

Problem 2. What is the rate if the interest for one year on $5000 is $275?

$$5000\overline{)275.00} = 5\overline{).275}^{\;.055 = 5.5\%}$$

We find that $275 is 5.5% of $5000. Hence the required rate is 5½%.

ORAL EXERCISES

Find the rate in each of the following:

	Principal	Interest		Principal	Interest
1.	$1000	$60	10.	$3000	$240
2.	$800	$40	11.	$400	$32
3.	$1500	$150	12.	$600	$30
4.	$2000	$100	13.	$3000	$210
5.	$4000	$240	14.	$5000	$250
6.	$6000	$300	15.	$2000	$120
7.	$600	$25	16.	$700	$56
8.	$1000	$50	17.	$400	$32
9.	$200	$12	18.	$900	$72

The problem of finding the rate of interest occurs very frequently. Anyone who saves money and invests it, thereby getting a certain income, is interested in knowing what rate of interest he is getting on his investment.

Problem. A man invested $1080 and received a yearly income of $60. What rate of interest did he receive on the investment? Find the result to the nearest tenth of one per cent.

$$\begin{array}{r} .0555 \text{ or } 5.6\% \\ 1080\overline{)60.00} \\ \underline{5400} \\ 6000 \end{array}$$

We find that 60 is 5.6% (nearly) of 1080. Hence, this is the required rate of interest.

WRITTEN EXERCISES

Find the rates of interest to the nearest tenth of one per cent in each of the following:

	Principal	Interest		Principal	Interest
1.	$6500	$480	8.	$5900	$460
2.	$1800	$100	9.	$14000	$1200
3.	$3600	$225	10.	$35000	$2800
4.	$450	$35	11.	$20000	$1800
5.	$2500	$300	12.	$7800	$600
6.	$4000	$360	13.	$4500	$400
7.	$5000	$640	14.	$1250	$150

15. A man invested $6500 in bank stock and received yearly income of $550. What was the rate of income on the investment?

16. A man invested $15000 in a business. His yearly share of the profits was $2000. What was the rate of income on this investment.

17. A man paid $150,000 for an office building. His net yearly income from it was $10,000. What was the rate of income on the investment?

WRITTEN WORK

1. What is the rate if the interest on $4200 for one year is $220?

2. What is the rate if the interest on $6500 for one year is $260?

3. A capitalist has investments amounting to $800,000 and the yearly income from them is $41,600. What is the average rate of interest?

4. In a recent year the total income-bearing property of a great university was $31,748,000, and the income from this property was $1,526,400. Find the rate to the nearest tenth of one per cent.

5. If an investment of $109.50 yields an income of $6 what is the rate of interest?

6. A man buys a property for $9850, which yields a net yearly income of $500. What is the rate of interest on the investment?

7. A farm is bought at $175 an acre. If the farm yields a net income of $12.50 per acre, what is the rate of income on the investment?

8. A house costing $7600 is rented for $50 a month. If taxes, repairs, depreciation and so on, amount to $175 a year, what is the rate of income on the investment?

9. An apartment building erected at a cost of $146,000 yielded a net income above all expenses of $11,500. What was the rate of income on the investment?

10. A security bringing in a yearly income of $8.00 is bought for $139.00. What is the rate of income on the investment?

11. A man improves a rented house at an expense of $2480, and the rent is thereby increased $10 a month. What is the rate of interest obtained on this investment?

12. Make and solve other problems on finding the rate of interest.

When the time for which interest runs is two or more years, the interest is found by multiplying the interest for one year by the number of years.

ORAL EXERCISES

1. At the rate of 5% what is the interest on $600 for one year? For 2 years?

2. At the rate of 8% what is the interest on $300 for one year? For 3 years?

3. At the rate of 6% what is the interest on $800 for one year? For 3 years? For 10 years?

Find the interest on each of the following:

	Principal	Rate	Time
4.	$300	5%	One year, 3 years, 6 years,
5.	$200	6%	One year, 4 years,
6.	$400	6%	One year, 2 years, 5 years.
7.	$800	5%	One year, 6 years

WRITTEN EXERCIS

Find the interest on the following:

	Principal	Rate	Time
1.	$7800	$5\frac{1}{2}$%	Two years, 5 years.
2.	$3200	$6\frac{1}{2}$%	One year, 6 years.
3.	$3800	$5\frac{3}{4}$%	Two years, 7 years.
4.	$5800	6%	Two years, 7 years.
5.	$3900	7%	Five years, 8 years.
6.	$25,000	$4\frac{1}{2}$%	Three years, 5 years
7.	$2700	8%	Four years, 9 years.
8.	$35,800	$4\frac{1}{2}$%	Two years, 5 years.

112. **Time Given in Months.** In computing interest it is customary to count 360 days to the year and 30 days to the month. One month is regarded as $\frac{1}{12}$ of a year, whether the month contains 28 days or 31 days. Two months equals $\frac{1}{6}$ of a year, and so on.

The time from any date in one month to the same date in the month following is counted as one month or thirty days.

Thus, from January 5 to February 5 is one month, and from February 5 to March 5 is also one month, although the latter is 28 days (except in a leap-year, when it is 29 days), while the former is always 31 days.

ORAL EXERCISES

1. What fraction of one year is there in 1 month? In 2 months? In 3 months? In 4 months? In 5 months? In 6 months? In 7 months? In 8 months? In 9 months? In 10 months? In 11 months?

To find the interest for one month take $\frac{1}{12}$ of the interest for one year; to find the interest for 2 months take $\frac{1}{6}$ of the interest for one year, and so on.

2. How may interest be found for 3 months? For 4 months? For 5 months? For 6 months?

3. How may interest be found for 7 months? For 8 months? For 9 months? For 10 months? For 11 months?

WRITTEN EXERCISES

Find the interest on the following:

	Principal	Rate	Time		Principal	Rate	Time
1.	$500.	6%	6 months	7.	$1600.	8%	10 months
2.	$1200.	5%	6 months	8.	$3700.	7%	11 months
3.	$800.	6%	4 months	9.	$5800.	7%	9 months
4.	$2500.	6%	3 months	10.	$4300.	8%	7 months
5.	$4500.	7%	8 months	11.	$7200.	6%	5 months
6.	$6400.	5%	9 months	12.	$5400.	5%	4 months

113. Time Given in Days. The simplest rule for finding interest for any time less than a year is to express the time as a fraction of a year and multiply the interest for one year by that fraction.

Since one year is regarded as 360 days, 20 days is $\frac{20}{360}$, or $\frac{1}{18}$ of a year.

Again, since every month is regarded as having 30 days, 1 month 20 days is 50 days or $\frac{50}{360}=\frac{5}{36}$ of one year.

Similarly, 18 days $=\frac{18}{360}=\frac{1}{20}$ of one year.

Example. At 7% find the interest for $800, for 3 mo. 15 da.

Solution:

$$\frac{7}{\underset{3}{\cancel{24}}}\times\overset{7}{\cancel{56}}=\frac{49}{3}=16.33\tfrac{1}{3}\text{ (dollars)}$$

3 mo. 15 da. is 105 da., or $\frac{105}{360}=\frac{7}{24}$ of a year. Hence, multiply the interest for one year ($56) by $\frac{7}{24}$.

WRITTEN EXERCISES

Express as fractions of a year and in the simplest form. If the time is given as months and days first reduce to days.

1. 70 days, 40 days, 50 days, 80 days, 100 days.
2. 65 days, 25 days, 160 days, 130 days.
3. 3 months and 15 days.
4. 2 months and 21 days.
5. 1 month and 5 days.
6. 4 months, 24 days.
7. 7 months, 18 days.
8. 9 months, 15 days.

Find the interest in each of the following:

	Principal	Rate	Time
9.	$1700.	5%	50 days
10.	$3500.	6%	2 mo. 15 da.
11.	$4200.	5%	3 mo. 10 da.
12.	$6800.	7%	4 mo. 20 da.
13.	$9500.	6%	7 mo. 6 da.

114. Fractional Per Cent. When the rates of interest contain fractions of one per cent, the fractions are usually very simple, such as $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{3}{4}$.

Example 1. Find the interest on \$3600 at $5\frac{1}{2}\%$ for 9 mo. 18 da.

$$\begin{array}{r|l}
\$36 & 00 = 1\% \\
18 & 00 = \frac{1}{2}\% \\
180 & 00 = 5\% \\
\hline
\$198 & 00 = 5\frac{1}{2}\% \\
4 & \\
\hline
5)\overline{792} & \\
\hline
158 & 40 = \text{interest in dollars.}
\end{array}$$

First find the interest for one year. To find $5\frac{1}{2}\%$ of \$3600 first find 1%, then $\frac{1}{2}\%$, and then 5%. Add the $\frac{1}{2}\%$ and 5%.

9 mo. 18 da. is $\frac{4}{5}$ of a year.

Multiply the interest for one year by $\frac{4}{5}$. In such examples the vertical line is used to separate dollars and cents.

Example 2. Find the interest on \$670 at $6\frac{3}{4}\%$ for 10 mo. 6 da.

$$\begin{array}{r|l}
\$6 & 70 \;\; = 1\% \\
1 & 675 = \frac{1}{4}\% \\
\hline
5 & 025 = \frac{3}{4}\% \\
40 & 20 \;\; = 6\% \\
\hline
\$45 & 225 = 6\frac{3}{4}\% \\
 & 17 \\
\hline
316 & 575 \\
452 & 25 \\
\hline
20)\overline{768} & 825 \\
\hline
38 & 44 = \text{interest in dollars.}
\end{array}$$

First find interest for one year. Find 1% of \$670, then $\frac{1}{4}\%$, then $\frac{3}{4}\%$, and finally 6%. Add the $\frac{3}{4}\%$ and 6%.

10 mo. 6 da. $= \frac{17}{20}$ of a year.

Multiply the interest for one year by $\frac{17}{20}$. Express the final result to the nearest cent.

WRITTEN EXERCISES

Find the interest on each of the following:

	Principal	Rate	Time
1.	\$4500	$5\frac{1}{4}\%$	8 months, 15 days.
2.	\$780	$5\frac{3}{4}\%$	4 months, 12 days.
3.	\$34,000	$4\frac{3}{4}\%$	9 months, 24 days.
4.	\$160	$6\frac{1}{2}\%$	3 months, 9 days.
5.	\$2400	$5\frac{3}{4}\%$	10 months, 7 days.

115. Drill in Interest. In the examples on this page the time is given as days, months or years, but not as combinations of these.

WRITTEN EXERCISES

1. What is the interest on \$750 at $5\frac{1}{2}\%$ for 3 years?
2. What is the interest on \$350 at 6% for 7 years?
3. What is the interest on \$9000 at 5% for 9 months?
4. What is the interest on \$1600 at $5\frac{1}{2}\%$ for 4 years?
5. What is the interest on \$1850 at $5\frac{1}{4}\%$ for 75 days?

Find the interest for each of the following:

	Principal	Rate	Time	Principal	Rate	Time
6.	\$1850	$5\frac{1}{2}\%$	3 yrs.	\$800	5%	126 da.
7.	\$900	$5\frac{3}{4}\%$	5 yrs.	\$220	6%	90 da.
8.	\$2400	6%	8 mos.	\$5300	$5\frac{1}{2}\%$	130 da.
9.	\$7500	$5\frac{1}{4}\%$	6 yrs.	\$5600	5%	85 da.
10.	\$9400	5%	3 yrs.	\$650	6%	260 da.
11.	\$5000	6%	7 mos.	\$3400	4%	180 da.
12.	\$245	6%	3 mos.	\$460	$3\frac{1}{2}\%$	230 da.
13.	\$760	$6\frac{1}{4}\%$	5 mos.	\$1500	5%	300 da.
14.	\$3400	$5\frac{1}{2}\%$	4 yrs.	\$940	$5\frac{3}{4}\%$	170 da.
15.	\$850	5%	11 mos.	\$8500	$5\frac{1}{2}\%$	135 da.
16.	\$460	$5\frac{1}{4}\%$	9 mos.	\$12,000	5%	280 da.
17.	\$3500	6%	5 mos.	\$16,500	$4\frac{3}{4}\%$	75 da.

Remark: To find the interest when the time is given as years and fractions of a year, first find the interest for the whole years and then for the fraction of a year, and add. Such problems occur rarely in practice.

Problem. A man borrows \$400 on October 10th at 6%, and pays it back on February 25th of the following year. How much must he pay?

To solve this problem we must find the number of months and days from October 10th to February 25th. From October 10th to February 10th is 4 months, and from February 10th to February 25th is 15 days. Hence, the time is 4 months 15 days or $\frac{3}{8}$ of a year.

The interest is $\frac{3}{8} \times \$24 = \$9$.

No one rule will give the shortest method for reducing months and days to a fraction of a year. Thus, 4 months 15 days is $\frac{1}{3}+\frac{1}{24}$ of a year $=\frac{9}{24}=\frac{3}{8}$ of a year. You should be on the alert to see short cuts of this sort.

WRITTEN EXERCISES

How many months and days are there between each of the following pairs of dates? Express each as a fraction of a year.

1. Jan. 15 to Sept. 24.
2. April 10 to Nov. 18.
3. Sept. 7 to May 1.
4. July 24 to Dec. 15.
5. May 12 to Nov. 21.
6. June 15 to Dec. 27.
7. April 24 to Aug. 27.
8. March 12 to Sept. 3.
9. A man borrowed \$550 Nov. 1st, and paid it back May 1st the next year. How much interest did he pay if the rate was 6%?
10. A man borrowed \$750 June 3, and paid it back December 1 of the same year. How much interest did he pay if the rate was $6\frac{1}{2}$%?
11. A man borrowed \$8500 to buy a lot and build a house. How much interest did he pay each year if the rate was $5\frac{1}{2}$%?
12. Find the interest on \$2500 at 6% from March 1st to July 10.
13. Make and solve other problems like these. See who can make the most interesting problems.

116. The Savings Bank. Money is frequently deposited in savings banks, which offer a high degree of security, but pay a low rate of interest, usually from 3% to 4%.

117. Interest Days. Interest Period. The savings bank computes the interest twice or four times a year. The days on which interest is computed are called *interest days*, and the periods between interest days are called *interest periods*.

Interest is usually computed on the smallest amount on deposit during the interest period; interest is not allowed on a fraction of one dollar.

Problem. In a certain savings account the smallest amount on deposit for the interest period from January 1st to April 1st was $284.60. Find the interest for this period, the rate being $3\frac{1}{2}\%$.

Solution: $3\frac{1}{2}\%$ of $\$284 = \$9.94$. The time is $\frac{1}{4}$ of a year.
$\frac{1}{4}$ of $\$9.94 = \$2.48$. The 60 cents in the principal is neglected.

WRITTEN EXERCISES

Find the interest:

	Minimum deposit	Interest period	Rate		Minimum deposit	Interest period	Rate
1.	$227.50	6 mo.	3%	5.	$124.20	6 mo.	4%
2.	$47.80	3 mo.	$3\frac{1}{2}\%$	6.	$680.50	3 mo.	$3\frac{1}{4}\%$
3.	$45.40	6 mo.	4%	7.	$362.70	3 mo.	3%
4.	$134	3 mo.	$3\frac{1}{4}\%$	8.	$145.60	6 mo.	4%

Solve as many examples as you can on this page orally.
Reduce each of the following fractions to per cent:

1. $\frac{1}{2}$	4. $\frac{1}{5}$	7. $\frac{4}{5}$	10. $\frac{1}{8}$	13. $\frac{7}{8}$
2. $\frac{1}{4}$	5. $\frac{2}{5}$	8. $\frac{1}{3}$	11. $\frac{3}{8}$	14. $\frac{1}{6}$
3. $\frac{3}{4}$	6. $\frac{3}{5}$	9. $\frac{2}{3}$	12. $\frac{5}{8}$	15. $\frac{5}{6}$

Reduce the following to decimals, giving each to the nearest ten-thousandth.

16. $1\frac{1}{2}\%$	19. $1\frac{1}{8}\%$	22. $4\frac{7}{8}\%$	25. $4\frac{1}{5}\%$
17. $2\frac{1}{4}\%$	20. $6\frac{3}{8}\%$	23. $4\frac{1}{3}\%$	26. $5\frac{3}{5}\%$
18. $3\frac{3}{4}\%$	21. $5\frac{5}{8}\%$	24. $7\frac{2}{3}\%$	27. $6\frac{4}{5}\%$

Fill in the blanks in each of the following:

	Base	Rate	Percentage		Base	Rate	Percentage
28.	460	6%	——	30.	8000	——	320
29.	540	$4\frac{1}{2}\%$	——	31.	2450	——	134.75

Express each of the following decimals in per cent:

32. .064	34. .438	36. .093	38. 1.04
33. .147	35. .001	37. .1942	39. 1.351

Below is a season's record of a school baseball team.

40. Reduce the standing of each team to a decimal correct to the nearest thousandth. Also express each in per cent.

	Games played	Games won	Standing
Lincoln School.	24	13	—
Grant School.	25	11	—
Marshall School.	23	10	—
Ward School.	24	12	—
Emerson School.	26	16	—
Calhoun School.	22	10	—

ORAL AND WRITTEN WORK

1. Tell how the rate of gain in making a sale is computed. Give an example.

2. Tell how the rate of loss in making a sale is computed. Give an example.

3. On what price is the rate of reduction from the marking price computed?

4. A man buys real estate for $10,000 and sells it for $11,500. Disregarding incidental expense, what is his gain per cent?

5. A merchant buys an article for $6.00 and marks it to sell 80% above the purchase price. At what price does he mark the article?

6. An article costing $10 is marked to sell at $14. What per cent of the purchase price is the gain?

7. A real estate man buys property for $7600 and sells it at a gain of 7%. What is the selling price?

8. A dealer buys goods for $730 and sells them at a gain of 12½%. What is the selling price?

9. A merchant buys goods for $100 and marks them to sell at a gain of 75%. What is the marking price?

10. At a special sale a merchant reduces the price of an article marked $175.00 by 50%. What is the selling price?

11. A housekeeper buys a box of soap containing 100 cakes of soap for $5.00. This soap sells at retail at 4 cakes for 30 cents. How much does the housekeeper save by buying the whole box instead of buying 100 cakes, 4 cakes at a time? How many per cent of the purchase price ($5.00) does she save?

12. Find the interest on $1740 for 7 mo. 18 da., the rate being 6%.

13. Find the interest on $4300 for 3 mo. 12 da. The rate of interest is 6½%.

WRITTEN WORK

1. A man invests \$7500 and makes a net gain of \$860. What is the rate per cent of gain on this investment?

2. A certain make of automobile depreciates 35% the first year. If it costs \$1350 when new, how much will it be worth at the end of one year?

3. An agent sells a second-hand automobile for \$850. How much does he remit to the owner if he deducts a commission of 10%?

4. A piece of real estate is sold for \$9600. How much does the owner get for it if the agent deducts a commission of 10%?

5. An agent buys a used automobile for \$1600. How much does it cost the buyer if the agent charges 3% as his commission?

6. A man buys land for \$165 an acre. This land yields a net income of \$11.60 an acre. What rate per cent does the man make on this investment.

7. Books are sold at $\frac{1}{2}$ off. What per cent reduction is this? If books are sold at $\frac{1}{3}$ off, what is the rate per cent discount?

8. $\frac{3}{4}$ is what per cent of $\frac{2}{5}$?

9. Find 45% of $1\frac{3}{4}$.

10. What is the interest on \$286 at $3\frac{1}{2}$% for 3 months?

11. Find the interest on \$14,500 at $6\frac{1}{2}$% for 2 months, 21 days.

12. Find the interest on \$3500 at $5\frac{3}{4}$% for 87 days.

13. Find the interest on \$780 at $6\frac{1}{2}$% for one year and 7 months.

14. A ship built at a cost of \$278,000 yielded a net return of \$97,400 the first year. What per cent of the cost was this?

WRITTEN WORK

1. A property costing $19,600 yields a net yearly income of $1650. What is the rate of interest on this investment?

2. In a school with 465 boys and 535 girls what per cent of the pupils are boys and what per cent are girls?

3. An automobile is bought for $675. If the cost of running it one year is $286, and depreciation 25% of the original price, what is the total expense of using this machine one year?

4. A county was bonded for $280,000 to build roads, the rate of interest being $4\frac{3}{4}$%. What was the yearly interest charged?

5. If roads costing $280,000 depreciate $6\frac{1}{4}$% a year, what is the yearly depreciation?

6. The population of a certain city in 1900 was 26,840, and in 1910 it was 41,352. What was the increase per cent during this decade?

7. If an investment of $147.50 yields an income of $9 a year, what per cent on the investment is this?

8. At an average rate of 4.87%, what is the income from an investment of $2,860,000?

9. If an investment of $84,280 yields a net income of $7360, what is the rate of income on the investment?

10. If an investment of $84 yields a net income of $5 a year, what is the rate on the investment?

11. Goods listed at $12.50 are sold for $8. What is the per cent of discount?

12. A firm has debts owing to it amounting to $14,850.60. Of this $12,500 is regarded as collectible. What per cent is regarded as collectible?

13. Goods listed at $65 are sold at discount of 25%. What is the selling price? If the buyer sells them again at a gain of 10% what is his selling price?

CHAPTER IV.

118. **The Solution of Problems.** In our study of Arithmetic we have learned to perform the operations of addition, subtraction, multiplication, and division on whole numbers and on fractions; and we have made use of these operations in the solution of problems. We will now make a more special study of the solution of problems.

Example. A farmer gets 560 bushels of wheat from a field containing 24 acres. At this rate, how much will he get from a field containing 64 acres?

Solution: $560 \div 24$ or $\frac{560}{24}$ = number of bushels from one acre.

$$\frac{560}{24} \times 64 = \text{number of bushels from 64 acres.}$$

The expression $\frac{560}{24} \times 64$ contains complete directions for solving the problem. All that is left is to carry out the operations indicated by this expression. This we do as follows:

$$\frac{560}{\underset{3}{\cancel{24}}} \times \overset{8}{\cancel{64}} \qquad \begin{array}{r} 560 \\ 8 \\ \hline 3)\overline{4480} \end{array} = 1493\tfrac{1}{3} = \text{number of bushels.}$$

First cancel 8 in 24 and 64. 3 is not a factor of 8 or of 560, so we multiply 560 by 8 and then divide by 3. Notice that we save trouble by multiplying by the 8 *before* dividing by 3.

WRITTEN EXERCISE

1. Solve the above problem by carrying out the operations as they arise. That is, divide 560 by 24 to get the yield per acre, and then multiply by 64. This gives you an idea of the importance of indicating all the operations before performing any of them.

ORAL AND WRITTEN WORK

1. If you know the number of acres in a field and the average yield per acre, how do you find the total yield?

The answer may be written as follows:

$$\textit{number of acres} \times \textit{yield per acre} = \textit{total yield}. \qquad (1)$$

2. If the number of acres and the total yield are given, how do you find the average yield per acre? (Compare with finding the missing number in $3 \times ? = 12$.)

3. The census of 1910 gives the total production of wheat in the United States for 1909 as 683,379,259 bushels, and the number of acres in wheat for that year as 44,262,592. The average yield in bushels per acre is given as 15.4. How was this average obtained by the Census Office? Find the average yourself, and see if it is correctly given.

4. If the total yield and the yield per acre are given, how do you find the number of acres? (Compare with $? \times 4 = 12$.) (Use equation 1.)

5. If you know the number of things bought or sold, and the price for each, how do you find the total cost?

The answer may be written:

$$\textit{number of articles} \times \textit{price} = \textit{cost}. \qquad (2)$$

6. If the total cost and the price per unit are given, how do you find the number bought? (Compare with $? \times 4 = 12$.)

7. If the total cost and the number of things bought are given, how do you find the price? (Compare with $3 \times ? = 12$.)

8. 435 horses were bought at a total cost of \$55,000. What was the average cost per head?

9. If horses for the army could be bought at an average price of \$175 per head, how many could be bought for \$500,000?

10. State in words the three problems which may be solved by using equation (2).

1. If you know the length and width of a rectangle, how do you find the area? The answer may be written:

$$\textit{length} \times \textit{width} = \textit{area}. \qquad \text{(A)}$$

2. If you know the length of a rectangle and its area, how do you find the width? (Compare $3 \times ? = 12$.)

3. If you know the width of a rectangle and its area, how do you find its length?

4. A farmer is plowing a field 120 rods long; how wide a strip must he plow to make 18 acres? Remember that one acre is 160 square rods. (Divide the area in square rods by the length.)

5. A builder wants 6400 square feet for a building. How wide a strip must he buy if the piece from which he buys is 98 feet deep?

6. A lot is 25 feet wide and 112 feet deep. Another lot is 28 feet wide. How deep must it be to have the same area as the first?

(*Suggestion:* $25 \times 112 =$ area in square feet of the first, and also of the second lot. Hence, $\frac{\text{area}}{\text{width}} = \frac{25 \times 112}{38} =$ depth in feet of the second lot.)

7. A room is 16 feet wide and 18 feet long. How wide must a room be which is to have twice the area of this one, if it is 24 feet long?

(*Suggestion*: $2 \times 16 \times 18 =$ area in square feet of the larger room).

8. A farmer wishes to plant 6 acres in apple trees. How long will the orchard be if it is planted on a strip of land 30 rods wide? (*Suggestion*: The area of the orchard is 6×160 square rods.)

9. A certain crew can put in 1250 square feet of pavement in one day. How long a street can they pave in one week if the street is 42 feet wide?

10. State in words the three problems that may be solved by using equation (A).

WRITTEN WORK

(Remember it is best to indicate the solution first.)

1. A lot 25 feet wide and 150 feet deep is sold for \$7500. What is the price per square foot?

2. At \$1.75 a square foot what is the value of a lot 26.8 feet wide and 94.3 feet deep? Find the value to the nearest cent.

3. At \$70 an acre what is the value of a farm 145 rods wide and 216 rods long?

(*Suggestion:* $\frac{145\times216}{160}$ is the number of acres in the farm.)

4. A farmer is offered \$10,000 for a piece of land 182 rods wide and 290 rods long. What price per acre is this?

5. A field 68 rods wide and 120 rods long yields 860 bushels of wheat. What is the average yield per acre from this field? Find the result to the nearest tenth of a bushel.

(*Suggestion:* The yield per acre is $860 \div \frac{68\times120}{160} = 860\times\frac{160}{68\times120}$.)

6. At 25 cents a square foot what is the cost of laying a hardwood floor in a room 26 feet by 36 feet?

7. A carpenter offers to put floors in a hall 80 feet by 105 feet for \$1800. What is the cost per square foot?

(*Suggestion:* Indicate the solution by writing $\frac{1800}{80\times105}$. This will give result in dollars.)

8. A sidewalk 5 feet wide and 350 feet long was laid at a cost of \$160. What was the cost per square foot? per square yard?

9. At \$1.80 a square yard what is the cost of paving a street 35 feet wide and 680 feet long?

10. A field 120 rods long and 80 rods wide was cultivated one season for a total cost of \$325.00. What was the cost per acre?

ORAL EXERCISES

1. A box is 8 inches long, 6 inches wide, and 4 inches deep. How many cubic inches does it hold?

2. If you know the length, the width, and the depth of a rectangular box, how do you find its volume? The answer may be written:

$$\textit{length} \times \textit{width} \times \textit{depth} = \textit{volume}. \qquad \text{(V)}$$

3. If you know the volume of a box and also its length and depth, how do you find its width? (Compare with finding the missing number in $2 \times 3 \times ? = 24$.)

4. If you know the volume of a box and also its length and depth how do you find its width?

5. If you know the volume of a box and also its width and depth, how do you find its length?

Problem. At 75c a cubic yard how much will it cost to excavate a basement 26 feet wide, 42 feet long, and 5 feet deep?

Solution: $\frac{26 \times 42 \times 5}{27}$ = number of cubic yards, and $\frac{26 \times 42 \times 5 \times 75}{27}$ = cost in cents.

That is, $$\frac{26 \times \overset{14}{\cancel{42}} \times 5 \times \overset{25}{\cancel{75}}}{\underset{\cancel{9}\ 3}{\cancel{27}}} = \frac{26 \times 14 \times 5 \times 25}{3} = 15167 = \text{cost in cents.}$$

Hence the cost to the nearest cent is $151.67.

Notice that the fraction $\frac{26 \times 42 \times 5}{27}$ is multiplied by 75 by multiplying its numerator by that number. It is also useful in indicating the solution of problems to note that a fraction is divided by a number by multiplying its denominator by that number.

WRITTEN EXERCISES

1. At 45 cents a cubic yard, find the cost to the nearest cent of making an excavation 49 feet wide, 50 feet long, and 4 feet deep.

2. A wheat bin in a grain elevator is 12 feet wide, 16 feet long, and 54 feet deep. How many bushels of wheat does it hold if one cubic foot is $\frac{4}{5}$ of a bushel?

(*Suggestion:* The required number of bushels is $12 \times 16 \times 54 \times \frac{4}{5}$.)

3. A farmer has a wheat bin 9 feet by 14 feet. How many bushels of wheat are there in it if the grain is $7\frac{1}{2}$ feet deep?

4. The excavation for a sewer is 7 feet wide and 10 feet deep. The contractor is removing 280 cubic yards each day. In how many days will he excavate one mile of sewer? Get result to the nearest day.

(*Suggestion:* $7 \times 10 \times 5280$ = number of cubic feet to be removed, and 280×27 = number of cubic feet removed in one day.

Hence, $\frac{7 \times 10 \times 5280}{280 \times 27}$ is the required number of days.)

5. A schoolroom 30 feet wide, 36 feet long, and 11 feet high seats 48 pupils. The large assembly hall, which is 84 feet wide and 96 feet long and 22 feet high, seats 678 pupils. Which room contains the larger number of cubic feet per pupil? How many?

(*Suggestion:* The number of cubic feet of air per pupil in the two rooms are indicated by: $\frac{30 \times 36 \times 11}{48}$ and $\frac{84 \times 96 \times 22}{678}$.)

6. A hall is 64 feet long, and 76 feet wide and 20 feet high. How many pupils may be seated in it if it is to contain 225 cubic feet of space per pupil? Get the result to the nearest whole number of pupils.

(*Suggestion:* The number is $\frac{64 \times 76 \times 20}{225}$.)

ORAL EXERCISES

Now consider the three problems:

(*a*) $\frac{3}{8}$ of $\frac{4}{5}$ = what? This is equivalent to finding the missing number in $\frac{3}{8}\times\frac{4}{5}=?$

(*b*) $\frac{3}{8}$ of what number $=\frac{3}{10}$? Or $\frac{3}{10}$ is $\frac{3}{8}$ of what number? This is equivalent to finding the missing number in $\frac{3}{8}\times ?=\frac{3}{10}$.

(*c*) $\frac{3}{10}$ is what part of $\frac{4}{5}$? This is equivalent to finding the missing number in $?\times\frac{4}{5}=\frac{3}{10}$.

In (*a*) the product of two fractions is required. In (*b*) and (*c*) the product and one factor are given and the other factor is required.

WRITTEN EXERCISES

Indicate the solution of each of the following:

1. Find $\frac{4}{7}$ of $\frac{3}{14}$.
   Indicated solution: $\frac{4}{7}$ of $\frac{3}{14}=\frac{4}{7}\times\frac{3}{14}$

2. $\frac{2}{3}$ is what part of $\frac{3}{4}$?
   Indicated solution: From $?\times\frac{3}{4}=\frac{2}{3}$ we have that the missing number is $\frac{2}{3}\div\frac{3}{4}$ or $\frac{2}{3}\times\frac{4}{3}$.

3. $\frac{1}{4}$ is what part of $\frac{7}{8}$?
4. $\frac{2}{3}$ is $\frac{1}{2}$ of what number?
5. $\frac{1}{2}$ is $\frac{5}{4}$ of what number?
6. $\frac{3}{4}$ is $\frac{1}{2}$ of what number?
7. $\frac{4}{5}$ is what part of $2\frac{1}{3}$?
8. $2\frac{1}{3}$ is what part of $6\frac{4}{5}$?
9. $1\frac{1}{2}$ is $\frac{2}{3}$ of what number?
10. $4\frac{1}{3}$ is $\frac{2}{3}$ of what number?
11. $12\frac{1}{2}$ is what part of 25?
12. $\frac{3}{4}$ is what part of $\frac{7}{8}$?
13. $1\frac{1}{2}$ is what part of $12\frac{2}{3}$?
14. $4\frac{1}{2}$ is $\frac{2}{3}$ of what number?
15. Find $\frac{3}{8}$ of $\frac{8}{15}$.
16. Find $\frac{5}{16}$ of $8\frac{2}{3}$.
17. $7\frac{2}{3}$ is $\frac{5}{6}$ of what number?
18. $4\frac{5}{6}$ is $\frac{4}{5}$ of what number?
19. $5\frac{2}{3}$ is what part of $17\frac{1}{2}$?
20. $2\frac{7}{8}$ is what part of $41\frac{1}{2}$?

Example. $\frac{5}{6}$ is what part of $2\frac{1}{3}$? Reduce the result to a decimal correct to the nearest thousandth. Also give it in per cent correct to the nearest tenth of one per cent.

Solution: From $? \times 2\frac{1}{3} = \frac{5}{6}$, we have that the required number is:

$$\frac{5}{6} \div 2\frac{1}{3} = \frac{5}{6} \div \frac{7}{3} = \frac{5}{\cancel{6}_{2}} \times \frac{\cancel{3}}{7} = \frac{5}{14} = .357 = 35.7\%.$$

We now see that the above problem is the same as "$\frac{5}{6}$ is how many per cent of $2\frac{1}{3}$?" The only difference is that in one case the result is given as a common fraction, and in the other as a decimal read as per cent.

WRITTEN EXERCISES

In each of the following give the result as a common fraction or mixed number and also as a decimal correct to the nearest thousandth, and in per cent correct to the nearest tenth of one per cent.

1. $1\frac{3}{4}$ is what part of $\frac{2}{3}$?
2. $7\frac{1}{2}$ is what part of 47?
3. $2\frac{1}{3}$ is what part of $24\frac{3}{8}$?
4. $1\frac{5}{8}$ is what part of $3\frac{5}{16}$?
5. $\frac{3}{4}$ is what part of $\frac{5}{16}$?
6. $3\frac{4}{5}$ is $\frac{5}{8}$ of what number?
7. A school year consists of 10 months. $3\frac{2}{5}$ months have passed. What fraction of the school year has passed? How many per cent of the year have passed?
8. The one-cent piece weighs 48 grains, $2\frac{2}{5}$ grains of which is tin and zinc and the rest is copper. What fraction of the one-cent piece is pure copper and what fraction is tin and zinc? Also reduce these fractions to per cents.
9. Brass is made of copper and zinc. 50 pounds of a certain variety of brass contains $32\frac{1}{2}$ pounds of copper and $17\frac{1}{2}$ pounds of zinc. What fraction of this brass is copper and what fraction is zinc? Also reduce these fractions to per cents.
10. Try to make other problems like these.

ORAL EXERCISES

1. What is meant by *per cent, base, rate, percentage?*

2. If the base and rate are given, how do you find the percentage? This may be written:

$$\textit{rate} \times \textit{base} = \textit{percentage}. \qquad \text{(P)}$$

3. State how you find the rate when the base and percentage are given. This problem is written:

$$? \times \textit{base} = \textit{percentage}.$$

4. State how you find the base when the rate and percentage are given. This problem is written:

$$\textit{rate} \times \; ? = \textit{percentage}.$$

5. What is meant by principal, rate, interest?

6. If the principal and rate are given how do you find the interest if the time is one year? This may be written:

$$\textit{rate} \times \textit{principal} = \textit{interest}. \qquad \text{(I)}$$

The rate and principal may then be found the same as the rate and base were found above.

7. State in words the problems that may be solved by using equation (P).

8. State in words the problems that may be solved by using equation (I).

WRITTEN EXERCISES

Find the missing numbers in the following:

	Base	Rate	Percentage		Base	Rate	Percentage
1.	450	5%	?	5.	240	?	14.4
2.	3900	$6\frac{1}{4}$%	?	6.	52,600	?	3419
3.	1250	?	50	7.	7900	?	335.75
4.	6400	?	352	8.	94,200	$4\frac{3}{4}$%	?

ORAL WORK

1. If the principal, rate and time are given, how do you find the interest? This may be written:

$$principal \times rate \times time = interest.$$

2. If the interest, principal and time are given, how do you find the time? (Compare with finding the missing number in $2 \times 3 \times ? = 24$. Also compare with problems on volume on pages 90, 91.)

3. If the interest, principal and time are given, how do you find the rate? (Again compare with $2 \times ? \times 4 = 24$ and with problems on volume, pages 90, 91.)

4. If the interest, rate and time are given, how do you find the principal?

5. If you know the list price and the rate of discount, how do you find the selling price?

6. If you know the list price and the selling price, how do you find the rate of discount?

7. If you know the commission and also the rate, how do you find the selling price?

8. If you know the buying price and the selling price, how do you find the rate of gain?

9. If you know the selling price and the rate of gain, how do you find the buying price?

10. If in a problem on commission you know the amount involved in the transaction and the rate, how do you find the commission?

11. State the four problems which can be solved by means of the equation:

$$length \times width \times depth = volume.$$

119. Equations in Arithmetic. It is important to see how few are the different ideas which really enter into the solution of problems. To assist you in this, we collect here the equations which we have used.

(S) *speed* $\times$ *time* = *distance.*

(C) *number of articles* $\times$ *price* = *cost.*

(A) *length* $\times$ *width* = *area.*

(V) *length* $\times$ *width* $\times$ *depth* = *volume.*

(P) *rate* $\times$ *base* = *percentage.*

(I) *principal* $\times$ *rate* $\times$ *time* = *interest.*

The letters S, C, A, V, P, and I indicate speed, cost, area, volume, percentage, and interest.

The ideas represented by these equations are absolutely fundamental in arithmetic and in practical life, and you must make sure that you master them perfectly. It is not enough that you remember them. You must see clearly why they are what they are. By means of these equations and others like them, which you can make as you need them, you can solve a large variety of problems. In general, a problem will give all but one of the numbers in such an equation, and your task will be to find that one. If you have trouble in finding the missing number just study the similar problem of supplying missing numbers in $3 \times ? = 12$, and $2 \times ? \times 4 = 24$.

ORAL WORK

1. Ask and answer the questions which can be answered from equation (C). (See page 211.)

2. Ask similar questions for equations (A) and (V).

3. Ask and answer similar questions for equations (P) and (I).

4. Ask and answer questions which can be answered from equation (S). (See page 28.)

WRITTEN WORK

1. A piece of land 100 rods wide and 150 rods long is sold for $8400. What is the price per acre?

2. If a carpet costs $1.80 per square yard; allowing for waste, what will be the cost of carpeting a room 20 feet wide and 24 feet long?

3. If ice weighs 57 pounds to the cubic foot, how many tons of ice are there in a car 38 feet long, $8\frac{1}{2}$ feet wide, if it is piled 5 feet deep?

4. A farm in Illinois is bought for $215 an acre. If taxes amount to $1.40 an acre, and if interest on money is 6%, how much per acre do the interest on the purchase price and the tax amount to?

5. In New York city a lot 40 feet wide and 86 feet deep is sold for $1.15 per square foot. If the purchase price is borrowed at $5\frac{1}{2}$% interest what is the yearly interest on the purchase price?

6. Goods sold at a reduction of 15% from the marking price are sold for how many per cent of the marking price?

7. A machine sold at 25% below list price is sold for $67.50. What was the list price?

8. A house costing $6500 to build is rented at $50 a month. If repairs and taxes amount to $175 yearly what is the net income from the house? What per cent of the cost of the house is this net income?

9. At $6\frac{1}{2}$% find the interest on $3400.00 from May 1st to November 15.

10. On April 19, 1919, Captain E. F. White made the first non-stop trip between Chicago and New York in an aeroplane, flying 727 miles in 6 hours and 50 minutes. Find his average speed in miles per hour.

WRITTEN WORK

In many of the following problems the complete solution should first be indicated. Then cancel as far as possible, and finally obtain the required answer. Discuss the advantages of first indicating the solution. Are these advantages equally great for all problems?

If it costs $2.40 to pave one square yard, how much does it cost to pave a street 40 feet wide and 1280 feet long?

2. If one bushel of grain is counted as $1\frac{1}{4}$ cubic feet, how many bushels are there in a bin 8 feet long, 6 feet wide and 5 feet deep?

3. A man buys 140 sheep at $9.50 apiece and sells them at a gain of 15%. How much does he gain? What is the selling price?

4. If 36 tons of hay are sufficient to winter 19 cows, how many tons will be required to winter 53 cows?

5. At 32 pounds of oats to the bushel, how many bushels are there in a load weighing 2380 pounds?

6. If oats are selling for 65 cents a bushel, what is the value of a load weighing 2640 pounds.

7. At $8.25 a ton, what is the value of a load of coal weighing 6700 pounds?

8. 272 is how many per cent of 531? Find the result to the nearest tenth of one per cent.

9. What is the rate of per cent of income on an investment of $648,000 if the income is 54,000 yearly?

10. An article listed at $40 is sold for $25. What is the rate of discount?

11. In the first non-stop aeroplane flight across the Atlantic Alcock and Brown flew 1900 miles in 16 hours, 20 minutes. What was their speed in miles per hour?

WRITTEN WORK

1. A furniture dealer bought a table for \$20, and marked it to sell at a gain of 40%. What was the marking price?

2. If the dealer of the preceding problem was obliged to sell the table at a reduction of 50% from the marking price, what was his selling price? Did he gain or lose?

3. If one loaf of bread requires 12 ounces of flour, how many loaves can be made from a barrel containing 196 pounds? How much does the flour in each loaf cost if the barrel costs \$4.15?

4. At 60¢ a square yard, how much will it cost to sod a lawn 40 feet by 55 feet?

5. $\frac{1}{2}$ is how many per cent of $\frac{3}{4}$?

6. $\frac{2}{3}$ is 50% of what number?

7. $4\frac{1}{3}$ is $12\frac{1}{2}$% of what number?

8. What is the interest on \$260 at 6% for 1 year, 5 months and 12 days?

9. What is the interest on \$3650 at $5\frac{1}{2}$% for 2 years, 7 months and 21 days

10. A potato field 18 rods by 40 rods yields 670 bushels. What is the yield per acre?

11. If the tax rate on personal property is $\frac{7}{8}$ of 1 per cent of its value, how much tax does a man pay whose personal property is valued at \$7400?

12. If the tax rate on farm land is .92% of the value, what must a man pay on a farm containing 170 acres, valued at \$95 an acre?

13. A very high wind blowing 45 miles an hour exerts a pressure of 8.1 pounds per square foot. How many tons pressure will it exert on a wall 54 feet by 22 feet?

120. Units of Measure. The unit of measure used depends upon the kind of thing to be measured.

Thus, in answer to the questions in Exercise 1 below, you say you weigh so many *pounds*, that you are so many *inches* tall, that you are so many *years* old, and that you live so many *blocks* or *rods* or *miles* from the school.

That is, in measuring any quantity whatever, use is made of a certain quantity as a *unit*, and we then find out how many times that unit is contained in the quantity to be measured.

ORAL EXERCISES

1. What is your weight? your height? How old are you? How far from school do you live?

2. Name some units used in measuring time, weight, length, area, volume.

121. Importance of Measurement. The art of measuring is one of the most important that man has learned, and you will find it very interesting to observe how much use is made of it.

Have you ever seen a stone building in the process of construction? How are the stones made to fit into their places? Are they cut near the building? If not, how does it happen that they are cut just the right size?

In digging tunnels, such as those under the Hudson River in New York, it is customary to begin from both sides and dig toward the middle. The engineers, by making careful measurements and calculations, are able to direct the work so that the tunnels meet exactly. In such cases as the building of these tunnels the measurements made are very exact, and very fine instruments are used.

Without careful measuring it would be impossible to construct large buildings, railways, steamships, or any sort of complicated machinery. Indeed, if the art of measuring were lost we should be reduced practically to the level of the savage, so far as our material comforts are concerned.

122. **Reasons for Studying Measurements.** It is necessary that we should study measurements both because of their practical usefulness to each of us and also because they are of such fundamental importance in our civilization that every intelligent and well-informed person should have some understanding of them.

123. **Denominate Numbers.** A number expressed in terms of a standard unit of measure, such as 5 gallons, 3 feet, etc., is called a *denominate number.*

A number expressed in terms of more than one unit of measure, such as 3 gallons and 2 quarts, or 2 feet 6 inches, is called a *compound denominate number.*

A number expressed in terms of one unit is a *simple denominate number.*

It is frequently necessary to reduce a compound denominate number to a simple denominate number. Thus, we may need to reduce yards and feet to feet; pounds and ounces to ounces. We may also need to reduce inches to a compound number, such as feet and inches.

ORAL EXERCISES

1. Give a number of examples of the use of compound denominate numbers. Can you state your height as a compound denominate number? Your age?

2. Express 2 minutes, 30 seconds in terms of seconds. Is the result a simple or compound denominate number?

3. Express 3 feet 8 inches as a simple denominate number.

4. Express 36 hours as a compound denominate number.

5. Express 13 feet as a compound denominate number.

Drill in Fundamentals. Play game No. 3, page 30. Multiply integers for the first event, divide integers for the second event, solve examples in interest for the third event, solve examples in finding the rate in percentage for the fourth event.

ORAL AND WRITTEN WORK

1. What things do you know that are sold by liquid measure? by dry measure?

2. Give the table of liquid measure; of dry measure.

3. How many cubic inches are there in one gallon? In one quart? (See page 288.)

4. How many cubic inches are there in one bushel? in one peck? in one dry quart?

5. One cubic foot of water weighs 62.5 pounds. From this find the weight of one gallon of water. Give result to the nearest hundredth of a pound.

Suggestion: One cubic inch of water weighs $\frac{62.5}{1728}$ pounds.

$$\text{Hence, one gallon weighs } \frac{62.5}{\underset{576}{\cancel{1728}}} \times \overset{77}{\cancel{231}} \text{ pounds} = ?$$

Milk delivered at a creamery is weighed instead of measured with a liquid measure. Why is this? One gallon of milk weighs 8.62 pounds.

6. During one week a farmer delivered milk as follows to a creamery:

Sunday,	384.3 pounds	Thursday	412.4 pounds
Monday,	406.6 pounds	Friday	405.8 pounds
Tuesday,	414.2 pounds	Saturday	416.3 pounds
Wednesday	397.5 pounds		

How many gallons did he bring in during the entire week? At $18\frac{2}{3}$¢ a gallon, how much was this milk worth?

7. A milk can filled with milk weighs 90.4 pounds. When empty it weighs 20.3 pounds. How many gallons of milk did it contain? Give the result to the nearest tenth of a gallon.

ORAL AND WRITTEN WORK

1. A bushel of potatoes should weigh 60 pounds. What should be the weight of a peck of potatoes?

2. If a driving horse is fed 12 quarts of oats a day, how many bushels of oats will he consume in one year? At 45 cents a bushel how much will these oats cost?

3. If a farmer sows $3\frac{1}{2}$ quarts of timothy seed to the acre, how many bushels will he require to seed a meadow containing 52 acres?

4. A grocer buys green peas by the bushel at $1.30 a bushel, and sells them at 6¢ a quart. How much does he gain by selling 4 bushels of green peas?

5. By how many cubic inches does .8 of a bushel differ from one cubic foot? (For the number of cubic inches to one bushel see page 288.)

In estimating the contents of boxes and bins, it is customary to regard one cubic foot as containing .8 of a bushel.

6. How many bushels are there in a bin 14 feet long and 10 feet wide, if the grain in it is $5\frac{3}{4}$ feet deep?

7. A bin is 12 feet long and 8 feet wide. How deep must the grain be in it to make 400 bushels? Get result to the nearest hundredth of a foot. Also, find the result in feet and inches to the nearest tenth of an inch.

8. Sometimes coal is sold by the bushel. The bushel used, however, is different from the ordinary bushel. One bushel of bituminous (soft coal) weighs 76 pounds. How many bushels of coal are there in a ton? At 25¢ a bushel, how much does one ton cost?

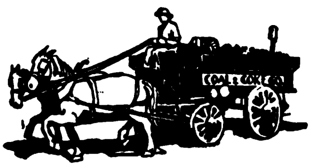
9. Measure the inside dimensions of a chalk box. Will this box hold more or less than one quart? How much? (See problem 3 on the opposite page.)

WRITTEN WORK

Hay is conveniently weighed by weighing the wagon and load together and then weighing the wagon separately.

1. A farmer hauled six loads of hay which, with the wagon weighed 3480 lbs., 3520 lbs., 3360 lbs., 3630 lbs., 3580 lbs. and 3430 lbs. respectively. The wagon, including the hayrack and the man, weighed 1480 lbs. How many tons of hay did the farmer haul? Give result to nearest hundredth.

2. At $12.50 a ton what was the value of the hay in problem 1?



Coal at the mine is usually weighed by the long ton. A long ton is 2240 pounds. It is sold by the ordinary short ton, which contains 2000 pounds.

3. A car at the mine is loaded with 46 long tons of coal. How many tons (short) can a dealer sell from this car if he deducts 3% from the weight at the mine for loss in transportation?

4. At $7.80 per ton (short tons) what is the value of ten loads of coal which weigh 3880 lbs., 4060 lbs., 3850 lbs., 3820 lbs., 3960 lbs., 3930 lbs., 4020 lbs., 4040 lbs., 3970 lbs., and 3890 lbs.

5. If one ton of coal occupies 35 cubic feet of space, how many tons are there in a car $8\frac{1}{2}$ feet wide and 42 feet long, if the coal is $4\frac{1}{2}$ feet deep?

Flour, live cattle, hogs, etc., are often bought and sold by the hundredweight. If you look up the market reports, in the daily paper, you will see that hogs and cattle are quoted at so much per hundred pounds. This means live weight.

Cattle

Beef steers, good to prime	$8.25a	$9.50
Beef steers, medium to good	7.40a	8.25
Beef steers, common to medium	6.65a	7.40
Yearlings	7.50a	9.35
Beef cows	5.00a	7.25
Fat heifers, good to selected	6.00a	8.00
Stock steers	4.70a	6.85
Feeders	5.75a	7.40
Canners and cutters	3.75a	4.70
Good to prime veals	10.25a	11.50

Hogs

Bulk of sales	$8.40a	$8.55
Common to good mixed	8.35a	8.50
Fair to choice medium weights	8.50a	8.55
Light weights	8.40a	8.55
Fair to choice butchers	8.45a	8.60
Select 200a 300 packers	8.45a	8.50

The above is a clipping from the market report of a Chicago paper. The prices given are normal prices existing before the war caused sudden and exceptional changes.

WRITTEN WORK

If a steer weighs 1000 pounds before killing, and if the dressed meat weighs 580 pounds, the steer is said to dress off 420 pounds or 42% in butchering.

1. What will be the average cost per pound of a dressed steer, which dresses off 38% in butchering, if the live animal is sold at $8.50 per hundredweight?

2. Poorer grades of steers dress off 45% in butchering. What will be the average cost per pound of the meat from such a steer if the live animal is sold at $7.25 per hundredweight?

3. Fine hogs dress off about 25% in butchering. What will be the average cost per pound of a dressed hog, if the live animal sells for $8.45 per hundredweight?

4. Using a newspaper report of prices of live cattle and hogs, make other problems and solve them. Remember that hogs dress off about 25%, while cattle dress off from 35% to 50%.

ORAL WORK

1. In measuring the width of this page, what unit of measure do you use? What unit do you use in measuring the length of your schoolroom?

2. What unit of measure is used in giving the distance between two cities like Chicago and New York?

3. Name lengths which are measured by the inch; by the foot; by the rod; by the mile.

4. Give the table of linear measure. (See page 288.)

5. How many rods are there in one mile? How many feet are there in one rod?

WRITTEN WORK

1. How many yards are there in one rod? In one mile? If you step $2\frac{1}{2}$ feet, how many steps must you take to walk one mile?

2. If a military step is $2\frac{1}{2}$ feet, how many steps must a soldier take on a day's march of 22 miles?

3. If fence posts are set 12 feet apart, how many posts are used in one mile of fence, including one post at each end of the fence?

4. If the cost of the posts, including the setting, is 23 cents apiece, what is the cost for the posts for this fence?

5. The distance between railway ties from center to center is 2 feet. How many ties are used to the mile? At 80 cents apiece what is the cost of ties for one mile?

6. A heavy railway rail weighs 105 pounds to the yard. How many tons of such rails are required for a mile of track? At $28.50 a ton what is the cost of these rails?

7. Find the average length of your step by making a distance of known length and counting the steps.

ORAL AND WRITTEN WORK

Answer as many as you can of the following questions without using pencil and paper:

1. How many square feet are there in one square yard?

2. How many square inches are there in one square foot?

3. How many square yards are there in one square rod? If you do not know this, how can you find out?

4. How many square rods are there in one square mile?

5. How many square rods are there in one acre?

6. How many acres are there in one square mile?

A township is a square piece of land six miles on each side.

7. How many square miles are there in a township?

A section of land is one mile square. A quarter-section is one-fourth of a section.

8. How many rods of fence are required to fence in a section of land? A quarter-section?

If this question causes trouble, draw a figure to represent a section and a quarter-section of land.

9. Give the table of square measure. (See page 288.)

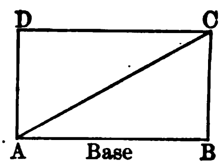
10. A field is 80 rods long and 60 rods wide; how many acres are there in it?

11. The total number of miles of railway in the United States in 1912 was 244,180 miles. The average width of the *right-of-way* was $4\frac{1}{4}$ rods. How many square miles of land was occupied by these railways?

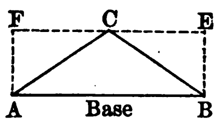
12. A contract is let to cover a lawn with sod for 65 cents a square yard. What is the cost if the lawn is $34\frac{1}{2}$ feet wide and 94 feet long?

124. **Area of Triangle.** We know that the area of a rectangle is equal to the product of the length and width. We sometimes call one side of a rectangle the *base* and the other side the *altitude*. That is,

area of rectangle = base × altitude.



In the first figure, opposite corners of a rectangle are connected, forming two equal triangles, ABC, and ACD. Each of these triangles is equal to one-half of the rectangle.



In the second figure, a triangle ABC is given, and the lines BE, EF, and FA are drawn so as to form a rectangle. It is clear that the triangle ABC is one-half of the rectangle ABEF.

One side of the triangle is called its base, and the line drawn from the opposite corner to the base and making a right angle with it is called the altitude. Then we have:

area of triangle = one-half base × altitude.

Notice that in the upper figure BC is the altitude of the triangle and also of the rectangle. In the lower figure a line drawn from C to the base AB and making right angles with it is the same length as BE, which is the altitude of the rectangle.

The rule given here for finding the area of any triangle is important, and you should make sure you understand it.

ORAL EXERCISES

Find the areas of triangles whose bases and altitudes are:

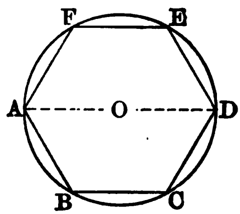
	Base	Altitude		Base	Altitude		Base	Altitude
1.	12	6	4.	16	10	7.	15	8
2.	8	10	5.	20	10	8.	18	7
3.	10	6	6.	40	12	9.	13	6

125. Circle, Diameter, Radius. A curve all of whose points are the same distance from one point, called the *center*, is a *circle*.

The word circle is also used to mean the area inside the curve. There is nothing strange in this, however. There are plenty of words with more than one meaning.

The distance from the center to the curve is the *radius* of the circle. The distance straight across the circle through the center is the *diameter*.

In the figure, OD is a radius and AD is a diameter. The figure ABCDEF placed inside the circle has all its sides equal to a radius. Hence the distance around this figure is 6 times the radius or 3 times the diameter.



The distance around the circle is called the *circumference*.

ORAL WORK

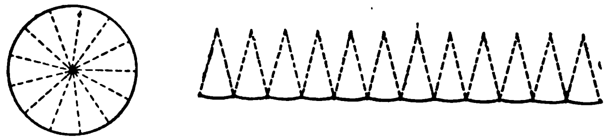
1. Is the circumference of a circle more or less than 3 times its diameter?

2. Measure the diameter of a circular object such as a tin can and also its circumference. Divide the circumference by the diameter to find the number by which the diameter must be multiplied to get the circumference. Make a number of such measurements, using different circular objects. Then, take the average of the quotients.

> *If the diameter of the circle is multiplied by $3\frac{1}{7}$ the product is very nearly its circumference.*

3. If the diameter of an automobile wheel is 34 inches, what is the length of the outside of the tire? (Multiply $3\frac{1}{7}$ by 34.) How far will this wheel run in making one revolution? (When a wheel turns around once it is said to make a revolution.)

4. If you have an automobile in your family find how many revolutions its wheels make in going one mile.



126. Area of Circle. From these figures we see that the interior of a circle may be cut into small pieces, each shaped nearly like a triangle. If they are cut narrow enough, we may regard them as triangles. The altitude of each triangle will then be the same as the radius of the circle, and the bases will be parts of the curve itself. The sum of the bases is the same as the circumference of the circle. So the area of all these triangles will be the sum of these bases, or the circumference of the circle, multiplied by half its radius. That is,

area of circle = circumference × half the radius.

ORAL AND WRITTEN WORK

1. If the diameter of a circle is given, how do you find the circumference? Then how do you find the area?
2. Find the circumference and the area of a circular flower bed whose diameter is 6 feet.
3. The distance across a circular pond is 36 feet. What is the distance around it? What is its area?
4. The diameter of the earth is about 8000 miles. Find the distance around it to the nearest thousand miles.
5. The diameter of the moon is about 2160 miles. How many miles are there around the moon?
6. The diameter of the sun is about 866,300 miles. How many miles are there around the sun?
7. Find the area of a circular disk one foot in diameter. How much less than a square foot is this?
8. Draw a circle on the board with radius 8 inches. Find its area.

WRITTEN WORK

1. A large gas tank is 180 feet in diameter. How many feet are there around it?

2. Find the number of square feet of ground covered by the gas tank of the preceding problem.

3. The diameter of the wheels of a certain automobile is 35 inches. How many revolutions do the wheels of this machine make in going from New York to Boston, a distance of 250 miles?

(*Suggestion:* The distance from New York to Boston in inches is $250 \times 5280 \times 12$, and the circumference of the wheel is $35 \times 3\frac{1}{7}$ or $35 \times \frac{22}{7}$. Hence, the required number of revolutions is:

$$\frac{250 \times 5280 \times 12}{35 \times \frac{22}{7}} = \frac{250 \times 5280 \times 12 \times 7}{35 \times 22}.)$$

NOTE: While for practical purposes the distance from New York to Boston is never given in inches, this problem is very much simplified by so expressing it, since the circumference of wheel with which the distance is to be compared is most naturally expressed in inches.

4. The diameter of a standard wheel on a railway passenger coach is 36 inches. How many times will this wheel turn around in going from Chicago to Boston a distance of 1120 miles?

5. During one season an automobile was driven 8480 miles. How many revolutions did its wheels make if the tires are 34 inches in diameter?

ORAL AND WRITTEN WORK

1. What is meant by a cubic foot? A cubic yard?

2. How many cubic inches are there in one cubic foot? How many cubic feet are there in one cubic yard?

3. The interior measurements of a box are: Length, 4 feet 5 inches; width, 3 feet 2 inches; depth, 2 feet 3 inches. Find its volume in terms of cubic feet.

Solution: 4 feet 5 inches $=4\frac{5}{12}$ feet $=\frac{53}{12}$ feet $=$ length.
3 feet 2 inches $=3\frac{1}{6}$ feet $=\frac{19}{6}$ feet $=$ width.
2 feet 3 inches $=2\frac{1}{4}$ feet $=\frac{9}{4}$ feet $=$ depth.

Hence the volume in cubic feet $=$

$$\frac{53}{12}\times\frac{19}{6}\times\frac{9}{4}=\frac{1007}{32}=31.47,$$

which is the result within one-hundredth of a cubic foot.

4. What measurements must be made to find the volume of a rectangular solid? When these measurements have been made, how do you find the volume?

The answer to this question may be stated.

$$\textit{length} \times \textit{width} \times \textit{depth} = \textit{volume}.$$

5. If the dimensions of the solid are measured in inches in what terms will the volume be given?

6. If the dimensions of a solid are measured in feet in what terms will the volume be given? In what terms will the volume be given if the dimensions are measured in yards?

7. If the dimensions of the solid are given in feet, how do you find the volume in terms of cubic yards?

8. How many cubic yards of dirt must be taken out in making an excavation 40 feet long, 25 feet wide and 5 feet deep? Get the result to nearest integer.

WRITTEN WORK

The exact volume of a bushel is 2150.42 cubic inches.

1. If you know the number of cubic inches in a box, how can you find the *exact* number of bushels it will hold?

2. If you know the number of cubic feet in a box, how do you find the number of cubic inches in it?

Hence, we see that to find the exact number of bushels a box will hold, we must multiply the number of cubic feet by 1728, and divide the result by 2150.42. This is the same as multiplying by $\frac{1728}{2150.42} = .8036$. (See note under example 3, page 231.)

For practical purposes it is sufficiently accurate to multiply the number of cubic feet by .8 to obtain the number of bushels. (See page 223.)

3. Find the number of bushels in a bin containing 1000 cubic feet by multiplying by .8 and then by .8036. By how many bushels do the results differ?

A grain elevator is a great building used for the storage of grain. Grain elevators are very common in the central and western parts of the United States.

4. A bin in a grain elevator is 16 feet long and 14 feet wide. How many bushels of wheat are there in it if the wheat is 48 feet deep? (Use .8036 as multiplier.)

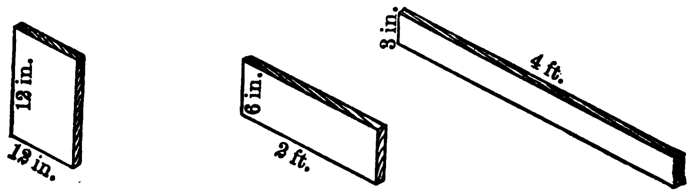
5. One cubic foot of clear water weighs 1000 ounces. How many pounds is this?

6. A tank used for hauling water to a threshing machine is 12 feet long, $3\frac{1}{2}$ feet wide, and 18 inches deep. How many pounds of water will it hold? How many tons?

7. The concrete foundation of a tall chimney is 10 feet 8 inches square and 7 feet deep. What is its cost at 35¢ a cubic foot?

127. The Board Foot. The unit for measuring lumber is the *board foot.* A board foot consists of a piece of board, one foot long, one foot wide, and one inch thick, or of any other piece of lumber having the same volume. A board foot is usually called a *foot* of lumber.

A board 1 inch thick, 6 inches wide, and 2 feet long contains one board foot, as does a board 1 inch thick, 3 inches wide, and 4 feet long.



Rule. To find the number of board feet in a piece of lumber, express two dimensions in feet and one in inches, and multiply.

Lumber dealers say: *"To get board feet, multiply feet by feet by inches."*

Thus, to get the number of board feet in a piece of studding 14 feet by 4 inches by 2 inches change the 4 inches to $\frac{1}{3}$ feet and then find the product, $14 \times \frac{1}{3} \times 2 = 9\frac{1}{3}$.

A board less than one inch thick is regarded as being one inch thick, while in pieces more than one inch thick, the actual thickness in inches and fractions of an inch is used.

In giving the dimensions of lumber, a foot is indicated by $'$ and an inch by $''$.

Thus the dimensions of a piece of lumber 2 inches thick, 8 inches wide and 16 feet long are given by $2'' \times 8'' \times 16'$.

Problem. The dimensions of a piece of lumber are: $3'' \times 7'' \times 14'$. Give two of these dimensions in feet and one in inches.

Solution: 3 inches is more readily expressed in feet than is 7 inches. But $3'' = \frac{1}{4}'$. Hence, $\frac{1}{4}' \times 7'' \times 14'$ are the required dimensions.

ORAL EXERCISES

Express two dimensions of each of the following pieces of lumber in feet and one dimension in inches:

1. $1\frac{1}{2}'' \times 12'' \times 10'$
2. $1'' \times 8'' \times 14'$
3. $2'' \times 4'' \times 8'$
4. $4'' \times 4'' \times 10'$
5. $1'' \times 4'' \times 14'$
6. $1'' \times 5'' \times 12'$
7. $1'' \times 8'' \times 16'$
8. $2'' \times 8'' \times 14'$
9. $3\frac{1}{2}'' \times 8'' \times 12'$

Problem 1. Find the number of board feet in a piece of lumber $4'' \times 10'' \times 16'$.

Solution: First: express $4'' \times 10'' \times 16'$ as $\frac{1}{3}' \times 10'' \times 16'$.
Second: multiply $\frac{1}{3} \times 10 \times 16 = 53\frac{1}{3}$ (board feet).

Problem 2. Find the number of board feet in 150 pieces, $1\frac{3}{4}'' \times 10'' \times 14'$.

Solution: The number of board feet is $\frac{7}{\underset{2}{\cancel{4}}} \times \frac{5}{\cancel{6}} \times \overset{7}{\cancel{14}} \times \overset{25}{\cancel{150}} = 3062\frac{1}{2}$.

WRITTEN EXERCISES

Find the number of board feet in the following pieces of lumber:

1. $6' \times 12'' \times 1''$, $8' \times 6'' \times 2''$, $10' \times 8'' \times 1\frac{1}{2}''$.
2. $12' \times 14'' \times 2''$, $14'' \times 8'' \times 2''$, $15' \times 12'' \times 4''$.
3. Fill out the blank spaces in the following:

Description	No. pieces	Dimension	Board feet
Rough boards	360	$12' \times 12'' \times 1''$	
Rough boards	120	$16' \times 10'' \times 1''$	
Oak flooring	580	$8' \times 3'' \times 1''$	
Oak flooring	340	$6' \times 3'' \times 1''$	
Pine planks	84	$14' \times 10'' \times 2\frac{1}{8}''$	
Studding	142	$10' \times 4'' \times 2''$	
Siding	275	$8' \times 6'' \times 1''$	
Sills	14	$12' \times 6'' \times 6''$	
Rafters	40	$10' \times 6'' \times 2''$	

128. Plastering and Painting. Plastering and painting are usually undertaken at a certain price per square yard. Sometimes allowance is made for doors and windows, and sometimes not. Only seldom is full allowance made for these, because working around them is slower than working over a clear surface.

1. Find the area in square yards of a wall 16 feet long and 9 feet high.

2. Find the area in square yards of a ceiling 18 feet long and 15 feet wide.

3. A room is 15 feet long, 12 feet wide and 9 feet high. How many square yards are there in its four walls, making no allowance for openings? How many square yards are there in the ceiling? What is the total area of the walls and ceiling?

4. Find the area in square yards of a ceiling 16 feet long and 14 feet wide.

5. Find the total area in square yards of the walls and ceiling of a room 20 feet long, 16 feet wide and 9 feet high, allowing 12 square yards for doors and windows.

6. What is the area in square yards of the walls and ceiling of a room 22 feet by 18 feet and 9 feet high? At 15 cents a square yard what does it cost to plaster this room?

7. The first story of a house is 9 feet high (above the base-board). At 12 cents a square yard, how much does it cost to plaster rooms (walls and ceiling) in this house having the following dimensions: $14' \times 12'$; $18' \times 15'$; $14' \times 10'$; $16' \times 12'$; $12' \times 8'$? No allowance is made for openings.

8. Make other problems like these and solve them.

129. Estimating Wall Paper. Wall paper is sold in single and double rolls, which are 8 yards and 16 yards long, respectively. The standard width is 18 inches.

Following are the steps in estimating the number of rolls required for papering a room:

First: Find the distance around the room, omitting the width of the doors and windows.

Second: Find how many strips ($1\frac{1}{2}$ feet wide) will cover this distance.

Third: Figure the number of rolls required to furnish this number of strips, keeping in mind that pieces will be needed to fill in above and below windows and above the doors, and that some allowance must be made for matching the patterns.

Problem. How many rolls of paper are needed for the walls of a room $18' \times 14'$, the length of the strips being 8 feet (distance from base-board to moulding), allowing $13'$ for doors and windows.

Solution: The length around the room, making allowance for doors and windows, is $18+18+14+14-13=51$ (ft.).

$51 \div 1\frac{1}{2} = 34$, which is the number of strips.

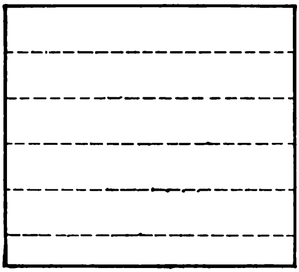
Only 5 strips can be cut from a 16-yard roll because of allowance for matching the pattern. Hence, 7 rolls will be needed.

WRITTEN EXERCISES

Find the number of rolls of paper required for each of the following rooms:

	Size of rooms	Length of strips	Allowance for openings
1.	$18' \times 13'$	8 feet	14 feet
2.	$19' \times 14'$	$8\frac{1}{2}$ feet	16 feet
3.	$23' \times 17'$	9 feet	18 feet
4.	$17' \times 16'$	$8\frac{1}{2}$ feet	11 feet

130. Estimating Carpets. In estimating the number of yards o carpet needed for a room the first thing to be kept in mind is that one cannot buy a fractional width of carpet. Thus, using carpet 1 yard wide, 6 strips will be needed for a room 16 feet wide, if the carpet is laid lengthwise.



Problem. How many yards of carpet one yard wide, are needed for a room 20 feet long and 17 feet wide, if the carpet is laid lengthwise of the room? Allow 6 inches on the length of each strip for matching the pattern.

Solution: First, find the number of strips. Since 17 feet is more than 5 yards (15 feet) and less than 6 yards, 6 strips are needed.

Second, the length of each strip must be 20 ft.+6 inches, or $20\frac{1}{2}$ feet. Hence, the total length of carpet is $6\times20\frac{1}{2}=123$ feet, or 41 yards.

WRITTEN WORK

1. How many yards of carpet one yard wide are needed for a room 17 feet long and 14 feet wide, if the strips are laid lengthwise of the room? Allow 6 inches on each strip for matching the pattern.

2. How many yards of carpet, one yard wide, are needed to carpet a lodge hall 64 feet long and 48 feet wide, no allowance being made for matching pattern, and the carpet being laid lengthwise of the hall?

3. How many yards of carpet will be needed to carpet the hall in Example 2 if the carpet is laid crosswise of the hall?

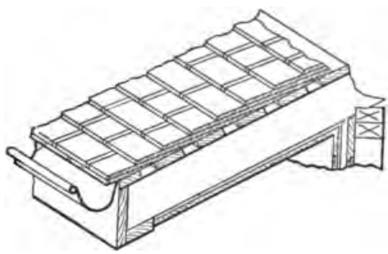
4. How many yards of carpet 27 inches $\frac{3}{4}$ yards) wide are required to carpet a room 21 feet long and 18 feet wide, if 8 inches ($\frac{2}{3}$ foot) is allowed on each strip for matching the pattern? The carpet is to be laid lengthwise of the room.

131. **Estimating Lumber for a Floor.** In estimating the number of board feet of lumber needed for a floor it is customary to add $\frac{1}{6}$ of the area of the floor to allow for waste and matching the boards.

Problem. How many board feet of oak flooring are required to cover one floor of a house 54 feet by 36 feet?

Solution: The total floor area is $54 \times 36 = 1944$ square feet. Adding $\frac{1}{6}$ of the area, we have 2268, the required number of board feet.

$$\begin{array}{r} 54 \\ \underline{36} \\ 324 \\ \underline{162} \\ 6)\overline{1944} \\ \underline{324} \\ 2268 \end{array}$$



132. **Shingles for Roof.** In estimating shingles for roof, 100 square feet of roof is called *one square.* The number of shingles required to cover a square of roof varies from 800 to 950. One *bunch* of shingles contains 250 shingles.

WRITTEN WORK

1. How many board feet are needed to lay one floor in a building 45 feet long and 32 feet wide? At $85 a thousand, what is the value of this lumber?

2. The first and second floors of a building 42 feet wide and 48 feet long are to be laid with oak flooring costing $78.00 a thousand feet. What is the cost of this flooring?

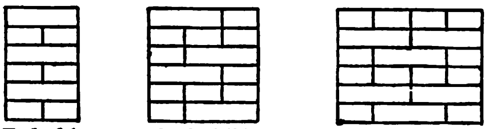
3. The two sides of the roof of a barn are $24' \times 46'$ each. How many squares of roofing are there in this roof? At 935 shingles to the square, how many shingles will be needed for this roof?

4. At $3.25 a thousand, find the cost of shingles for a roof containing $24\frac{1}{2}$ squares, if 950 shingles are used for a square?

133. Number of Bricks in a Wall. Ordinary building bricks are $8\frac{1}{4}$ inches long, 4 inches wide and 2 inches thick. The following are a builder's estimates of the number of bricks needed per square foot of wall for various thicknesses:

$8\frac{1}{4}$-inch wall, 14 bricks per square foot of wall.
$12\frac{3}{4}$-inch wall, 21 bricks per square foot of wall.
17-inch wall, 28 bricks per square foot of wall.
$21\frac{1}{2}$-inch wall, 35 bricks per square foot of wall.

The thicknesses of the walls are determined by the size of the brick. Thus, in the $8\frac{1}{4}$-inch wall one brick is laid crosswise or two

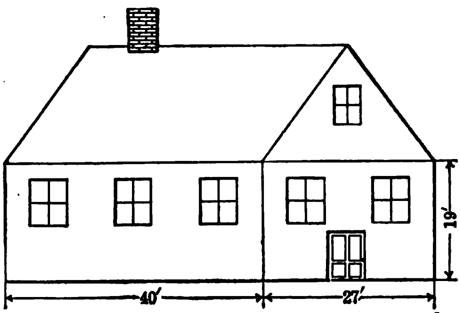


End of 1 brick wall — End of 1½ brick wall — End of 2 brick wall

side by side, with $\frac{1}{4}$ inch of mortar between. How are the bricks laid in the $12\frac{3}{4}$-inch wall? In the 17-inch? In the $21\frac{1}{2}$-inch?

WRITTEN WORK

The total length of wall around the house shown in the figure is 134 feet. Find the number of bricks needed to build this house if the walls are $12\frac{3}{4}$ inches thick and the house is 19 feet high. (No allowance is made for doors and windows, since some extra brick will be needed for chimneys and gables.)



ORAL AND WRITTEN WORK

See how many of the following you can solve orally:

1. How many hours are there from noon to noon of the next day?

2. What is the number of days in a common year? In a leap-year?

The real length of a year (solar year) is the time it takes the earth to complete its journey around the sun. This is nearly $365\frac{1}{4}$ days. The error caused by calling 365 days a year is rectified by making every fourth year a leap-year.

The years whose numbers are divisible by 4, such as 1916, 1920, 1924, are leap-years. However, years whose numbers are divisible by 400, such as 1600, 2000, are not leap-years.

Ten years is called a decade, and 100 years a century.

3. Make a table of time measure, including seconds, minutes hours, days, and weeks, months, years, leap-years, decade and century.

4. How many Sundays are there in one year? Add to these the number of the holidays you can think of, and then find the number of working days in a year.

5. How many days do you go to school each year? What fraction of the year is this? What per cent?

6. Give the number of days in each month of the year. Recall the device suggested on page 166 of the primary arithmetic for remembering this.

7. How many days are there from January first to July first in a common year? In a leap-year?

Find the number of days in each of the following:

8. April 3d to August 17th

9. May 9th to December 3d

10. July 27th to January 4th

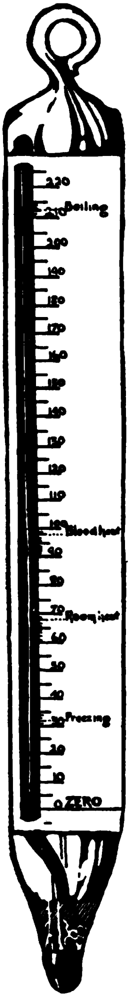
11. March 20th to October 14th

12. February 3d to August 23d

13. June 15th to March 7th

ORAL EXERCISES

1. What is the name of the thermometer in common use in the United States?



2. At what temperature does water freeze? At what temperature does it boil?

3. How many degrees above freezing is the boiling-point?

4. The boiling-point of fat is 315 degrees. How much above the boiling-point of water is that?

5. What is the highest temperature you have ever seen? What is the lowest? What is the difference between these temperatures?

6. What is the highest temperature you have ever read about? What is the lowest? What is the difference between these temperatures?

On account of its higher boiling temperature, fat is used in cooking certain things which cannot be cooked in water.

The amount of heat required to raise the temperature of one pound of water one degree Fahrenheit is called a *heat unit*. Thus, 180 heat units are required to change one pound of water from the freezing-point to the boiling-point.

You are acquainted with the fact that even on a hot day ice will thaw out but slowly. To change one pound of ice into water, the water being just the same temperature as the ice, requires 144 heat units.

After a pound of water is just brought to a boil it requires 966 heat units to boil it away entirely.

Thus, to thaw one pound of ice, bring it to a boil, and boil it entirely away, requires $144+180+966=1290$ heat units.

WRITTEN WORK

1. How many heat units are required to bring a kettle containing 12 pounds of water to a boil, if it starts at a temperature of 54 degrees?

2. How many heat units are required to boil away these 12 pounds of water after the water is brought to a boil?

3. How many heat units are required to melt 16 pounds of ice?

It is because the ice uses up so much heat in melting that it keeps the ice-box cool.

4. Which requires more heat, to melt 20 pounds of ice or to bring 15 pounds of ice-cold water to a boil?

5. Which requires more heat, to bring 20 pounds of ice-cold water to a boil or to boil away 4 pounds of water after it just starts boiling?

6. How many units of heat are required to melt a block of ice weighing 65 pounds? How many pounds of water could be changed from 70° to the boiling-point with this amount of heat?

Because of decreased atmospheric pressure, water boils at a ower temperature as the altitude increases.

Altitude	Boiling-point	Altitude	Boiling-point	Altitude	Boiling-point
Sea level	212°	4169	204°	10685	192°
512 ft.	211°	5225	202°	11799	190°
1025	210°	6304	200°	12934	188°
1539	209°	7381	198°	14075	186°
2065	208°	8481	196°	15221	184°
3115	206°	9575	194°		

ORAL EXERCISES

1. At about what temperature does water boil at your altitude?
2. Can you think of a place where water boils at 206 degrees? At 200 degrees? At 190 degrees? At 184 degrees?

134. Examples of Practical Reductions. Compound denominate numbers may be reduced to simple denominate numbers as in the following:

Example 1. Reduce 5 yards, 2 feet, 8 inches to inches.

Solution: 5 yards = 15 feet. 15 feet + 2 feet = 17 feet = 204 inches. 204 + 8 = 212 = number of inches in 5 yards 2 feet 8 inches.

A simple denominate number may be reduced to another simple denominate number, as are the following:

Example 2. How many square feet are there in one acre?

$$\begin{array}{r} 16.5 \\ 16.5 \\ \hline 825 \\ 990 \\ 165 \\ \hline 272.25 \end{array} \qquad \begin{array}{r} 272.25 \\ 160 \\ \hline 1633500 \\ 27225 \\ \hline 43560.00 \end{array}$$

16.5 × 16.5 = 272.25 = number of sq. ft. in one square rod.

272.25 × 160 = 43,560 = number of sq. ft. in one acre.

Example 3. Reduce 788 feet to feet and yards.

$$3\overline{)788}\;-2 \quad \text{quotient } 262$$

Dividing 788 by 3 gives a quotient of 262 and a remainder of 2. Hence 788 feet = 262 yards 2 feet.

In some cases a simple denominate number is expressed in a higher denomination, even when that requires fractions, as in the following:

Example 4. Reduce 186470 square feet to acres.

$$\begin{array}{r} 4.28 \\ 43560\overline{)186470} \end{array}$$

1 acre = 43560 square feet and 186470 ÷ 43560 = 4.28 = number of acres in 186470 square feet.

This problem occurs very frequently in foresters' measurement.

Example 5. Reduce 17840 pounds to tons.

$$\begin{array}{r} 8.92 = \text{number of tons.} \\ 2\overline{)17.840}\phantom{= \text{number of tons.}} \end{array}$$

First divide by 1000 and then by 2. How do you divide 17840 by 1000?

This problem occurs frequently in selling coal, hay, and other things sold by the ton.

135. Special Reductions. Frequently it is necessary to reduce one kind of denominate number to another kind. Thus, we need to find the number of gallons in a given number of pounds of milk. We need to reduce cubic feet to gallons and to bushels, and so on.

8.6 pounds of milk	= 1 gallon (nearly)
8.4 pounds of water	= 1 gallon (nearly)
231 cubic inches	= 1 gallon
$\frac{4}{5}$ or .8 cubic feet	= 1 bushel (nearly)
36 cubic feet of soft coal	= 1 ton (nearly)

WRITTEN EXERCISES

1. Reduce 43 feet 7 inches to inches.

2. Reduce 1340 inches to feet and inches.

3. Reduce 286,400 square feet to acres.

4. Reduce 42,650 square feet to square yards.

5. Reduce 6840 cubic feet to cubic yards.

6. 3840 pounds of milk are how many gallons?

7. How many gallons of water are there in a tank which holds 27.8 cubic feet. (First reduce to cubic inches.)

8. How many bushels does a bin hold which contains 480 cubic feet?

9. At $7.25 a ton, what is the value of 12,480 pounds of coal?

10. At 13.75 a ton what is the value of 27 360 pounds of hay?

11. How many cubic yards are there in an excavation 42 feet long, 34 feet wide and $5\frac{1}{2}$ feet deep?

12. How many square yards are there in a surface $18\frac{1}{2}$ feet by $13\frac{1}{3}$ feet?

13. A tank $6\frac{1}{2}' \times 3\frac{1}{2}' \times 1\frac{1}{2}'$ will hold how many gallons?

Example 1. Add 8 ft. 6 in.   We first add the inches.
9 ft. 9 in.   The sum is 15 inches, which equals
18 ft. 3 in.   one foot and 3 inches.

Write 3 in. in inches column and carry 1 ft. The sum is 18 ft.3 in.

Example 2. Multiply 4 gal. 2 qt. 1 pt. by 7.

4 gal. 2 qt. 1 pt.
7
32 gal. 1 qt. 1 pt.

7×1 pt. $=7$ pts. or 3 qts. and 1 pt. We write 1 pt. in the pints column and carry 3 qt. 7×2 qt. $=14$ qt. 14 qt. $+3$ qt. (carried) $=17$ qt., which is 4 gal. and 1 qt. Write 1 qt. in the quarts column and carry the 4 gal. The product is 32 gal. 1 qt. 1 pt. Note that the carrying in multiplication is just the same as in addition.

EXERCISES

Add the following, doing as many as you can orally.

1. 2 gal. 2 qt.
1 gal. 1 qt.

4. 2 mi. 140 rd.
3 mi. 280 rd.

7. 10 yd. 10 in.
24 yd. 8 in.

2. 15 yd. 2 ft.
12 yd. 1 ft.

5. 18 gal. 3 qt.
7 gal. 1 qt.

8. 4 mi. 80 rd.
7 mi. 300 rd.

3. 12 bu 3 pk.
7 bu. 2 pk.

6. 5 T. 980 lbs.
7 T. 270 lb.

9. 12 cu. ft. 104 cu. in.
7 cu. ft. 93 cu. in.

Multiply the following:

10. 3 bu. 1 pk.
6

13. 9 T. 150 lbs.
80

16. 5 yd. 8 in.
16

11. 7 yd. 2 ft.
16

14. 8 cu. ft. 120 cu. in.
72

17. 6 gal. 3 qt.
27

12 3 mi. 115 rds
8

15. 3 qt. 1 ft.
10

18. 17 ft. 5 in.
24

Example 1. Find the difference between 8 gal. 2 qt. and 4 gal. 3 qt.

FIRST METHOD

8 gal. 2 qt.	Add 4 qt. (1 gal.) to both numerator and denominator. Then 6 qt. = 3 qt. + 3 qt. 8 gal. = 5 gal. + 3 gal
4 gal. 3 qt.	
3 gal. 3 qt.	

SECOND METHOD

8 gal. 2 qt. = 7 gal. 6 qt.	First reduce 8 gal. 2 qt. to 7 gal. 6 qt. and then subtract. Why must 8 gal. 2 qt. be reduced to 7 gal. 6 qt.?
4 gal. 3 qt. = 4 gal. 3 qt.	
3 gal. 3 qt.	

Example 2. Divide 13 bus. 3 pk. 6 qt. by 3.

$$\begin{array}{r} 4 \text{ bu. } 2 \text{ pk. } 4\tfrac{2}{3} \text{ qt.} \\ 3\overline{)13 \text{ bu. } 3 \text{ pk. } 6 \text{ qt.}} \end{array}$$

Dividing 13 by 3 gives a quotient 4 and remainder 1. Reduce the remainder (1 bu.) to the next lower denomination, which is pecks. We then have 7 pk. in all. Dividing gives a quotient 2 and remainder 1. Reduce the remainder (1 pk.) to qt. We then have 14 qt. in all. Dividing by 3 gives $4\frac{2}{3}$ qt. The result is 4 bu. 2 pk. $4\frac{2}{3}$ qt.

WRITTEN EXERCISES

Subtract the following:

1. 27 bu. 5 qt.
   8 bu. 7 qt.
2. 160 a. 40 sq. rd.
   90 a. 64 sq. rd.
3. 17 rd. 6 ft.
   8 rd. 12 ft.
4. 35 cu. ft. 264 cu. in.
   18 cu. ft. 396 cu. in.
5. 84 cu. yd. 23 cu. ft.
   30 cu. yd. 26 cu. ft.
6. 20 hr. 35 min. 23 sec.
   4 hr. 20 min. 45 sec.

Divide each of the following:

7. 6)8 T. 340 lb.
8. 9)36 cu. yd. 24 cu. ft.
9. 7)24 bu. 2 pk.
10. 12)65 A. 84 sq. rd.
11. 8)14 hr. 38 min.
12. 6)21 mi. 145 rd.

136. Bills and Receipts. Before making out the bills on this and the next page, read again page 92.

A bill is receipted by writing "Paid" across the face of it and signing the name or initials of the person receiving the payment.

ORAL AND WRITTEN EXERCISES

1. What are the principal items that should be contained in a bill?

2. What is meant by "extending" and "footing" a bill? by "receipting" it?

3. Copy, extend and foot the following bill:

Mr. A. C. Blaine, Chicago, Ill.,
214 Lake Street, City. April 8, 19—

Bought of LAKESIDE FRUIT CO.

Terms: 30 days.

140 boxes oranges	$4.00	$560				
120 doz. pineapples	.75					
20 bbls. greenings	3.25					
50 bbls. baldwins	3.80					
25 bbls. Kings	4.50					
75 bunches bananas	1.25					
20 crates strawberries	5.25					
45 bbls. sweet potatoes	3.63					
30 bbls. potatoes	3.50					
14 crates lemons	5.20					
15 crates grapefruit	3.00					
15 bbls. Northern spy	4.75					
17 boxes Floridas	3.12					
190 lbs. dates	.08					
125 bbls. wealthies	4.25					
560 baskets of Concord grapes	.15					
350 baskets California grapes	.20					
80 boxes tangerines	1.50					

Paid March 3rd 19

The first line in this bill indicates 140 boxes of oranges at $4 a box. The total cost of these oranges is entered in the next column. The total amount of the bill should be entered in the last column to the right.

WRITTEN WORK

Arrange the following in the form of bills and find the amounts:

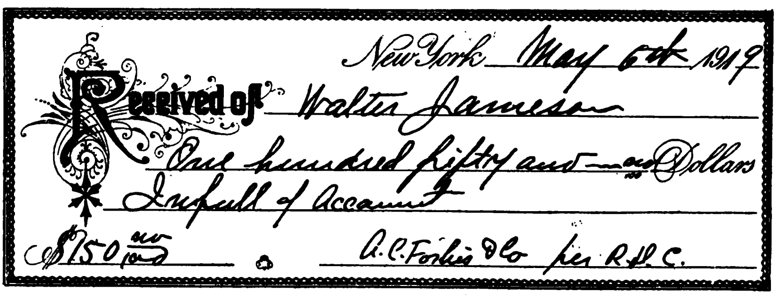
1. February 8, 1916, the following articles were bought of the Edwards Co. by Mrs. Frank Morse, both of Springfield, Mass.: 10 lbs. of sugar at $7\frac{1}{2}$¢ a lb., 1 sack of flour at $2.20 a sack, $1\frac{1}{2}$ bushels of potatoes at $1.25 a bu., 2 bags of salt at 6¢ a bag, 2 lbs. of lettuce at 16¢ a lb., 2 loaves of bread at 5c a loaf, $2\frac{1}{4}$ dozen eggs at 45¢ a dozen, 2 lbs. of Elgin butter at 40¢ a lb., $1\frac{3}{4}$ dozen oranges at 35¢ a dozen, and 2 cans of peas at 20¢ a can.

2. Walter Simons, Syracuse, N. Y., bought of the Rudd Co., Buffalo, 1400 bushels of apples at 72¢ a bu., 580 boxes of oranges at $3,14 a box, 15 bunches of bananas at $3.98 a bunch. Date, August 9th, 1917.

3. Johnson Co., of Pittsburg, sold William Earle, Avon, Ohio, Jan. 17, 1917. 20 bushels of cloverseed at $11.25 per bu., 18 bushels of alfalfa seed at $11.75 per bu., 8 bushels of timothy seed at $4.30 per bu.

Receipt the bill as having been paid, Feb. 8, 1917.

4. Allen, Frye & Co., St. Paul, sold to Johnson and Williams, of Fargo, North Dakota, 860 yards of gingham at $6\frac{1}{4}$¢ per yd., 1450 yds. of cashmere at 62¢ a yd., 580 yards of silk at $1.15 a yd., 1840 yards toweling at $12\frac{1}{2}$¢ a yd.


New York May 6th 1917
Received of Walter Jameson
One hundred fifty and no/100 Dollars
In full of Account
$150 00/100
A.C. Forbes & Co per R.H.C.

In some cases a receipt of the form shown here is given for money paid. What does this receipt tell you?

137. **The Cash Account.** People who are in business are obliged to keep accounts, so they may know at any time the condition of their business. Different accounts are used for different purposes. In the *cash account* is put down every item of cash taken in or paid out. The cash received is placed on the left side of the account and the cash paid out on the right side of the account.

The account below shows the following receipts of cash: April 2d, $12.40, April 3d, $16.50; April 4th, $25.65; April 5th, $31.45; April 6th, $25.25, and April 7th, $46.45.

It also shows that at the beginning of this week there was $246.78 on hand.

During this week cash was paid out as follows: April 2d, $4.60; April 3d, $10.70; April 4th, $14.85; April 5, $7.65; April 6th, $10.72; April 7th, $124.25. The total paid out was $172.77, and at the end of the week there was $231.71 on hand.

CASH

Dr.					Cr.
April 2	On hand	$246.78	April 2	By cash	$4.60
" 2	To cash	12.40	" 3	" "	10.70
" 3	" "	16.50	" 4	" "	14.85
" 4	" "	25.65	" 5	" "	7.65
" 5	" "	31.45	" 6	" "	10.72
" 6	" "	25.25	" 7	" "	124.25
" 7	" "	46.45			
		$404.48			$172.77
April 7	On hand	$231.71		By balance	231.71
					$404.48

The left side of an account is called the debit side, and is marked Dr. The right side is called the credit side, and is marked Cr. Whenever cash is received the amount is placed on the Dr. side of the cash account. We say we debit this account. Whenever cash is paid out we credit the cash account by entering the amount on the Cr. side.

138. **Account with Real Estate.** The account below is designed to show the expenditures and receipts in buying and selling a house. Copy the account and fill in the numbers as you solve the problems.

WRITTEN WORK

1. Bought a house for $9600. At 7% what is the interest on this sum for one year?
2. Taxes for one year $161.28.
3. At $1.75 a month water tax, what was this tax for one year?
4. Paid for repairs and improvements: $3.50, $46.50, $8.25, $12.65, $27.40. What is the total of these amounts?
5. The house was rented 11 months at $55.00 a month. What was the total rent?
6. At the end of one year, the house was sold for $11,000. What was the net gain?

House 9246 Langley Avenue.

Dr.				Cr.	
Purchase Price			Rent 11 mo.		
Interest			Selling price		
Taxes					
Water Taxes					
Repairs					
	$10552	58		$11605	
Balance	1052	42			
			Gain	$1052	42

7. Make up problems like the above, using data which you can find in your neighborhood. Find out for how much a building or a farm was bought, how much it was rented for, and how much was again sold for.

See if you can make up this problem without help from the teacher.

139. Account with Selling Papers. The *Morning Sun*, the *Morning American*, the *News* and the *Star* are sold to the newsboys at the rate of 5 papers for 3 cents.

The *Evening Sun* is sold to the newsboys at the rate of 5 papers for 2 cents.

All these papers are sold at a cent apiece.

When the account on the next page is completed it will show just how much money James made in one week selling papers. The left side of the account shows what he paid for his papers, and the right side shows what he received for them.

During one week James sold afternoon papers as follows:

Monday		Tuesday		Wednesday	
Evening Sun	45	*Evening Sun*	40	*Evening Sun*	40
News	35	*News*	45	*News*	35
Star	30	*Star*	35	*Star*	35

Thursday		Friday		Saturday	
Evening Sun	40	*Evening Sun*	40	*Evening Sun*	50
News	35	*News*	30	*News*	45
Star	35	*Star*	35	*Star*	40

1. How much did he pay for his papers each day?

Copy the account on the next page and fill in the numbers as you solve the problems.

2. How much did he get for his papers each day?

3. On Sunday morning James sold 36 Sunday *Americans* and 85 Sunday *Suns*. At 2 cents apiece what did he pay for these papers? How much did he get for them if he sold 24 papers at 5 cents apiece and the rest at 3 cents apiece? (The regular selling price for these Sunday papers was 3 cents, but early in the morning they were sold at 5 cents.)

4. On Saturday morning James sold 55 copies of the *Morning Sun* and 40 copies of the *Morning American*. How much did these papers cost him? How much did he get for them?

Extras are sold to the newsboys at the regular price, but the boys charge all they think they can get for them.

5. On Friday evening James sold 14 copies of extras at 5 cents apiece, 12 copies at 2 cents apiece, and 24 copies at 1 cent apiece. How much did the papers cost him? How much did he get for them.

The Saturday Evening Post is sold to the newsboys for 3 cents apiece. Unsold copies are returnable. The *Post* is sold everywhere at five cents a copy.

6. This week James sold 17 copies of the *Saturday Evening Post*. How much did they cost him? How much did he get for them

Dr. James' Account with his Papers Cr.

Mon.	Paid for eve. pap.			Mon.	Rec. for eve. pap.		
Tues.	" " " "			Tues.	" " " "		
Wed.	" " " "			Wed.	" " " "		
Thurs.	" " " "			Thurs.	" " " "		
Thurs.	*Sat Eve. Post*			Thurs.	*Sat. Eve. Post*		
Fri.	Paid for eve. pap.			Fri.	Rec. for eve. pap.		
Fri.	" " Specials			Fri.	" " Specials		
Sat.	" " " "			Sat.	" " " "		
Sat.	" " Morn. "			Sat	" " Morn. "		
Sun.	" " " "			Sun.	" " " "		
	Total				Total		
	Balance				Gain		

The prices used on these pages were taken from the city of Baltimore before the war. Make up the above account supposing that the papers cost the boys $1.40 per hundred and sold for 2 cents apiece.

140. Account with a Vegetable Garden. Walter planted a vegetable garden, and sold vegetables from it. Copy the account on the next page, and fill in the items as you work the problems.

1. He bought a hoe for 80 cents, a rake for 50 cents, a hand-weeder for 15 cents, and a watering-can for 40 cents. How much did all these tools cost him?

2. Walter bought 5 packages of lettuce seed at 5 cents a package, 3 packages of radish seed at 5 cents a package, and 2 pounds of onion sets for 25 cents. How much did all these cost him?

3. Walter worked 3 hours a day for 8 days, and one hour a day for 57 days. How many hours did he work altogether? At ten cents an hour, how much did this work amount to? We must enter the total pay for this work on the debit side of our account.

4. What was Walter's total expenditure on his garden, including his labor and $2.50 for a load of manure.

Walter sold vegetables from his garden as shown by the following table:

Crop	No. of days sale	Average sale per day	Price
Radishes	15	2 bunches	2 bunches for 5 cents
Lettuce	26	6 "	3 cents a bunch
Onions	16	4 "	2 bunches for 5 cents
Beet Greens	14	3 "	3 cents a bunch
Beets	17	6 "	2 bunches for 5 cents

How much did Walter get for each of his crops?

If his tools were worth $1.25 in the fall, how much did Walter make on his garden, besides getting paid for his work?

Dr.			Cr.		
Cost of tools			Sold radishes for		
" " seeds			" lettuce for		
" " labor			" onions		
" " manure	$2	50	" beet greens		
			" beets		
			Value of tools	$1	25
Total balance			Total gain		

141. Account with Belgian Hares. Make up an account like the above, using the following data:

Tommy Weeks raised Belgian hares for sale.

From April to September he bought 12 bales of alfalfa at 90 cents a bale; 650 lbs. of bran at $1.10 per cwt. (hundredweight); 500 lbs. ground barley at $1.60 per cwt.

Tom believes that his work taking care of the hares is worth 10 cents a day. How much does this amount to for these 6 months? (Find how many days there are in these months.) Enter this amount on the debit side.

His sales were:

April,	37 lbs. of dressed meat at	38 cents
May,	34 lbs. at	26 cents
June,	47 lbs. at	25 cents
July,	49 lbs. at	35 cents
August,	34 lbs. at	35 cents
September,	34 lbs. at	35 cents

Enter all expenses on the Dr. side, and all receipts on the Cr. side. From the account tell how much money he made these six months, besides receiving 10 cents a day for his work.

142. Classes of Mail Matter. Mail matter addressed to any post office in the United States or her possessions is divided into four classes: first, second, third, and fourth.

Ordinary letters, and any written matter whatsoever, is first class matter, and the rate is two cents an ounce. Second class matter includes newspapers and periodicals. The rate of postage is 1 cent a pound if mailed by the publishers or their agents; otherwise 4 cents a pound. Third class matter includes circulars, printed matter (not books or complete newspapers and magazines); the rate of postage is 1 cent for two ounces.

The rate for first, second and third class matter are the same for all distances. Fourth class matter (parcel post) includes books, merchandise and other articles in very large variety. (For rates and weight of packages permitted, see page 259.)

A letter may be *registered* or a *special delivery* secured for an extra 10 cents in postage.

On October 3, 1917, a law levying special war taxes went into effect. By this law the rates of postage on nearly all classes of mail matter were increased.

It was understood, however, that these increases were purely temporary, and on July 1, 1919, the old rates were restored.

WRITTEN EXERCISES

1. Assuming that the average weight of a letter is $\frac{5}{8}$ of an ounce, how many letters weigh one ton? At 2 cents apiece, how much does the government get for transmitting one ton of such mail?

2. How much does the government get for carrying one ton of newspapers and periodicals in the mails?

3. How much does the government get for carrying one ton of third class matter in the mails?

4. During one month a large business house sent out 148,600 sealed letters (2c) and 56,400 open letters (1c). How much did this house pay for postage?

143. Money Orders. Money may be sent to practically any post-office in the world by means of postal money orders.

The fees for money orders, payable in the United States are:

For orders from			$0.01	to	$2.50	3 cents
"	"	"	2.51	"	5.00	5 cents
"	"	"	5.01	"	10.00	8 cents
"	"	"	10.01	"	20.00	10 cents
"	"	"	20.01	"	30.00	12 cents
"	"	"	30.01	"	40.00	15 cents
"	"	"	40.01	"	50.00	18 cents
"	"	"	50.01	"	60.00	20 cents
"	"	"	60.01	"	70.00	25 cents
"	"	"	75.01	"	100.00	30 cents

The fees for foreign money orders payable in any country on which a money order can be drawn, other than those named above, may be ascertained upon inquiry at the post office.

The express companies sell money orders at the same rate as the post office.

ORAL EXERCISES

1. What is the cost of a money order for $20.01?

2. What is the cost of a money order for $3.50? For $6.00? For $1.75? For $9.65? For $17.45?

The total income of the Post Office Department for the years 1850, 1880, 1916 were:

	1850	1880	1916
Income	$5,499,985	$33,315,479	$312,057,688
Population	23,191,876	50,155,783	102,000,000

WRITTEN EXERCISES

1. Find the average amount expended for postage by each person in the United States for each of these years. Give each result to the nearest cent.

How do you account for the very large increase in the amount spent for postage?

144. **Fourth class matter** embraces that known as domestic parcel post mail, and includes merchandise, farm and factory products, seeds, cuttings, bulbs, roots, scions, and plants, books, (including catalogs), miscellaneous printed matter weighing more than 4 pounds, and all other mailable matter not embraced in the first, second, and third classes.

145. **Insurance.** Fourth class mail (*but no other*) may be insured against loss, upon payment of a fee of 5 cents, in addition to the postage, for value not exceeding $25, or 10 cents for value not exceeding $50. Damages are paid only in cases of total loss or destruction of the insured articles. No damage is paid for partial destruction.

The rates are given in the table on the opposite page.

ORAL EXERCISES

1. What is the postage on a parcel weighing 2 lbs. sent a distance of 25 miles? 100 miles? 500 miles? 800 miles?

2. What is the postage on a parcel weighing 20 pounds sent a distance of 250 miles? 450 miles? 700 miles? 950 miles?

3. What is the cost of sending a package from New York to Chicago (912 miles) if it weighs 3 lbs.? 5 lbs.? 12 lbs.? 16 lbs.? 20 lbs.?

4. What is the rate from St. Paul to Boston (7th zone) of a package weighing 3 lbs.? 10 lbs.? 12 lbs.? 16 lbs.? 18 lbs.? 20 lbs.?

5. What is the rate from St. Louis to Cleveland (4th zone) on a package weighing 5 lbs.? 7 lbs.? 19 lbs.?

6. In what zone is New York from your home? St. Louis? Boston? Chicago? Denver? San Francisco? Estimate these as closely as you can. What is the postage on a 7-pound package from your home to each of these places?

7. Find out what you can about express rates, and compare them with parcel post rates.

Weight in pounds	Local	1st Up to 50 miles	2d 50 to 100 miles	3d 100 to 300 miles	4th 300 to 600 miles	5th 600 to 1000 miles	6th 1000 to 1400 miles	7th 1400 to 1800 miles	8th Over 1800 miles
1	$0.05	$0.05	$0.05	$0.06	$0.07	$0.08	$0.09	$0.11	$0.12
2	.06	.06	.06	.08	.11	.14	.17	.21	.24
3	.06	.07	.07	.10	.15	.20	.25	.31	.36
4	.07	.08	.08	.12	.19	.26	.33	.41	.48
5	.07	.09	.09	.14	.23	.32	.41	.51	.60
6	.08	.10	.10	.16	.27	.38	.49	.61	.72
7	.08	.11	.11	.18	.31	.44	.57	.71	.84
8	.09	.12	.12	.20	.35	.50	.65	.81	.96
9	.09	.13	.13	.22	.39	.56	.73	.91	1.08
10	.10	.14	.14	.24	.43	.62	.81	1.01	1.20
11	.10	.15	.15	.26	.47	.68	.89	1.11	1.32
12	.11	.16	.16	.28	.51	.74	.97	1.21	1.44
13	.11	.17	.17	.30	.55	.80	1.05	1.31	1.56
14	.12	.18	.18	.32	.59	.86	1.13	1.41	1.68
15	.12	.19	.19	.34	.63	.92	1.21	1.51	1.80
16	.13	.20	.20	.36	.67	.98	1.29	1.61	1.92
17	.13	.21	.21	.38	.71	1.04	1.37	1.71	2.04
18	.14	.22	.22	.40	.75	1.10	1.45	1.81	2.16
19	.14	.23	.23	.42	.79	1.16	1.53	1.91	2.28
20	.15	.24	.24	.44	.83	1.22	1.61	2.01	2.40
21	.15	.25	.25						
22	.16	.26	.26						
23	.16	.27	.27						
24	.17	.28	.28						
25	.17	.29	.29						
26	.18	.30	.30						
27	.18	.31	.31						
28	.19	.32	.32						
29	.19	.33	.33						
30	.20	.34	.34						
31	.20	.35	.35						
32	.21	.36	.36						
33	.21	.37	.37						
34	.22	.38	.38						
35	.22	.39	.39						
36	.23	.40	.40						
37	.23	.41	.41						
38	.24	.42	.42						
39	.24	.43	.43						
40	.25	.44	.44						
41	.25	.45	.45						
42	.26	.46	.46						
43	.26	.47	.47						
44	.27	.48	.48						
45	.27	.49	.49						
46	.28	.50	.50						

Special Rates:

(*a*) Parcels weighing 4 ounces or less, except books, seeds, plants, etc., 1 cent for each ounce any distance.

(*b*) Parcels weighing 8 ounces or less containing books, seeds, plants, etc., 1 cent for each 2 ounces any distance.

(*c*) Parcels weighing more than 8 ounces containing books, seeds, plants, etc., parcels of miscellaneous printed matter, weighing more than 4 pounds, and all other parcels weighing more than 4 ounces, are chargeable according to distance as shown on the table on this page.

47	.28	.51	.51
48	.29	.52	.52
49	.29	.53	.53
50	.30	.54	.54

146. Drawing to Scale. We are now fairly familiar with simple drawings representing simple objects to a definite scale. The scale used depends, among other things, upon the size of the object to be represented. In a drawing representing objects such as a door or a window 1 inch may represent one foot, while in a map of the United States one inch may represent 1000 miles.

ORAL EXERCISES

What scale would you use in making a drawing representing the floor of your schoolroom? The top of your desk? A farm? Your state?

WRITTEN EXERCISES

1. Draw a figure representing the floor of your schoolroom. Let 1 inch of your drawing represent 4 feet of space. Put in marks to show the location of the seats and the teacher's desk. Take great care to get each object in its proper place. Thus if the teacher's desk stands 4 feet from the wall, it must be just 1 inch from the edge of your drawing. Underneath your drawing write very neatly:

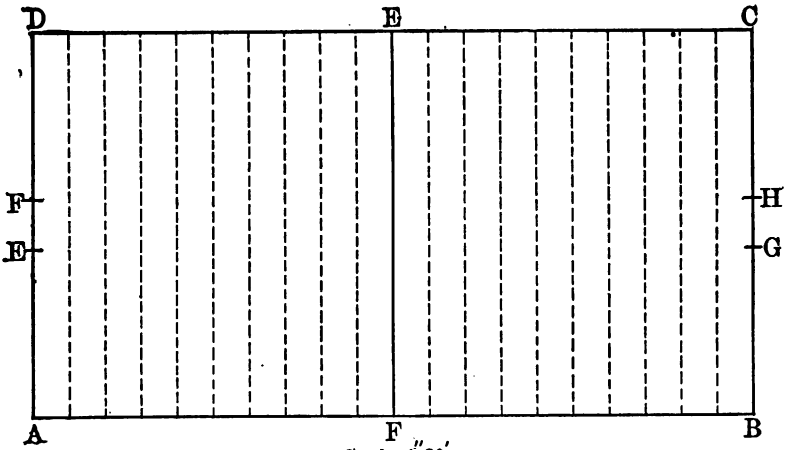
   Scale: 1 inch represents 4 feet.

2. Draw a figure representing the wall of your schoolroom. Put in all doors and windows. Make all the necessary measurements. Use the same scale as in Problem 1.

The drawings of buildings and rooms made by architects are always carefully drawn to scale.

3. Draw carefully to scale a map of your school grounds. By means of your map determine the distance between opposite corners of the yard.

4. If you live in a city make a drawing representing several blocks and streets about your schoolhouse. If you live in the country make a drawing representing the farm on which you live. Be careful to select a convenient scale.



Scale 1″:80′

The above is a diagram of a football field drawn to a scale in which 1 inch represents 80 feet.

WRITTEN EXERCISES

1. Measure the distances AB and BC carefully with a ruler, and then determine the dimensions of a football field.

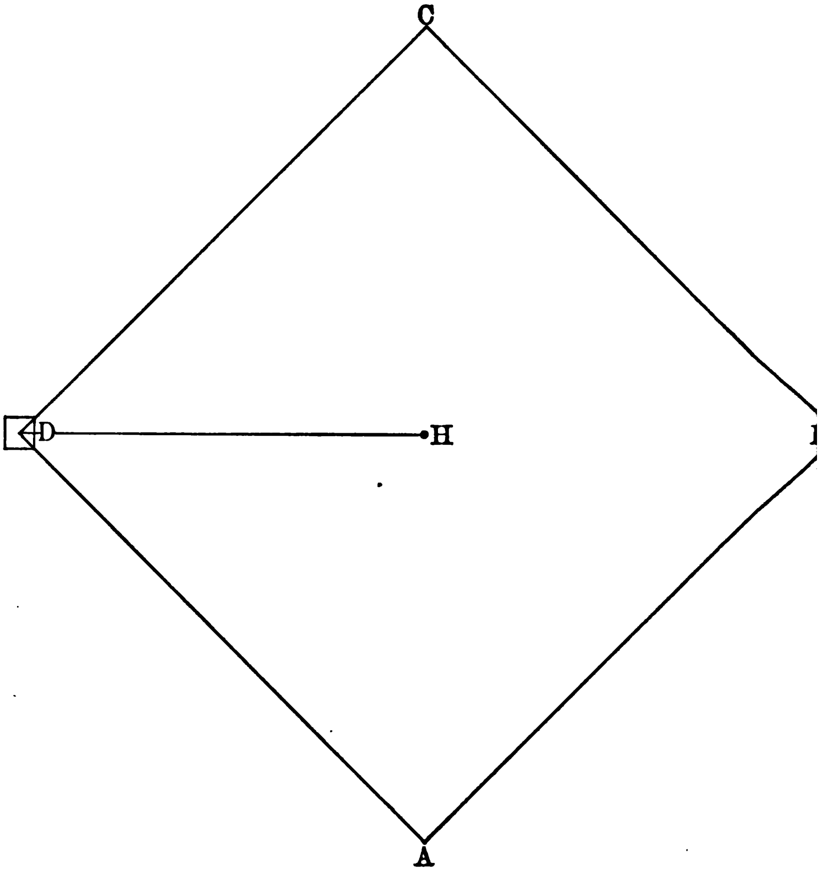
2. What is the distance from the center of the field to the goal-line? Give the distance in yards.

3. The goal-posts are $18\frac{1}{2}$ feet apart. How far is it from the corner A to the nearest goal-post?

4. On the board draw a diagram of a football field, letting 1 inch represent 10 feet. Put in the cross-lines, the goal-posts, and the side-lines. Make it represent a real football field as completely and accurately as you can.

5. Make a drawing to a convenient scale representing a door in your schoolroom. Put in as many details as you can.

6. Make a drawing to a convenient scale representing a window in your schoolroom, showing the sizes of the window panes.



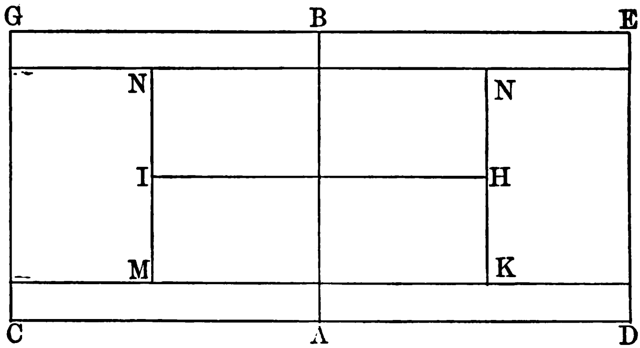
Scale 1"—30'.

The above drawing represents a baseball field. The point D marks the home plate. Name the other points that are lettered.

1. From the scale given on the drawing give the distances DA, AB, BC, CD, DB, and DH.

2. Draw this field to the scale 1" to 15'.

This drawing represents a tennis court of standard size.



Scale 1″ = 24′.

ORAL EXERCISES

1. From the scale given on this drawing find the distances CD, DE, HN, IH.

2. If this drawing is "1″ to 6′," what will be the length of the line CD in the drawing? What will be the lengths of the lines DE, HN, IH?

WRITTEN EXERCISES

1. Draw a tennis court to the scale 1″ to 12′.

2. On the blackboard draw a tennis court to the scale 1″ to 6′. Make this drawing as nearly accurate as you can.

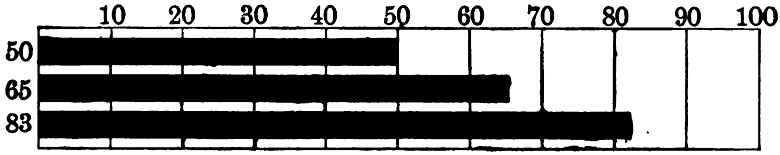
3. On the blackboard draw a baseball diamond to the scale "1 foot to 45 feet."

4. Draw a good map of your county, selecting a convenient scale, and find distances from your home to important points.

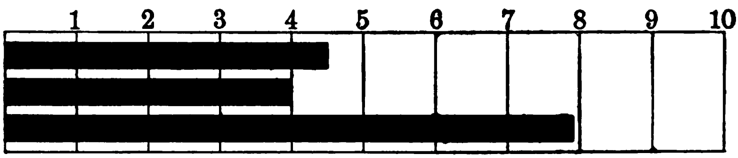
5. On a good map of your state find the straight line distances from your home to important points in the state. (First find the scale of the map.)

6. Draw to scale a good map of your state.

147. Practical Uses of Graphic Representation. Pictures of many kinds are used to represent numbers and sizes of things. All such pictures are called *graphs.* There are temperature charts which show the number of degrees of temperature for each hour of the day. The sizes of the armies and navies of different countries, prices of goods as they change from month to month and from year to year, populations of countries as they increase from decade to decade, and an endless number of other things are represented by graphs of one kind and another. Indeed, graphs are in such general use that popular magazines cannot be read intelligently without some understanding of them. On this and the next three pages one important kind of graph is explained.



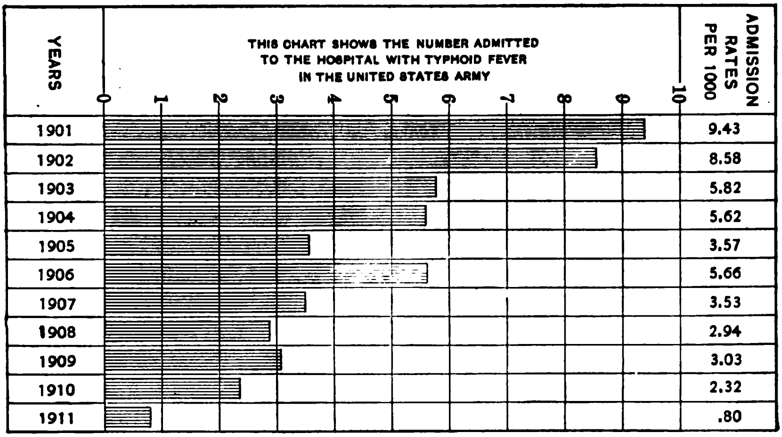
In the above figure the first black rectangle represents 50, the second 65, and the third 83.

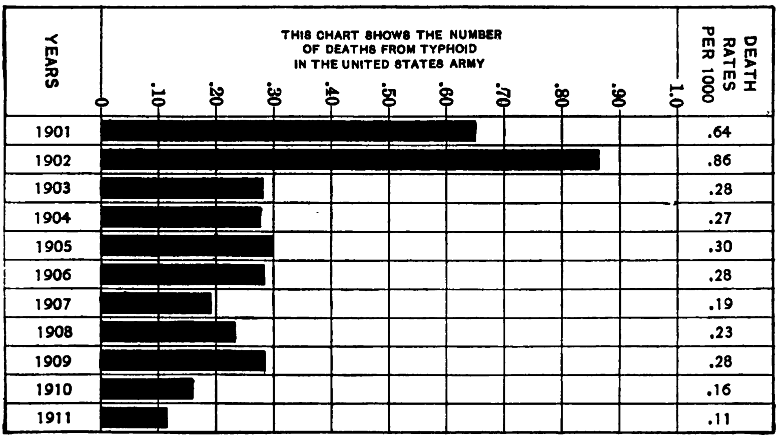


1. What number is represented by each of the black rectangles in the second figure?

2. Draw a rectangle like the first figure above, and by means of small rectangles within it represent each of the numbers 20, 80, 83, 57.

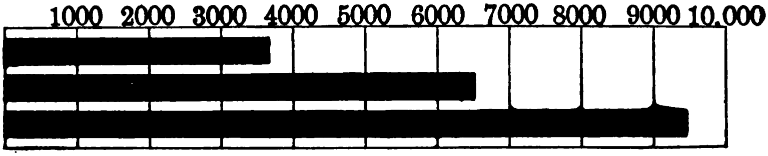
3. By means of a figure like the second one above represent the numbers 3, 5, 5, 7, 3, 9, 8.





During these years the sanitation in the army was greatly improved, and vaccination against typhoid came into general use.

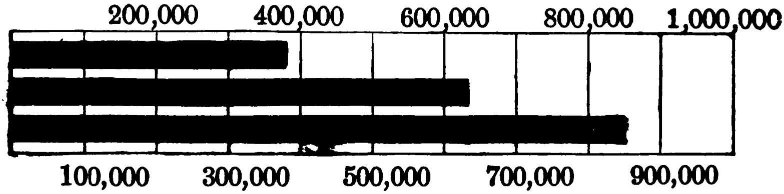
Which shows the facts more plainly, the column of figures to the right or the drawing?



1. State what numbers are represented by the black rectangles in the above graph.

A number like 3846 can be represented only approximately on such a graph. On the other hand, 4000 can be represented exactly, and 6300 may be represented very nearly exactly. That is, we can estimate or measure one-tenth of the spaces quite accurately. On a graph like the above we can therefore represent a number to the *nearest hundred.*

2. By means of such a figure represent 2500, 4600, 3900. To represent a number like 1680, in this figure, we simply take the number to the nearest hundred. That is, we take 1700.



In this graph numbers may be represented to the nearest 10,000.

3. On such a graph represent 480,000, 820,000, 270,000.

4. On such a graph represent approximately 148,560, 257,942, 573,300, 911,841. Use the approximations 150,000, 260,000, 570,000, 910,000.

5. Find the number of people in your city now, 10 years ago, 20 years ago, and 30 years ago. Represent these four numbers on a graph like the above. Your teacher will help you find these numbers. Your graph will show the growth of your city during the last 30 years.

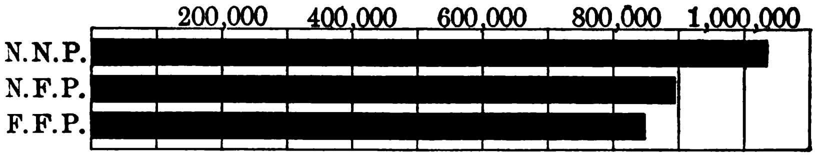
6. Find the number of pupils attending your school for each of the last few years, and make a graph to represent these numbers. What does this graph tell you about the growth of your school?

In the graph on this page the numbers are represented to the nearest 10,000.

In 1900 there were in Massachusetts, 1,032,264 persons who were native born of native parents, 897,386 were native born of foreign or mixed parents, and 840,114 were foreign born.

This is shown graphically as follows:

(N of N. P. means native born of native parents, N of F. P. means native born of foreign or mixed parents, and F. of F. P. means foreign born of foreign parents.)



Make graphs representing the information given below.

Compare these graphs to see what you can learn from them as to the changes in the character of the population in these states.

		1910	1900	1890
Mass.	N. of N. P.	1,103,429	1,032,264	995,430
	N. of F. P.	1,170,447	897,386	606,440
	F. of F. P.	1,051,050	840,114	653,503
III.	N. of N. P.	2,600,555	2,271,764	1,882,690
	N. of F. P.	1,723,847	1,498,473	1,044,804
	F. of F. P.	1,202,560	964,635	840,975
Ca.	N. of N. P.	2,568,382	2,179,395	1,804,778
	N. of F. P.	25,672	24,913	19,683
	F. of F. P.	15,072	12,021	11,892
N. D.	N. of N. P.	162,461	65,811	37,712
	N. of F. P.	251,236	133,311	63,347
	F. of F. P.	156,158	112,590	81,238

Make a graph to show the number of girls and the number of boys in your school.

See if you can find graphs like these in some magazine. If you do, bring it to school and explain what the graphs tell you.

1. A large wagon box is 10 feet 6 inches long, 3 feet 6 inches wide, and 26 inches deep, all inside measure. How many bushels will it hold? One cubic foot = .8 bu.

2. A man bought lots for \$1850, and sold them for \$1926. What was his loss if his expenses, including taxes and interest on the money invested, was \$245? What was his loss per cent?

3. At the rate of $6\frac{1}{4}\%$ commission, what is an agent's commission for selling a pleasure boat for \$4600?

4. The drivers on a passenger locomotive are 6 feet in diameter. How many revolutions will these drivers make in going from Milwaukee to Chicago, a distance of 85 miles?

5. A piece of furniture is bought by a dealer for \$25, and marked 75% above the buying price. It is then sold at a discount of 25%. What is the marking price? What is the selling price?

6. If 85% of butter is butter fat, how many pounds of butter fat are required to make 45 pounds of butter?

7. At \$2.15 a square foot, what is the value of an oriental rug 9 feet 6 inches wide and 11 feet 8 inches long?

8. A field contains 48.5 acres. How wide is it if its length is 120 rods?

9. A family spends 18% of their total income for rent. What is the income if they pay \$580 a year rent?

10. In October a farmer put 2360 bushels of corn into his cribs. He sold it in February at 71 cents a bushel. How much did he receive for it if the corn shrank 8%?

In a loaded freight car the weight of the car is called "dead weight," while the weight of the freight is called "live weight."

1. A wooden freight car weighs about 30,000 pounds, and carries 30 tons of freight. What per cent of the weight of a loaded car is "dead weight," and what per cent is "live weight"?

2. A steel car weighs 36,000 pounds, and has a capacity of 50 tons. What per cent of the total weight is dead weight and what per cent is live weight?

3. A steel coal car (gondola) weighs 46,000 pounds, and has a capacity of 70 tons. What per cent is live weight and what per cent is dead weight?

4. A house which rents for \$600 a year can be bought for \$8000. Which is cheaper, to buy the house or to rent it if money can be borrowed at $5\frac{1}{2}\%$, and if taxes and repairs amount to \$180 a year?

5. An auctioneer sold \$5630 worth of goods, charging a commission of $2\frac{1}{2}\%$ of the selling price. How much did the owner get for the goods?

6. An agent remits \$323 to his customer after deducting 5% of the sales price as his commission. What was the sales price?

7. A lot 25 feet wide and 95 feet deep sold for \$1450. At this rate what is the value of a lot 45 feet wide and 85 feet deep?

8. A dealer sold a furnace for \$402.50, which was a gain of 15% over the buying price. What was the buying price?

9. A farmer bought 860 feet of steel fence at \$.95 per foot. If he received discounts of 20% and 5%, what was the net cost?

10. A coal bin 14 feet long, $10\frac{1}{2}$ feet wide, is filled with coal to a depth of $5\frac{1}{2}$ feet. How many tons of coal are there in the bin? (One ton of coal occupies 36 cu. ft.)

1. During one year an agent averages \$2450 monthly sales. What is his yearly income if his commission is $16\frac{2}{3}\%$ on all of his sales?

2. The inside measurements of an ice-box are 18 inches by 14inches by 12 inches. How many cubic feet of ice does it hold, allowing one inch in each dimension because the ice does not fit the box?

   What is the weight of this ice if one cubic foot weighs 57 pounds? At 35 cents a hundred pounds how much does it cost?

3. A certain street in a large American city is 29 miles long and 66 feet wide. How many acres are there in this street? At 56 bushels to the acre, how much corn could be raised on this area?

4. A cattle dealer buys 78 head of cattle averaging 1173 pounds at $11\frac{1}{4}$ cents a pound. He sells them a few days later at $12\frac{3}{4}$ cents a pound. What was his buying price? What was his selling price if the cattle shrank 650 pounds in weight? His expenses were \$38.50. Did he gain or lose on the transaction?

5. In a city containing 246,900 inhabitants there were 4024 deaths in one year. What was the death-rate per thousand? Give rate to the nearest tenth.

   (*Suggestion:* Divide 4024 by 246.9.)

6. How many tons of ice can be cut from one acre of lake surface, if the ice is 16 inches thick. Allow 10% for waste in cutting and hauling. (One cubic foot of ice weighs 57 pounds.)

7. Recently an American-made racing car was driven 1 mile in 24.02 seconds. How many miles per hour was this?

   (*Suggestion:* At this speed the car would go $\frac{1}{24.02}\times 3600$ miles in 1 hour (1 hour equals $60\times 60=3600$ seconds).

8. In his report of the first aeroplane journey across the Atlantic Lieut.-Commander A. C. Reed says that the time from Trepassey to the Azores was 15 hours 18 minutes, and the average speed 81.7 knots. What was the distance in miles?

1. Find the interest on \$1400 at $6\frac{1}{2}\%$ for 3 years 4 months.

2. A farmer fed 1180 bushels of corn to a drove of hogs, thereby making them gain 10,316 pounds. How many pounds of corn were required to make the hogs gain one pound? Give result to the nearest tenth of a pound. (1 bushel of corn weighs 56 pounds.)

3. If milk contains 4.2% butter fat, how many pounds of milk will yield 75 pounds of butter fat?

4. Find the interest on \$850 at $5\frac{1}{2}\%$ for 4 months 15 days.

5. A real estate agent sold a house and lot for \$5600. How much did the owner get if the agent deducted 5% as his commission? How much did the agent make if his expenses connected with the same were \$84.75?

6. A lot in a city is valued at \$2300, and the house on it is valued at \$12,500. What is the tax on this property if it is 1.952% of the total value?

7. Find the circumference of a circle 16 inches in diameter. What is the area of this circle? By how much does it differ from the area of a square whose sides are 13 inches?

8. A stationery bill sold by a wholesale house amounting to \$1240 is discounted 26%. What is the net amount of the bill?

9. Find the interest on \$180 at $3\frac{3}{4}\%$ for 4 months 27 days.

10. What is the area of a triangle whose base is 16 inches and altitude 10 inches?

11. In making an excavation 45 feet wide, 72 feet long and $4\frac{1}{2}$ feet deep how many tons of earth must be removed if 18 cubic feet of solid earth weigh one ton?

12. If there are 11 ounces of flour in a 14-ounce loaf of bread, how many pounds of flour will make 1000 loaves?

1. If bricks weigh on an average $4\frac{1}{2}$ pounds, how many bricks are there in a load of $4\frac{1}{4}$ tons?

2. At $65 per thousand board feet, what is the cost of 560 boards 10 feet long 4 inches wide and 1 inch thick?

3. A water tank is 7 feet 6 inches long, 3 feet 4 inches wide, and 1 foot 10 inches deep. How many cubic feet does it hold? How many gallons of water does it hold?

4. Find the number of board feet in 60 planks $2\frac{1}{2}$ inches thick, 10 inches wide, and 14 feet long.

5. A steel rail expands $\frac{1}{3}$ of an inch in length when the temperature changes from our lowest to our highest. What per cent of the total length does such a rail expand if it is 30 feet long?

6. A circle is constructed inside a square to touch all four of its sides. Find the area of both the square and the circle if the diameter is 12 inches.

7. A farmer selling his milk is paid at the rate of $18\frac{1}{2}$ cents a gallon. How much does he get for the milk from one cow that yields 8940 pounds of milk for the year? (One gallon of milk weighs 8.6 pounds.)

8. How long will it take a man to dig 80 rods of ditch 2 feet wide and $3\frac{1}{2}$ feet deep, if he removes 320 cubic feet of earth a day?

9. If it costs $960 to pave a block of street 40 feet wide and 480 feet long, how much will it cost to pave a street 45 feet wide and 1800 feet long?

10. Find the number of bricks required to build a 17-inch wall, 160 feet long and 7 feet high. (See page 240.)

11. Loads of coal weighing 5760 lbs., 6050 lbs., 5810 lbs., 5370 lbs., 5040 lbs. are delivered. At $7.40 a ton, what is the value of this coal? How much would be saved by buying the coal three months earlier at $7.10 a ton?

1. A lot is 175 feet deep. How wide is it if it contains 5337.5 square feet?

2. In a certain town the tax on real estate is 1.06% of the value of the property. A man pays $156.88 tax on real estate. What is the value of his holdings?

(*Suggestion:* $156.88 is 1.06% of what number?)

3. What is the circumference of a wagon wheel 4 feet 4 inches in diameter? How many revolutions does this wheel make in going 1 mile?

4. How many bushels of potatoes will a bin hold that is 12 feet wide, $14\frac{1}{2}$ feet long, and $5\frac{1}{2}$ feet deep, if one cubic foot is .7 bushels of potatoes? (Bushels of different grains vary, depending on how much the measure is to be heaped up.)

5. After being used one year an automobile was sold for $1755. What was its original cost if the first year's depreciation was figured at 35%?

6. If 3.8% of milk is butter fat, and 85% of butter is butter fat, how many pounds of butter can be made from a cow yielding 11,860 pounds of milk?

7. How many heat units are required to melt 15 pounds of ice and then raise the water to 80 degrees. (It requires 144 heat units to melt one pound of ice. See page 242.)

8. Which requires more heat, to melt 35 pounds of ice or to raise 30 pounds of water from 60 degrees to the boiling point?

9. Using a tape measure, a boy finds that the circumference of a round log at its middle is 85 inches. Find the diameter of the log.

10. If it costs $56 to carpet a room 16 feet wide and 18 feet long, how much will it cost to carpet a room 25 feet wide and 36 feet long?

1. A brick barge on the Hudson River carries 650 tons of brick. What is the value of this barge load of brick at $6.75 a thousand if bricks weigh on an average $4\frac{1}{2}$ pounds apiece?

2. The total area of the State of Illinois is 56,650 square miles. How many acres are there? In 1909 the total area used for raising grain in this state was 16,536,500 acres. What per cent of the area of Illinois was used for raising grain?

3. What is the circumference of a wagon wheel 4 feet in diameter? How many revolutions will this wheel make in going one hundred miles?

4. A train travels 100 miles in 2 hours and 45 minutes. How many miles an hour is it going? How long will it take the train to go from St. Paul to Chicago, a distance of 420 miles?

5. In a certain country school the salary of the teacher is $450 for the year. Other expenses for the school are $245 for the year. The building cost $3500, on which the school district is paying interest at the rate of $5\frac{1}{2}\%$. Find the total expense per year of this school. What is the average expense per pupil if there are 28 pupils in the school?

6. If it takes 4 days to plow a piece of land 160 rods long and $10\frac{1}{2}$ rods wide, how long should it take to plow a piece of land 160 rods long and 54 rods wide?

7. The contract price of a house is $8640. What is the cost of this house, including architects' fees, which are 7% of the contract price?

8. Give convenient dimensions for a coal bin to hold 20 tons of soft coal (1 ton of soft coal measures about 36 cubic feet).

9. A man invested $12,500 and received a net annual income of $1050. What rate per cent did he receive? Find result to the nearest tenth of one per cent.

10. The value of a certain forest increases 4% each year. If it is worth $150,000 now, how much will it be worth in 3 years?

1. A new automobile costs $1560. If it depreciates 35% in value the first year and 25% the second year, for how much could it be sold at the end of the second year?

(*Suggestion:* First deduct 35% of the original cost to find the value at the end of the first year, and then deduct 25% of this value to find the value at the end of the second year.)

2. If a farmer gets $18\frac{1}{4}$¢ a gallon for milk, what is the value of the milk from one cow which yields 10,435 pounds in one year? (See page 245.)

3. The milk from a certain cow contains an average of 4.3% butter fat. During one year this cow yields 7830 pounds of milk. How many pounds of butter can be made from this milk if 85% of the butter is butter fat?

4. At 35¢ a square yard, how much does it cost to plaster the walls and ceiling of a room 18 feet long, $15\frac{1}{2}$ feet wide, and 9 feet high, making no allowance for doors or windows?

5. The milk from a certain cow contains an average of 3.6% butter fat. During one year this cow yields 11,895 pounds of milk. How many pounds of butter can be made from this milk if 85% of the butter is butter fat?

6. At $84 per thousand, what is the cost of 2460 linear feet of flooring $2\frac{1}{2}$ inches wide and $\frac{3}{4}$ inches thick? (A board less than 1 inch thick is always counted as 1 inch.) By linear feet is meant the sum of the length of all the boards. The length is given in this way because the pieces of flooring are of different lengths.

7. A grain dealer bought 45,000 bushels of oats at 37¢ a bushel, and sold them two months later at 41¢ a bushel. If the oats shrank 2%, and if his expenses were $340, did he gain or lose, and how much?

8. See who can bring in the most interesting problems for the class to solve.

148. Discount Series. Sometimes two or more discounts are given on the same goods.

Thus, goods may be sold at a 25% discount because the market price is lower than the list price. Another discount of 10% may be given because of an unusually large order, and still another discount of 5% because of prompt payment.

Example. Find the selling price of an article listed at $100.00 if discounts of 25% and 10% are allowed:

$100 less 25% of itself equals $75.
$75 less 10% of itself equals $75 − $7.50 = $67.50, which is the selling price.

The first discount is computed on the list price as a base; the second discount is computed on the amount left after the first discount has been deducted.

Solve this problem by first deducting 10% of the list price and then 25% of the remainder.

Notice that the *order* in which the discounts are deducted makes no difference.

WRITTEN EXERCISES

Find the selling price in each of the following examples. In the first four get the results in two ways:

	List price	Discounts		List price	Discounts
1.	$48.50	20%, 15%	9.	$248.00	25%, 15%
2.	$760.00	15%, 8%	10.	$8670.75	30%, 20%
3.	$240.00	20%, 12%	11.	$1450	10%, 5%
4.	$94.50	15%, 5%	12.	$4600	15%, 10%
5.	$345.00	30%, 10%	13.	$890	20%, 5%
6.	$18.60	20%, 10%	14.	$1860	25%, 15%
7.	$360.00	15%, 10%	15.	$2470	30%, 20%
8.	$125.00	20%, 12½%	16.	$6150	35%, 15%

WRITTEN EXERCISES

Find the selling price in the following. See how many you can do in 6 minutes:

	List price	Discounts		List price	Discounts
1.	$45.00	25%	7.	$250.00	$12\frac{1}{2}$%
2.	$60.00	30%	8.	$28.00	$16\frac{2}{3}$%
3.	$1.40	40%	9.	$35.00	$8\frac{2}{3}$%
4.	$18.00	10%	10.	$75.00	45%
5.	$160.00	$33\frac{1}{3}$%	11.	$125.00	15%
6.	$24.00	45%	12.	$64.00	$37\frac{1}{2}$%

Find the selling price of goods reduced as follows:

	List price	Discounts		List price	Discounts
13.	$80.00	15%, 10%	19.	$1.75	15%, 5%
14.	$45.00	10%, 5%	20.	$3.50	40%, 10%
15.	$18.00	20%, 15%	21.	$2.75	60%, 5%
16.	$60.00	30%, 10%	22.	$12.50	40%, 30%
17.	$45.00	25%, 10%	23.	$65.00	15%, 8%
18.	$85.00	10%, 5%	24.	$750.00	25%, 20%

Find the rates of discount in the following to the nearest per cent:

	List price	Selling price		List price	Selling price
25.	$45.00	$35.00	31.	$40.00	$28.00
26.	$2.40	$1.80	32.	$50.00	$32.00
27.	$6.50	$4.50	33.	$125.00	$85.00
28.	$8.25	$6.00	34.	$12.50	$8.75
29.	$5.00	$2.40	35.	$62.50	$37.50
30.	$0.75	$0.40	36.	$90.00	$55.00

149. **Why Taxes are Collected.** Money derived from taxes is needed for many purposes. The teachers in your school, county officials, such as judges and other officers of the courts, state officials, such as the Governor and members of the legislature, are all paid with money obtained from taxes. If you live in a city, you have a police department and a fire department. There are schoolhouses, and maybe a courthouse. The streets are paved and sewers built. All these things are paid for with money obtained from some kind of taxation or with borrowed money, which is to be paid back from taxes.

150. **Kinds of Taxes.** There are many ways of levying taxes. One of the most common ways is to make the owners of houses and lands (called *real estate*) pay a certain per cent of their value each year.

151. **Tax Rate, Valuation.** The rate per cent of the value of property which is paid as taxes is called the *tax rate.*

WRITTEN EXERCISES

1. If the tax rate on real estate is $\frac{3}{4}$ of one per cent, what is the tax on a house and lot valued at \$12,000? (*Suggestion:* $\frac{3}{4}\% = .0075$.)
2. If the tax rate is $\frac{1}{2}$ of one per cent, what is the tax on a farm containing 140 acres valued at \$65 per acre, and buildings valued at \$3400?
3. If the tax rate is $1\frac{1}{2}\%$, what is the tax on a city property valued at \$45,000? ($1\frac{1}{2}\% = .012$.)
4. If the tax rate is $\frac{7}{8}$ of one per cent, what is the tax on a lot 50 by 120, valued at 45¢ a square foot, and a house valued at \$8500? ($\frac{7}{8}\% = .00875$.)
5. If the tax rate is $\frac{4}{5}$ of one per cent, what is the tax on vacant city lots 150 by 140 feet, valued at 25¢ a square foot?

Sometimes the tax rate is not conveniently expressed in decimals, Thus, a tax rate of .00894 on the dollar means that the number of dollars' valuation of the property is multiplied by .00894 to obtain the amount of the tax.

In each of the following find the result correct to the nearest cent:

1. If the tax rate is .00927 on the dollar, what is the tax on property valued at $4750?

2. If the tax rate is .0103 on the dollar, what is the tax on a house and lot valued at $16,000?

3. A man owns two lots, each 25 feet by 120 feet, valued at 55¢ a square foot. On one of them stands a building valued at $7000. What is the tax on this property, if the rate is .00734 on the dollar?

4. A farm contains 350 acres, valued at $85 an acre. The buildings are valued at $6400. What is the tax on this property if the rate is .00825 on the dollar?

Personal property is any property not real estate. The tax rate on personal property is different from that on real estate.

5. A man has personal property valued at $12,400, and real estate valued at $10,000. What is his tax if the rate on personal property is .0054 on the dollar, and on real estate .0075 on the dollar?

6. Find the tax of a man whose personal property is valued at $8600 and real estate at $14,000, if the rate on personal property is .00925 on the dollar, and on real estate .0125 on the dollar.

7. What is the tax of a man who pays a personal tax (poll tax) of $3.00 and a rate of .00945 on property valued at $24,800?

8. Find the tax rate in your own town, and the valuation of some building, and then find how much tax is paid on it.

152. **Fire Insurance Policy.** A contract in which a company agrees to pay a certain amount in case a property is destroyed or damaged by fire is called *a fire insurance policy.*

153. **Premium Rate.** The amount paid by the insured to the company for insuring his property is called the *premium.* The premium is usually given as a certain rate per cent of the amount insured. This rate is called the *insurance rate.*

The insurance rate depends on a large number of conditions. The rate is higher on a wooden house than on a brick or stone house. It is higher where the houses are crowded close together, or where the protection against fire is poor.

The rate is lower when the insurance is taken out for a long time, say three years, than it is for a shorter period.

Problem. Find the annual premium on a fire insurance policy for $3500 if the rate is .6%.

$$\begin{array}{r} \$3500 \\ \underline{.006} \\ 21.000 \end{array}$$

Multiplying $3500 by .006, we get $21, which is the yearly premium.

WRITTEN EXERCISES

Find the annual premiums on the following:

	Amount of insurance	Rate		Amount of insurance	Rate
1.	$5000	.5%	3.	1500	.7%
2.	$2000	.6%	4.	$32,000	.3%

5. A man carried $8000 insurance on his home and $5000 on personal property. What was his yearly premium, if the rate on the house was .4% and on the personal property .5%?

6. The insurance rate on a factory building was decreased from .45% to .275% by the installation of automatic sprinklers. How much was saved in yearly premium if the factory was insured for $225,000?

154. **Long and Short Term Rates.** The standard period for which fire insurance rates are quoted is one year. Three-year policies are usually issued for a premium equal to $2\frac{1}{2}$ times the yearly premium, and 5-year policies for a premium equal to 4 times the yearly premium. For terms less than one year higher rates are charged.

1. Find the three-year premium on a policy for $4000, its yearly rate being .5%, if the three-year premium equals $2\frac{1}{2}$ times the yearly premium.

2. Find the five-year premium on a policy for $8500, the yearly premium being .45%, if the five-year premium equals 4 times the yearly premium. How much is saved by taking out 1 five-year policy instead of 5 one-year policies?

3. Household goods are placed in storage, and insured at $2500 for 90 days. What is the premium if the yearly rate is .56%, and the rate for 90 days is 40% of the yearly premium?

4. A building worth $16,000 is insured for 80% of its value. What is the yearly premium if the rate is .375%?

5. Furniture is insured for $3400, the annual rate being .6%. What is the premium for 6 months if it is 70% of the annual rate?

6. Find the premium on a five-year policy for $25,000, the annual rate being .35%, if the premium for the five years is 4 times the annual premium?

7. A building is insured for $12,000. What is the total of premiums for 20 years if five-year policies are used, the annual rate being .45%, the premium for 5 years being 4 times the annual premium.

8. Find the insurance rates on different kinds of property in your locality, and then find how much is paid in premium on certain buildings.

9. Is your schoolhouse insured? For how much? What is the premium?

155. **Life Insurance Policy.** A life insurance policy is a contract whereby a company agrees to pay a certain sum at a certain time, or on the death of the person insured. There are several kinds of personal insurance, such as accident insurance, health insurance, and so on. Only regular life insurance is considered in this book.

156. **Kinds of Life Insurance.** There is a large variety of life insurance policies, such as *straight life, limited payment life,* and *endowment policies.*

157. **Annual Premiums.** The premium paid per thousand dollars of life insurance varies with the age of the person insured, and the kind of policy.

158. **Straight Life.** Thus, in a certain company a healthy person 21 years of age may insure his life in straight life policy for a yearly premium of $19.62. This means that the insured is to pay $19.62 to the company every year as long as he lives. In consideration of this, the company agrees to pay $1000 to the heirs of the insured at the time of his death, whether his death occurs immediately after the policy is taken out or 60 years later.

159. **Twenty Year Endowment.** For a yearly premium of $48.63 a person 21 years of age may obtain a $1000 policy by which the company agrees to pay $1000 at the expiration of 20 years or earlier in case the insured dies.

160. **Twenty Payment Life.** In the same company a person 21 years of age may get a life policy for $1000 on the payment of $29.84 for 20 years. On this policy, the insurance company will pay $1000 on the death of the insured, but his payments cease at the end of 20 years.

161. **Table of Rates.** The table on the next page shows how the premium increases with the age of the insured, at the time the policy is taken out.

Age	Straight life	20-payment life	20 years endowment
21	$19.62	$29.84	$48.63
25	21.49	31.83	49.33
30	24.38	34.76	50.43
35	28.11	38.34	51.94
40	33.01	42.79	54.06
45	39.55	48.52	57.34
50	48.48	56.17	62.25
55	60.72	66.69	70.81
60	77.69	81.60	83.82

Use this table in solving the following problems:

1. At 30 years of age, what is the yearly premium on a straight life policy for $2500?

2. At 25 years of age, what is the yearly premium on a $2000 twenty-payment life policy?

3. At 35 years of age, what is the yearly premium on a $5000 twenty-year endowment policy?

4. A man takes out a $5000 straight life policy at the age of 35. If he dies after making 17 payments how much more will he get from the company than he paid to it?

5. At 40 years of age a man takes out a $4000 twenty-year endowment. If he lives twenty years and received the $4000 from the company, will he receive more or less than the sum of the premiums paid in?

6. At 45 years of age a man takes out a straight life policy for $10,000. If he dies after making 28 payments, by how much will the premiums paid exceed the amount paid at the time of his death, not counting interest on the premiums.

7. Make and solve other problems on life insurance, using the above table.

162. Promissory Note. When a man borrows money from a bank, he makes out a note, payable to the bank. This note is a promise to pay so much money to the bank on a certain day. The form of such a note is as follows:

\$200.00	New York, Dec. 15, 1916.
March 3, 1917.	I PROMISE
TO PAY TO THE ORDER OF............Myself.................	
TWO HUNDRED..............................DOLLARS (\$200.00)	
AT THE CORN EXCHANGE BANK, UNIVERSITY BRANCH,	
BROADWAY AND 113TH STREET, NEW YORK CITY, N. Y.	
VALUE RECEIVED	
No. 1034 DUE March 3, 1917.	*James B. Noble,* 542 West 124th St. (Residence or place of business.)

The amount written in the note, viz., \$200.00, is called the *face* of the note. James B. Noble is called the *maker* of the note.

The bank computes interest on this note from Dec. 15, 1916, to March 3, 1917, at some agreed rate of interest, say 6% per annum, deducts this from the face of the note, and credits James B. Noble with the difference. In this case the time is 78 days, and the interest amounts to $200 \times .06 \times \frac{13}{60} = 2.60$ dollars.

163. Bank Discount, Proceeds. The remainder, when the \$2.60 is deducted from the face of the note, is called the *proceeds* of the note. The interest deducted is called the *bank discount.*

Find the bank discount and the proceeds of each of the following:

	Face of note	Rate of discount	Time
1.	\$500	6%	90 days
2.	\$450	6%	75 days
3.	\$1280	$5\frac{1}{2}$%	80 days
4.	\$2740	$5\frac{1}{2}$%	103 days
5.	\$7860	5%	45 days

164. Discounting Bills. If you have debt due two months from now, the man to whom you owe it may need the money, and hence may be willing to take a little less than the full amount, if you pay it at once. The amount by which the bill is reduced on account of immediate payment is called *discount.*

In discounting a bill in this manner the discount is stated as a certain per cent of the amount or *face* of the bill, and is not computed as interest for a certain time, as in the case of bank discount.

Find the proceeds in each of the following:

	Amount or face of bill	Rate of discount		Amount or face of bill	Rate of discount
1.	$260.00	2%	4.	$48.20	3%
2.	$356.70	3%	5.	$86,140	$2\frac{1}{4}$%
3.	$160.25	$2\frac{1}{2}$%	6.	$4370	$2\frac{3}{4}$%

Problem. A debt of $8500 is settled for $8000 on account of immediate payment. What was the rate of discount?

```
       0588 = 5.88%
8500 )5.0000
       425
        750
        680
         700
```

500 is how many per cent of 8500? The result to the nearest tenth of 1% is 5.9%.

Find the rate of discount in the following:

	Face of bill	Proceeds		Face of bill	Proceeds
7.	$4000	$3800	12.	$560	$530
8.	$12,000	$11,600	13.	$2800	$2700
9.	$840	$800	14.	$4300	$4100
10.	$375	$350	15.	$5760	$5400
11.	$1800	$1700	16.	$3940	$3700

165. **Standard Time.** If the difference in the longitude of two places is 15, the time of day differs by one hour. Thus, noon comes about an hour earlier in Philadelphia than in St. Louis, and one hour earlier in St. Louis than in Denver. For convenience the United States has been divided into four regions, in each of which the same time is used throughout the region. Thus Boston, New York, and Philadelphia all have the same time. Similarly, Cleveland, Chicago, and St. Paul have the same time.

1. When it is 6 P. M. in Chicago, what time is it in Boston? In Denver? In San Francisco?

2. When it is noon at your home, what time is it in Philadelphia? In Los Angeles?

3. A newspaper in San Francisco prints a story at 1 P. M. of an event which occurrred in New York at 3 P. M. Explain.

4. From what part of the country do election returns come in first? Why?

ORAL WORK

1. Bring any book or magazine to class in which there are graphs. Get the teacher to help you find the meaning of such graphs.
2. What is meant by discounting a bill? If you know the full amount of a bill and the rate of discount, how do you find the proceeds?
3. What is meant by the *proceeds*?
4. If you know the face of a bill and the proceeds, how do you find the rate of discount?
5. What is a promissory note? What is the face of a note? Who is the maker of a note?
6. What is meant by bank discount? Give an example to show how a bank discounts a note.
7. What is a life insurance policy? Describe several different kinds of life insurance policies?
8. What is a fire insurance policy? Describe several different kinds of fire insurance policies.
9. What is meant by premium? Premium rates? Give examples.
10. What are taxes? Give several reasons why taxes are collected? Describe two kinds of taxes.
11. What is meant by tax rate? Give two different forms of expressing tax rates.
12. Make and solve a problem on taxation. If possible, get the material from your neighborhood.
13. Do New York and San Francisco have noon at the same time? If not, why not?
14. Which has noon earlier, and how much, Boston or Seattle?
15. Which have noon earlier, people in your home, or people living in St. Louis? In Denver?

Lineal Measure

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
$5\frac{1}{2}$ yards	= 1 rod (rd.)
$16\frac{1}{2}$ feet	= 1 rod
40 rods	= 1 furlong (fur.)
8 furlongs	= 1 statute mile
5280 feet	= 1 statute mile
6080 feet	= 1 nautical mile (knot)

Liquid Measure

4 gills (gi.)	= 1 pint (pt.)
2 pints	= 1 quart (qt.)
4 quarts	= 1 gallon (gal.)
$31\frac{1}{2}$ gallons	= 1 barrel (bbl.)
2 barrels	= 1 hogshead (hd.)
1 gallon	= 231 cubic inches

1 gallon of water or petroleum weighs 8.355 pounds.
1 gal. of milk weighs 8.62 lb.

Square Measure

144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards	= 1 square rod (sq. rd.)
160 square rods	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.) or one section (sec.)
36 square miles	= 1 township (T.)

Cubic Measure

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)
128 cubic feet	= 1 cord (cd.)

1 cubic foot of water weighs 62.5 pounds
1 cubic foot of ice weighs 57.5 pounds

Dry Measure

2 pints (pt.)	= 1 quart (qt.)
8 quarts	= 1 peck (pk.)
4 pecks	= 1 bushel (bu.)
11 pecks	= 1 barrel of apples
1 bushel	= 2150.42 cubic inches

1 English bushel = 2,218.92 cubic inches
1 English bushel of water weighs 80 pounds
1 bushel of wheat weighs 60 pounds
1 bushel of corn weighs 56 pounds
1 bushel of potatoes weighs 60 pounds

Time

60 seconds (sec.)	= 1 minute (min.)	365 days	= 1 year (yr.)
60 minutes	= 1 hour (hr.)	12 units	= 1 dozen (doz.)
24 hours	= 1 day (da.)	12 dozen	= 1 gross (gr.)
7 days	= 1 week (wk.)	12 gross	= 1 great gross

Avoirdupois Weight

$437\frac{1}{2}$ grains (gr)	= 1 ounce (oz.)
16 ounces	= 1 pound (lb.)
25 pounds	= 1 quarter (qut.)
4 quarters	= 1 hundredweight (cwt.)
20 hundredweights	= 1 ton (T.)
2000 pounds	= 1 ton (short)
2240 pounds	= 1 ton (long)

Money

10 mills	= 1 cent	25 cents	= 1 quarter
10 cents	= 1 dime	50 cents	= 1 half-dollar
10 dimes	= 1 dollar	5 dollars	= 1 half-eagle
10 dollars	= 1 eagle	20 dollars	= 1 double eagle

There are other tables, which are of little general use, and which are not given here. The so-called Apothecaries' table of weights is used in weighing drugs, and the *Troy* table of weights is used to some extent in weighing precious metals.

Both the Troy and the Apothecaries' ounce contains 480 grains, and the pound 12 ounces.

The Avoirdupois pound contains 7000 grains, and the Troy and Apothecaries' pound contains 5750 grains.

The same table of time measure is used nearly the world over. In England the measures of length and weights are practically the same as those used in the United States. The English ton is 2240 pounds, the same as our long ton. In the United States the long ton is used for weighing coal at the mines, for shipping purposes, and in the United States Customs House.

The meter, which is about 39.37 inches, is used in France, Germany, Russia and many other countries.

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